IV. *Of the* Moon's *Diſtance and Parallax: A Letter to* Andrew Reid, *Eſq; from* P. Murdoch, *D. D. and F. R. S.* 12 Nov. 1763.

S I R,

Read Jan. 26, 1764.

I Have at your deſire wrote out what I was mentioning to you in our laſt converſation; of an eaſy rule for determining the Moon's diſtance, from the received theory of central forces: which I wiſh may merit your approbation: it will at leaſt ſerve as a teſtimony of the eſteem and regard with which I am, &c.

I.

Sir Iſaac Newton inveſtigated the law of gravitation, in the duplicate ratio of the diſtance of the central body inverſely, from the following *data*.

1. The length of a ſimple pendulum which vibrates in one *ſecond* of time, gave him, by Huygens' theorem, a determinate meaſure of the force of gravity, at the place of obſervation. And, by his own theory, he could thence infer the like meaſure for any other place, of a given latitude *.

2. The Earth's ſemidiameter was computed from the Abbé Picard's meaſure of a degree of the terreſtrial meridian.

* See one of the Eſſays prefixed to *Buſching's* Geography.

3. The

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3. The Moon's parallax, as determined by the moſt ſkilful aſtronomers, gave him the Moon's diſtance in ſemidiameters of the Earth.

4. The time of a periodical month gave him the ratio of the verſed ſine of the arc of the Moon's orbit which ſhe deſcribes in one *ſecond*, to the radius.

And from theſe his concluſion was; that the gravitation at the Earth's ſurface, being diminiſhed as the ſquare of the diſtance from the Earth's centre increaſes, would, at the diſtance of the Moon, produce a fall from reſt, in one *ſecond*, preciſely equal to that verſed ſine. Or, that the gravitation of the Moon toward the Earth, being increaſed as the ſquare of that diſtance is diminiſhed, would, at the Earth's ſurface, be of the ſame quantity as that of falling bodies is (by the experiment of the pendulum) actually found to be.

II.

But the law of gravitation, thus deduced, being found to hold univerſally, and reciprocally, amongſt all the great bodies of our ſyſtem, ſo that even the minute anomalies of their motions are explained from it; we may now aſſume *it* as given, and make the *Moon's diſtance* the *quantity ſought*.

Thus, writing F for the number of feet which a body falling from reſt, deſcribes, *in vacuo*, at the equator, in one *ſecond*, V for the verſed ſine of the arc of the moon's orbit deſcribed in the ſame time, to the radius unity, D for the ſemidiameter of the equator in feet, and the ratio of the diſtance of the centers of the earth and Moon, to the ſemidiameter of

of the earth, that of X to 1: We ſhall have, by the general law, the Moon's fall in 1″, equal to $\frac{F}{X^2}$; but the ſame fall is equal to $V \times D \times X$; whence $X^3 = \frac{F}{V \times D}$, and $X = \sqrt[3]{\frac{F}{V \times D}}$ is the diſtance ſought, in ſemidiameters of the equator.

Now a ſimple pendulum which beats ſeconds, meaſuring, at London, 39.126 inches; if the uſual allowance is made for the weight of the air, and for the *Newtonian* figure of the Earth *, the weight $(\frac{1}{289})$ taken off by the centrifugal force being likewiſe reſtored, a *ſecond-pendulum* at the equator would be 39.154 inches long. And, by Huygens' rule, half this length is to the initial fall in one *ſecond*, in the duplicate ratio of the diameter of a circle to it's circumference: that fall therefore, at the equator, and *in vacuo*, is 16.10185 feet; the logarithm of which number is 1.2068645 = log. F.

The *toiſes* in a degree of the equator, or, which is the ſame, in a degree of the meridian at *lat.* $54\frac{3}{4}$, being nearly 57200, the logarithm of the number of feet Engliſh in the ſemidiameter of the equator,

that is log. D will be nearly — —	7.3211900,
And the log. verſed ſine of the Moon's arc in 1″, being — — —	—12.5492882,
Their Sum — — — —	—5.8704782

taken from log. F, leaves + 5.3363863, a third of which is 1.7787954, the logarithm of X = 60.08906 ſemidiameters of the equator.

* See the Eſſay quoted above.

And

And the arithmetical complement of this laſt logarithm, which is —2.2212046, is the log. tangent of the Moon's mean horizontal parallax at the equator; which therefore, is 57′ 12″, 34.

III.

Such would be the diſtance of the Earth's and Moon's centers, were the Earth immoveable: but it is ſomewhat increaſed by their revolution round their common centre of gravity.

Writing $x+1$ for that diſtance, divided by the centre of gravity in the ratio of x to 1; imagine a ſphere of the ſame dimenſions as our earth, placed at that centre, to exert the ſame attractive force on the Moon as our Earth actually does, the periodic time remaining unaltered: then muſt the denſity of this ſphere be diminiſhed in the ratio of x^2 to $\overline{x+1}|^2$, that its nearer diſtance from the Moon may be compenſated by the defect of denſity and attractive force. If now an inhabitant of the fictitious earth were ſuppoſed to compute its diſtance from the Moon, in the manner juſt now ſhewn; the quantities V and D would be the ſame as in the former calculation; but his f would be to our F, as x^2 to $x+1^2$, and thence, his x would be to our X as $x^{\frac{2}{3}}$ to $\overline{x+1}^{\frac{2}{3}}$, that is, $\mathrm{x} = \overline{\frac{x}{x+1}}|^{\frac{2}{3}} \times \mathrm{X}$.

This is the diſtance from the fictitious Earth, or from the common centre of gravity; but (T) the diſtance from our Earth, is $\frac{x+1}{x} \times \overline{\frac{x}{x+1}}|^{\frac{2}{3}} \times \mathrm{X}$, greater,

 as

as was ſuppoſed, in the ratio of $x+1$ to x; that is,

$$T = \overline{\frac{x}{x+1}}\Big|^{-1} \times \overline{\frac{x}{x+1}}\Big|^{-\frac{2}{3}} \times X = \overline{\frac{x}{x+1}}\Big|^{-\frac{1}{3}} \times X = \sqrt[3]{\frac{x+1}{x}} \times X.$$

Sir Iſaac Newton, from the *phænomena* of the tides, eſtimated the ratio of $x+1$ to x to be that of 40.788 to 39.788. In that caſe, the cubic root of $\frac{x+1}{x}$ will have for its logarithm 0.0035934; which added to 1.7787954, the logarithm of X computed for an immoveable earth, gives 1.7823888, the logarithm of 60.5883 ſemidiameters of the equator. And the Moon's horizontal parallax, for this diſtance, is 56′ 44″,07.

IV.

On the other hand, if we had obſervations of the Moon's parallax (and diſtance) which could be reckoned exact enough for the purpoſe, we might thence determine the ratio of x to 1, that is, the ratio of the quantities of matter in the Earth and Moon.

For having $\frac{T}{X} = \sqrt[3]{\frac{x+1}{x}}$, and $\frac{T^3}{X^3} = \frac{x+1}{x}$; likewiſe T being given from obſervation, and X computed as above; it is manifeſt that the ratio of $x+1$ to x, and, by diviſion, that of x to 1, or of the maſs of the earth to that of the Moon, is given.

For example, if it ſhould be concluded from good obſervations, that T, the Moon's mean diſtance, is $60\frac{1}{2}$ ſemidiameters of the equator; for the logarithm of this diſtance, which is 1.7817554, take the logarithm of X, or 1.7787954, thrice the remainder

 will

will be 0.00888, the logarithm of $\frac{x+1}{x} = 1.02066$; and the maſſes of the Earth and Moon would, on this ſuppoſition, be as 48.4027 to 1.

In all this, a ſmall variation from the law of attraction, ariſing from the ſpheroid-figure of the earth, is neglected as inconſiderable; which it will be found to be by whoever takes the trouble to compute its quantity and effects.

R E M A R K S.

1. If F and D were taken of their juſt quantities, the Moon's horizontal parallax for an immoveable Earth being, at the equator, $57' \ 12\frac{1}{3}''$, is a *limit* which the true mean parallax cannot exceed: and the correſpondent diſtance 60.08906 is a *limit* which the diſtance cannot fall ſhort of: both being computed upon the ſuppoſition that $x+1=x$, or that the matter of the Moon is as nothing in compariſon of the Earth. Nor can the parallax and diſtance be ſuppoſed to lye very near theſe limits, without leaving too little attractive force in the Moon to raiſe the tides.

2. If the Moon's mean apparent ſemidiameter is $15' \ 38\frac{1}{4}''$, and the diſtance of the centers 60.5883 ſemidiameters of the equator, according to Sir Iſaac's eſtimate of the maſſes; the ſemidiameter of the Moon will be 0.275601 parts of the ſemidiameter of the equator, or .2763 of a mean ſemidiameter of the Earth. And the magnitudes of the Moon and Earth being as the cubes of their ſemidiameters, if the inverſe ratio of their magnitudes is joined to the

 direct

direct ratio of their maſſes (1 to 39.788) the ſum will be the ratio of their denſities, that of 1.19143 to 1, a little leſs than 6 to 5.

3. Suppoſing ſtill the ſame ſemidiameter of the orbit as before, the force of gravity will be to the Earth's attractive force on the Moon as 3670.94 to 1, and to the Moon's force on the Earth as 40.788 times that number, or 149730.4, to 1.

Again, the force of the Moon upon that ſurface of the ocean to which ſhe is vertical, being to her force on the Earth's centre, as the ſquare of 60.5883 to that of 59.5883; and the difference of theſe ſquares being to the latter as 1 to 29.54623, this difference of the forces will ſupport the weight of one 4423968th part of the water at the vertex. And, becauſe the Earth's ſemidiameter is ſmall in compariſon to the Moon's diſtance, the like differences of force will decreaſe from the ſurface to the centre, nearly in an arithmetical progreſſion, as the weight of the water does; making the caſe analogous to the diminution of gravity by centrifugal force.

But it is likewiſe eaſily ſhewn, that half this quantity of lunar force exerts itſelf to depreſs the waters all around at the diſtance of 90 degrees from the vertex; $\frac{3}{2}$ therefore of the former fraction, that is one 2949312 part of the force of gravity, will be the total cauſe of the difference in heigth of the *flood* and *ebb*, in an open and boundleſs ocean.

Say therefore, if (in determining the figure of the earth) $\frac{1}{289}$ of gravity, ſuſpended by the centrifugal force, gave, for the difference of diameters $\frac{1}{231}$, what will one 2949312 part give? and the anſwer, in feet, will be 8.887.

4. In like manner, if we take $8''\frac{3}{4}$ for the Sun's parallax, and, thence, his diſtance from the Earth 23468,6 ſemidiameters of the equator, we ſhall find that his *whole* force to produce a difference of *Flood* and *Ebb*, is to his force at the Earth's centre, as 1 to $7823\frac{2}{3}$. But the Sun's diſtance being to the radius of the Moon's orbit as 387.34535 to unity, this laſt force will be to that of the Earth on the Moon, as 387,34535 to 178.7234 (by cor. 2. prop. princip. I.) And the Earth's force on the Moon is to gravity as 1 to the ſquare of 60 5883; whence, adding theſe ratios, the Sun's force to move the ſea will be to the force of gravity as the fraction whoſe logarithm is — 8.1778026 to 1; or gravity is to that force as 13249445 to unity. And therefore, by the ſame analogy as above, we find the difference of *Flow* and *Ebb*, from the Sun alone, to be 1.97824; one foot $11\frac{3}{4}$ inches.

The ſolar force therefore, in raiſing the tides, is to the lunar, as 1 to 4.4924, in a ratio ſomewhat leſs than that computed by Sir *Iſaac*. The ratio likewiſe of the ſum of the forces to their difference is but 7.869 to 5, inſtead of 9 to 5, which he aſſumes from comparing the *ſpring* and *neap*-tides at *Briſtol*. And it is indeed ſurprizing, how he could, from that *datum*, arrive at concluſions ſo near the truth, as his very probably are. He tells us he uſed the ratio of 9 to 5, only till a more certain could be procured, And therefore the foreign mathematicians, who have cenſured him on that head, and on ſome other articles of this doctrine, might have ſpared their reflexions; at leaſt till they could ſhew that their own deductions were more agreeable to nature and obſervation.

6. Unity

6. Unity reprefenting the force of gravity, *d* the Sun's diftance, the Earth's force on the Sun will be $\frac{1}{d^2}$, or the fraction whofe logarithm is — 9.259021.8. And the folar force on the Earth is (from the numbers in remark 4) to the force of gravity as 1 to 1673.1: whence the attractive forces (and maffes) of the Sun and Earth will be as 325172,3 to 1. Add to this the inverfe ratio of their magnitudes, collected from the Sun's mean apparent femidiameter 16′.6″, and the parallax 8″$\frac{4}{5}$; and the denfity of the Sun will be to that of the Earth, as 1 to 4.068.

All this upon the fuppofition that the maffes of the Earth and Moon are as 39.788 and 1. Hereafter, when the Moon's diftance fhall be more certainly known, that element may be corrected, and the operations repeated.

As to the Sun's parallax 8″$\frac{4}{5}$, it cannot be much affected by any future determination of the Moon's diftance. Nor is it here affumed of that quantity, at random; but from a theorem deduced from the eftablifhed principles. I am, however, too diffident of myfelf to communicate it at prefent: becaufe, altho' it agrees very well with Mr. *Short's* conclufion from the *Tranfit* of *Venus*, it differs confiderably from that which a very learned and juftly celebrated author hath lately publifhed.

Note, The periods, as affumed in this paper, are; a *fidereal year* of 365.2563923 days; the periodical month 27.32165835 days.

V. *An*

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THE END OF VOL. LIV.

ERRATA.

P. 33. l. 2. *read thus:* $T = \overline{\frac{x}{x+1}}\Big|^{-1} \times \overline{\frac{x}{x+1}}\Big|^{\frac{2}{3}} \times x$ &c,

14. *read* reckoned.

18. *for* $\frac{T^3}{\times 3}$ *read* $\frac{T^3}{x^3}$

36. 11. *from the bottom*, to 8 *put* to 5.

99. ult. *for* greatly deformed, *read* somewhat deformed.