

XV. *The Application of Dr. Saunderfon's Theorem for solving unlimited Equations, to a curious Question in CHRONOLOGY: By Mr. James Horsfall, F. R. S.*

Read March 24, 1768. **B**Y old tables it appears that Easter day happened on the 22d of March (which is the *soonest* it ever can happen), in the years of Christ 165, 697, 1229, and lastly in 1761.

Quest. 1. What is the next year of our Lord, when it will happen so again before 1900? For,

Note. From thence to 2199, the paschal full moon, or the golden number 14, which distinguished the years above, will be fixed on the 22d of March; consequently EASTER day cannot happen before the 23d of March in that period.

Answer. In the act for altering the stile, it appears by the table for finding Easter till 1899, that this can *never* happen in *that* period, but when the golden number, or lunar cycle, is 14, and the Sunday letter D.

Also, by making a *solar cycle* for that century, the first year of it will fall on 1812, the Sunday letters E D, wherefore all the years in that cycle, which have D for the Sunday letter, are 1, 7, 18, 24: and now the question is reduced to this.

Quest.

Quest. What year of our Lord in the 19th century will have the solar cycle either 1, 7, 18, or 24, *when* the lunar cycle is 14? Or, which is the same thing;

Quest. What number is there between 1800 and 1900, which divided by 28 leaves 1, 7, 18, or 24; and being divided by 19 leaves 14?

### SOLUTION.

Here then in the general theorem  $\frac{ar}{l} \times d - e + d$ ,  
 is  $a=28$ ,  $b=19$ ,  $D=1, 7, 18, \text{ or } 24$ ;  $E=14$ ,  $l=1$ .  
 To find  $r$ , the quotients are \* 1, 2, 9; \* 19)28(1  
 drop the first and last, because their  
 number is odd: then the series re- 9)19(2  
 quired will be 0, 1 2; therefore  $r=2$ .

N. B. If to any year of Christ be added 9, and the sum divided by 28; the remainder, or 28, if 0 remains, will be the *cycle of the sun* for that year; and if 1 be added to any year of Christ, and the sum divided by 19; the remainder, or 19, if 0 remains, is the *cycle of the moon*. Hence, if any year of Christ be severally divided by 28 and 19, and the remainders be  $d$  and  $e$  respectively; then  $d + 9$ , or  $d + 9 - 28 = D$ ; and  $e + 1$ , or  $e + 1 - 19 = E$ . In the present case, taking  $D=1$ ,  $d + 9 = 1$  cannot be; because  $d$  would be *negative*: but  $d$  must *not only be affirmative*, but also GREATER than  $e$ ; wherefore make  $d + 9 - 28 = D = 1$ ;  
 therefore

therefore  $d = 1 + 28 - 9 = 25$ ; and  $e + 1 = E = 14$ , therefore  $e = 13$ .

D and E are the *cyclar* numbers, and  $d$  and  $e$  are the *anno domini* numbers suited to the theorem.

Here then  $\overline{ar \times d - e} + d = \overline{28 \times 2 \times 7} + 20 = 412$ ; and  $412 + 532 + 532 + 532 = 2008$ . The first answer, therefore, A. D. 412, is too little, and when encreased by three *dionysian* periods, or *multiples* of 28 and 19 is too big, going beyond the century required. So, *when this solar cycle is 1, it will not do.*

Let  $D = 7$ , the rest as before. Then  $d + 9 - 28 = 7$ ; therefore  $d = 26$ . Here then  $\overline{ra \times d - e} + d = \overline{28 \times 2 \times 13} + 26 = 754$ ; and  $754 + 532 + 532 = 1818$ . So A. D. 1818 WILL ANSWER THE QUESTION.

Let  $D = 18$ , the rest as before. Then  $d + 9 - 28 = 11$ ; therefore  $d = 37$ . Here  $\overline{ra \times d - e} + d = \overline{28 \times 2 \times 24} + 37 = 1381$ ; and  $1381 + 532 = 1913$ . This goes beyond the century required; so *will not do.*

Let  $D = 24$ , the rest as before. Then  $d + 9 = 24$ ; therefore  $d = 15$ . Here  $\overline{ra \times d - e} + d = \overline{28 \times 2 \times 2} + 15 = 127$ ;  $127 + 532 \times 4 = 2255$ ; which goes *beyond* the century required.

So there is *but one year* in the 19th century, *viz.* 1818, that will have the conditions required. The cycle of the sun will then be 7; the cycle of the moon 14; and the Sunday letter D; and *Easter Day the 22d of March*

N. B. For

N. B. For *every century* a *new* solar cycle must be made; because, by the Act of Parliament\* for correcting the calendar, every 100th year for three centuries is *common*, and *not bissextile*; so that the same dominical letter stands against the same year, in the cycle only for 100 years in *three* successive centuries.

N. B. By a continual addition of 28 to 1700 or 1756†, we have the *first* year of each solar cycle; and when the *first* year of that cycle *next after the beginning* of any century is had, and its dominical letter found, by the rules and tables in the act, *the cycle for that century may be formed*, with the dominical letters answering to each year of it; whereby may be seen on *what* years of the cycle *the same Sunday letter recurs*. Thus;

Quest. 2. If it was required to find in what years between 2200 and 2300 Easter Day would again happen on 22d of March; I find by the hints above, that the *first* year of the solar cycle falls on 2204; and, being *leap-year*, I find by the rules and tables in the act, that the *dominical-letters* are A G: from thence I construct the solar cycle of 28 years, as in table I.

And from the table prefixed to the late Earl of Macclesfield's Letter to Martin Folkes, Esq; P. R. S. read May 10, 1750, and published in Phil. Transf. Vol. XLVI. p. 47. shewing the place of the golden

\* 24 George II.

† Vide Table I.

numbers in the calendar, and the paschal full moon, and the Sunday letter, answering thereto for that century (which stand as in table II). I construct table III, for finding Easter Day during *that* century; and observe it *never* happens on the 22d of March, but when the *golden number C*, and the *dominical letter D*.

And the dominical letter D happens only in the 4th, 9th, 15th, and 26th years of the solar cycle in that century, as appears by table I.

Now the question is reduced to this, *viz.*

What number is there between 2200 and 2300; which, being divided by 28, leaves either 4, 9, 15, or 26; and being also divided by 19, leaves 6?

## SOLUTION.

In the general theorem above, *viz.*  $\frac{ar}{l} \times d - e + d$  are given  $a = 28$ ,  $l = 1$ ,  $r = 2$ , as before; and to find the values of  $d$  and  $e$ ,

We have  $d + 9 - 28 = D = 4$ ; therefore  $d = 23$  } in the theorem :  
And because  $e + 1 = E = 6$ ; therefore  $e = 5$  }  
*viz.*  $28 \times 2 \times 18 + 23 = 1031$ ; and  $1031 + 532 + 532 = 2095$ : so that this cyclar number will *not* do, the year falling either below or beyond the century required.

2. Let  $D = 9$ ; the rest as before. Then since  $d + 9 - 28 = 9$ ; therefore  $d = 28$ , and  $e = 5$  as before; and  $28 \times 2 \times 23 + 28 = 1316$ ; and  $1316 + 532 + 532 = 2380$ . This cyclar number will *not* do, for the same reason as the last.

3. Let

3. Let  $D = 15$ ; the rest as before. Then since  $d + 9 = D = 15$ ; therefore  $d = 6$ , and  $e = 5$ , as before; and  $28 \times 2 \times 1 + 6 = 62$ ; and  $62 + 532 \times 4 = 2190$ . This cyclar number will *not* do, for the same reason as before.

4. Let  $D = 26$ ; the rest as before. Then  $d + 9 = 26$ ; therefore  $d = 17$ , and  $e = 5$ , as before; and  $28 \times 2 \times 12 + 17 = 682$ ; and  $689 + 532 \times 3 = 2285$ ; and this is the *only* year that will answer the question; because it has 6 for its golden number, and D for its dominical letter. Whence we may conclude, that after A. D. 1761, there will not be so long a Trinity-vacation again till 1818; and after that year, the like will not happen till 2285.

TABLE I.  
Solar Cycle,  
&c. for the  
23d Century.

1756	112 = 4 × 28
1868	112
1980	112
2092	112
2204	112
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32

TABLE III.  
For finding Easter from 2200 to 2299, by the Golden  
Number and Dominical Letters.

G. No	A	B	C	D	E	F	G
I Apr. 16	17	18	19	20	21	22	
II Apr. 9	10	11	5	6	7	8	
III Mar. 26	27	28	29	30	31	25	
IV Apr. 16	17	18	19	13	14	15	
V Apr. 2	3	4	5	6	7	8	
VI Mar. 26	27	28	22	23	24	25	
VII Apr. 16	10	11	12	13	14	15	
VIII Apr. 2	3	4	5	Mar. 30	31	Apr. 1	
IX Apr. 23	24	18	19	20	21	22	
X Apr. 9	10	11	12	13	7	8	
XI Apr. 2	Mar. 27	28	29	30	31	Apr. 1	
XII Apr. 16	17	18	19	20	21	15	
XIII Apr. 9	10	4	5	6	7	8	
XIV Mar. 26	27	28	29	30	24	25	
XV Apr. 16	17	18	12	13	14	15	
XVI Apr. 2	3	4	5	6	7	1	
XVII Apr. 23	24	25	19	20	21	22	
XVIII Apr. 9	10	11	12	13	14	15	
XIX Apr. 2	3	4	Mar. 29	30	31	Apr. 1	

TABLE II.  
Lunar Cycle,  
&c. for the  
23d Century.

Gold. Numbers.	D. of the Month for Paschal Full Moons.	Domin. Lett.
6	Mar. 21	C
22	22	D
14	23	E
3	24	F
11	25	G
19	26	A
8	27	B
5	28	C
13	29	D
2	30	E
10	31	F
18	1	G
7	2	A
15	3	B
4	4	C
12	5	D
1	6	E
9	7	F
17	8	G
	9	A
	10	B
	11	C
	12	D
	13	E
	14	F
	15	G
	16	A
	17	B
	18	C
	19	D
	20	E
	21	F
	22	G
	23	A
	24	B
	25	C