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XV. *Remarks on the Hadley's Quadrant, tending principally to remove the Difficulties which have hitherto attended the Use of the Back-observation, and to obviate the Errors that might arise from a Want of Parallelism in the two Surfaces of the Index-Glass. By Nevil Maskelyne, F. R. S. Astronomer Royal.*

Read May 28, 1772. **T**HE back observation with Hadley's quadrant being founded on the same principles, and in theory, equally perfect with the fore-observation, and being at the same time necessary to extend the use of the instrument up to 180 degrees (it being impracticable to measure angles with any convenience beyond 120 degrees with the fore-observation) it may seem surprizing that it hath not been brought equally into general use, more especially since the method of finding the longitude by observations of the Moon, has been practised at sea for some years past; since this method would receive considerable advantage from the use of the back-observation in taking distances of the Sun and Moon between the first and last quarter, could such

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observations be as much depended upon as the fore observations. The causes of this seem to have been principally these two, the difficulty of adjusting the back horizon glass, and the want of a method of directing the sight parallel to the plane of the quadrant. The back horizon glass, like the fore-one, requires two adjustments; the first, or common one, disposes it at right angles to the index glass, when the index stands at (o) upon the arch; which is usually performed by setting (o) of the index of the arch of the quadrant by double the dip of the horizon of the sea, and then holding the quadrant vertical with the arch downwards, and turning the back-horizon glass about, by means of its lever or perpetual screw, till the reflected back horizon appears to coincide with the fore-horizon seen directly. But this operation is so difficult in practice with the back-horizon glass wholly silvered, except a small transparent slit in the middle, as it has been usually made, that few (if any) persons have ever received proper satisfaction from it. If the back-horizon-glass was silvered in every respect like the fore-horizon-glass (which it ought to be) the upper part being left unsilvered, and a telescope was applied to it, perhaps this adjustment might be rendered somewhat easier and more exact; but it could not even thus be made so exact as the adjustment of the fore-horizon-glass may, by making use of the Sun's limbs.

The second adjustment of the back-horizon-glass, in the common construction of the quadrant, is still more troublesome, since it cannot be executed without setting the index 90 degrees off the arch, in order to place the index-glass parallel

allel to the back-horizon-glass; when this adjustment may be performed in the same manner as the corresponding adjustment of the fore-horizon-glass. But the bending of the index, that follows the setting it off the arch, is a very disagreeable circumstance, having a tendency, especially on board of ship, to expose both the index and centre work to damage; and may even, without extraordinary precautions taken by the instrument maker in placing the plane of the index-glass exactly according to the length of the index, disturb its perpendicularity to the plane of the quadrant: on these accounts it would be much better if this adjustment of the back-horizon-glass could be performed, like those of the fore-horizon-glass, with the index remaining upon the arch of the quadrant. Fortunately, this *desideratum* has been lately effected by an ingenious contrivance invented by Mr. Dollond, which he has given an account of in a letter addressed to me\*, which I have presented to this Society, by means of an additional index applied to the back-horizon-glass; whereby both the adjustments may be made by the same observations and with nearly the same exactness as those of the fore-horizon-glass: for a farther knowledge of which, see the account itself.

Besides the difficulty of adjusting the back-horizon-glass, the want of a method of directing the line of sight parallel to the plane of the quadrant has proved also a considerable obstacle to the use of the back-observation: this will easily appear from the following proposition, that the error of the angle measured arising from any small de-

\*. See the XIVth paper, which immediately precedes this.

violation of the visual ray from a parallelism to the plane of the quadrant, is to twice an arch equal to the verse-sine of the deviation; as the tangent of half the angle measured by the quadrant is to radius, very nearly. Thus a deviation of  $1^{\circ}$  in the line of sight, will produce an error of about  $1'$  in measuring an angle of  $90^{\circ}$ , whether by the fore or back observation; but the same deviation will produce an error of  $4'$  in measuring an angle of  $150^{\circ}$ , of  $6'$  in taking an angle of  $160^{\circ}$ , and  $12'$  in taking an angle of  $170^{\circ}$ . Hence a pretty exact adjustment of the line of sight, or axis of the telescope, is requisite in measuring large angles, such as those are taken by the back observation: and therefore a director of the sight ought by no means to be omitted in the construction of the instrument (as it commonly has been since Mr. Hadley's time, though recommended by him), except a telescope be made use of, which if rightly placed answers the same purpose better, especially in observing the distance of the Moon from the Sun between the first and last quarter. The director of the sight may be placed exact enough by construction; but the telescope cannot, and Mr. Hadley, not having been aware of the importance of an exact position of it, has accordingly given no directions for the placing it. I shall therefore endeavour to supply this defect in the following remarks.

In the first place, I would by all means recommend an adjusting piece to be applied to the telescope, whereby its axis may be brought parallel to the plane of the quadrant: in the next place, the back-horizon-glass ought to be silvered in the same manner as the fore-horizon-glass: and thirdly, two thick silver wires should be placed within the eye-tube

in the focus of the eye-glass parallel to one another and to the plane of the quadrant. If they were put at such a distance as to divide the diameter of the field of view into three equal parts, it might be as convenient as any other interval. In this manner wires were placed in the telescope by Mr. Hadley, as appears by his account of the instrument in *Philos. Trans.* N° 420. These wires are to be adjusted parallel to the plane of the quadrant, by turning the eye-tube round about which contains the wires, till they appear parallel to the plane of the quadrant. The axis of the telescope, by which is meant the line joining the centre of the object-glass and the middle point between the two wires, is to be adjusted parallel to the plane of the quadrant by either of the two following methods.

1st method. When the distance of the Moon from the Sun is greater than 90 degrees, by giving a sweep with the quadrant and moving the index, bring the nearest limbs to touch one another at the wire nearest the plane of the quadrant. Then, the index remaining unmoved, make the like observation at the wire farthest from the plane of the quadrant; and note whether the nearest limbs are in contact as they were at the other wire: if they are, the axis of the telescope is parallel to the plane of the quadrant: but if they are not, it is inclined to the same, and must be corrected as follows. If the nearest limbs of the Sun and Moon seem to lap over one another at the wire farthest from the plane of the quadrant, the object end of the telescope is inclined from the plane of the quadrant, and must be altered by the adjustment made for that purpose: but, if the nearest  
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limbs of the Sun and Moon do not come to touch one another at the wire farthest from the plane of the quadrant, the object end of the telescope is inclined towards the plane of the quadrant, and must be altered by the adjustment accordingly. Let these operations be repeated until the observation is the same at both the parallel wires, and the axis of the telescope will be adjusted parallel to the plane of the quadrant. In like manner, the axis of the telescope may be also adjusted parallel to the plane of the quadrant for the fore-observation.

Second method. Set the index to (o) and hold the plane of the quadrant parallel to the horizon of the sea, with the divided arch upwards, the two wires being parallel to, and including both the direct fore-horizon, and the reflected back-horizon, between them. Raise or lower the plane of the quadrant until the direct and reflected horizons coincide together: if the coincidence happens in the middle between the two wires, or rather, to be more exact, above the middle by such a part of the field of view as answers to the number of minutes in the depression of the horizon (which may be easily estimated if the angular interval of the wires be first found by experiment, in manner hereafter mentioned) the axis of the telescope is parallel to the plane of the quadrant; but if it does not, the line of sight is inclined to the plane of the quadrant, and must be corrected as follows. If the direct and reflected horizons, when they coincide, appear higher above the middle between the wires, than what the quantity of the depression of the horizon amounts to, the object end of the telescope is inclined from the plane of the quadrant, and must be altered by the adjustment made for that purpose;

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but if the two horizons appear to coincide in a lower part of the field of the telescope, the object end of the telescope is inclined towards the plane of the quadrant, and must be altered by the adjustment accordingly. Repeat these operations till the two horizons appear to coincide above the middle between the two wires, by the quantity of the depression of the horizon, and the axis of the telescope will be adjusted parallel to the plane of the quadrant. In order to find the angular interval between the wires, hold the quadrant perpendicular to the horizon, as in observing altitudes; and turn about the eye-tube with the wires until they are parallel to, and include, the direct fore-horizon and reflected back-horizon between them. Move the index from (o) along the divided arch, at the same time raising or lowering the telescope by the motion of the quadrant until the direct horizon appears to coincide with the upper wire, and the reflected back-horizon with the lower wire; the number of degrees and minutes shewn upon the arch, increased by double the depression of the horizon, will be the angular interval of the wires; its proportion to the depression of the horizon will be therefore known; and hence the space in the field of the telescope answering to the depression of the horizon, may be easily estimated near enough for adjusting the axis of the telescope in the manner before-mentioned. The first of the two methods here given for adjusting the position of the telescope will probably be found most convenient; and the greater the distance of the Sun and Moon is, the more nearly may the adjustment be made, because the same deviation of the axis of the telescope will cause a greater error.

The telescope should be fixed by the instrument-maker so as to command a full field of view when the instrument is placed at  $90^{\circ}$  if the instrument be an octant, or  $120^{\circ}$  if it be a sextant; because the index-glass then stands more oblique with respect to the incident and reflected rays, and consequently the field of view of the telescope, as far as it depends upon the index-glass, will be more contracted than in any other position of the index: but if there is a fair field of view in this case, there necessarily must be so in every other position of the index.

The two parallel wires will be very useful on many occasions, as well in the fore as the back-observation. In taking the altitude of the Sun, Moon, or star, direct the sight towards the part of the horizon underneath, or opposite to the object, according as you intend to observe by the fore or back observation, and hold the quadrant that the wires may constantly appear perpendicular to the horizon, and move the index till you see the object come down towards the horizon in the fore-observation, or up to it in the back-observation, and turn the instrument in order to bring the object between the wires; then move the index till the Sun or Moon's limb, or the star touch the horizon. The nearer the object is brought to an imaginary line in the middle between the wires (it is indifferent what part of the line it is brought to) and the truer the wires are kept perpendicular to the horizon, the more exact will the observation be. In the fore-observation, the object appears in its real position; but in the back-observation, the object being brought through the zenith to the horizon, the real upper-limb will appear the lowest;  
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and the contrary. Either limb of the Sun may be used in either observation; but it will be most convenient in general to make the Sun appear against the sky, and not against the sea; and then the objects appearing inverted through the telescope, the Sun will appear lowest, and the horizon highest. The observed altitude is to be corrected for dip, refraction, and Sun's semi-diameter, as usual.

In taking the distance of the nearest limbs of the Sun and Moon, whether by the fore or back-observation, having first set the index to the distance nearly, by the help of the Nautical Almanac, and brought the Moon to appear anywhere on or near the diameter of the field of view of the telescope, which bisects the interval between the wires, give a sweep with the quadrant, and the Sun and Moon will pass by one another; if in this motion the nearest limbs, at their nearest approach, just come to touch one another, without lapping over, on or near any part of the diameter of the field of the telescope which bisects the interval between the wires, the index is rightly set; but if the nearest limbs either do not come to meet, or lap over one another, alter the index, and repeat the observation till the nearest limbs come to touch one another properly. This method of observing will be found much more easy and expeditious than without the wires, since in that case it would be necessary to make the limbs touch very near the centre of the telescope, but here it is only necessary to make them do so anywhere on or near the diameter of the field of the telescope which bisects the interval between the two wires.

The same method may be used in taking the Moon's distance from a fixed star.

It may not be amiss here to make some remarks on the rules that have been usually given for observing the Sun's altitude, both with the fore and back-observation, which have all been defective, and to point out the proper directions to be followed, when a telescope is not used with two parallel wires to direct the quadrant perpendicular to the horizon, and to shew the principles on which these directions are founded.

Observers are commonly told, that in making the fore observation they should move the index to bring the Sun down to the part of the horizon directly beneath him, and turn the quadrant about upon the axis of vision; and when the Sun touches the horizon at the lowest part of the arch described by him the quadrant will shew the altitude above the visible horizon. I allow that this rule would be true, if a person could by sight certainly know the part of the horizon exactly beneath the Sun; but, as this is impossible, the precept is incomplete. Moreover, in taking the Sun's altitude in or near the zenith, this rule entirely fails, and the best observers advise to hold the quadrant vertical, and turn one's self about upon the heel, stopping when the Sun glides along the horizon without cutting it: and it is certain that this is a good rule in this case, and capable with care of answering the intended purpose. We have thus two rules for the same thing, which is a proof that neither of them is an universal one, or sufficient in all cases alone.

In taking the back-observation, observers have been advised either to turn the quadrant about upon the axis of vision, or, holding the quadrant upright, to turn themselves about upon the heel, indifferently. The true state of the case is this; that, in taking the Sun's altitude, whether by the fore or back-observation, these two methods must be combined together; that is to say, the observer must turn the quadrant about upon the axis of vision, and at the same time turn himself about upon his heel, so as to keep the Sun always in that part of the horizon-glass which is at the same distance as the eye from the plane of the quadrant: for, unless the caution of observing the objects in the proper part of the horizon-glass be attended to, it is evident the angles measured cannot be true ones. In this way the reflected Sun will describe an arch of a parallel circle round the true Sun, whose convex side will be downwards in the fore-observation, and upwards in the back-observation, and consequently, when, by moving the index, the lowest point of the arch in the fore-observation, or the uppermost point of the arch in the back-observation, is made to touch the horizon, the quadrant will stand in a vertical plane, and the altitude above the visible horizon will be properly observed.

The reason of these operations may be thus explained: the image of the Sun being always kept in the axis of vision, the index will always shew on the quadrant the distance between the Sun and any object seen directly which its image appears to touch; therefore, as long as the index remains unmoved, the image of the Sun will describe an arch everywhere equidistant from the Sun in the heavens, and consequently

quently a parallel circle about the Sun, as a pole ; such a translation of the Sun's image can only be produced by the quadrant being turned about upon a line drawn from the eye to the Sun, as an axis ; a motion of rotation upon this line may be resolved into two, one upon the axis of vision, and the other upon a line on the quadrant perpendicular to the axis of vision ; and consequently a proper combination of these two motions will keep the image of the Sun constantly in the axis of vision, and cause both jointly to run over a parallel circle about the Sun in the heavens ; but when the quadrant is vertical a line thereon perpendicular to the axis of vision becomes a vertical axis ; and, as a small motion of the quadrant is all that is wanted, it will never differ much in practice from a vertical axis ; therefore the observer, by properly combining and proportioning two motions, one of the quadrant upon the axis of vision, and the other of himself upon his heel, keeping himself upright (which gives the quadrant a motion upon a vertical axis) will cause the image of the Sun to describe a small arch of a parallel circle about the Sun in the heavens, without departing considerably from the axis of vision.

If it should be asked, why the observer should be directed to perform two motions rather than the single one equivalent to them on a line drawn from the eye to the Sun as an axis ? I answer, that we are not capable, while looking towards the horizon, of judging how to turn the quadrant about upon the elevated line going to the Sun as an axis, by any other means than by combining the two motions above-mentioned, so as to keep the Sun's image al-

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ways in the proper part of the horizon-glass. When the Sun is near the horizon, the line going from the eye to the Sun will not be far removed from the axis of vision; and consequently the principal motion of the quadrant will be performed on the axis of vision, and the part of the motion made on the vertical axis will be but small. On the contrary, when the Sun is near the zenith, the line going to the Sun is not far removed from a vertical line, and consequently the principal motion of the quadrant will be performed on a vertical axis, by the observer's turning himself about, and the part of the motion made on the axis of vision will be but small. In intermediate altitudes of the Sun, the motions of the quadrant on the axis of vision and on a vertical axis will be more equally divided. Hence appears the reason of the method used by the best observers in taking the Sun's altitude when near the zenith by holding the quadrant vertical and turning about upon the heel, and the defects of the rules that have been commonly given for observing altitudes in other cases.

As it may conduce to the setting this matter in a still clearer light, I shall here describe in order the several motions that will be given to the reflected image, by turning the quadrant about upon the axis of vision, a vertical axis, or the line drawn from the eye to the Sun, successively.

- I. If the quadrant is turned about upon the axis of vision, the same being directed to the point of the horizon exactly beneath or opposite the Sun, the image of the Sun will move from right to left, or from left to right, across the  
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the horizon-glass, the same way as the arch of the quadrant is carried, both in the fore and back-observations, with a velocity which is to the angular velocity of the quadrant as the sine of the Sun's altitude to the radius, describing an arch convex downwards in both cases; and when the motion of the Sun in this arch is parallel to the horizon, the quadrant is held truly perpendicular to the horizon, and consequently in a proper position for taking the Sun's altitude. But, if the axis of vision be directed to, and turned round a point in the horizon beside the vertical circle passing through the Sun, the Sun's image, when its motion is parallel to the horizon, will be neither in the axis of vision nor the Sun's vertical, but between both; at the same time, the plane of the quadrant will not be vertical, and the altitude found by bringing the Sun's image to touch the horizon will not be the true altitude.

- II. If the quadrant be held perpendicular to the horizon, and turned about upon a vertical axis, or one nearly so, the Sun will describe an arch convex downwards in the fore-observation, and upwards in the back-observation, the motion of the Sun being the same way as the axis of vision is carried in both cases, and being to the angular motion of the quadrant, as the verse-sine of the Sun's altitude to the radius in the fore-observation, but as the verse-sine of the supplement of the Sun's altitude to  $180^\circ$  to the radius in the back-observation. The Sun therefore will move slower than the axis of vision in the fore-observation, and consequently will be left behind, with

with respect to the axis of vision, or seem to move backwards; and the Sun will move quicker than the axis of vision in the back-observation, or will seem to get before it. When the motion of the Sun in this arch is parallel to the horizon, the plane of the quadrant coincides with the vertical circle passing through the Sun, and consequently the quadrant is in a proper position for taking the Sun's altitude. But if the quadrant be held a little deviating from the perpendicular position to the horizon, and turned about upon an axis, either vertical or only nearly so, the arch described by the Sun apparently will cut the horizon, but will never move parallel to it, and consequently the quadrant will not be brought into a proper position for observing the Sun's altitude.

III. If the quadrant be turned on the line going to the Sun as an axis, the reflected Sun will be kept constantly in the axis of vision, and will describe an arch of a parallel circle about the real Sun, with a velocity which is to the angular motion of the quadrant, as the sine of the Sun's altitude is to the radius; and when the motion of the reflected Sun is parallel to the horizon, the quadrant is vertical.

Hence naturally arise the three methods of taking an altitude, which have been mentioned before. In the first, the axis of vision is supposed always directed to one and the same part of the horizon, namely that which is in the Sun's vertical. In the second, the observer is required to hold the quadrant truly vertical, and to turn himself upon a vertical axis;

but it is evident neither of these motions can be accurately performed. In the third method, the observer is only required to move both himself and the quadrant, so as to keep the Sun always in or near the axis of vision, which may be performed very well, because the axis of vision is a visible and certain direction for it. One exception, however, should be made to this general rule, namely, in taking the Sun's altitude when very low, by the back observation; in which case it will be best to use the second method, or else to hold the quadrant perpendicular by judgment; which will be much facilitated by using a telescope containing wires in its focus parallel to the plane of the quadrant, as described in p. 106: for, in this case, the perpendicular position of the quadrant cannot be attained so near by the method of turning the quadrant on a line going to the Sun as an axis, as it can by the other method.

It remains to treat of the errors which may arise from a defect of parallelism in the two surfaces of the index-glass, and to point out the means of obviating them in the celestial observations. It is well known, that if a pencil of parallel rays falls upon a glass whose two surfaces are inclined to one another, and some of the rays are reflected at the fore-surface, and others passing into the glass and suffering a reflection at the back-surface and two refractions at the fore-surface emerge again from the glass, these latter rays will not be parallel to those reflected at the fore-surface, as they would have been if the surfaces of the glass had been parallel, but will be inclined to the same. I find that the angle of their mutual inclination, which may be called the deviation of the rays



rays reflected from the back-surface, will be to double the inclination of the surfaces of the glass (which is here supposed to be but small), as the tangent of the angle of incidence out of air into glass, is to the tangent of the angle of refraction. Hence, in rays falling near the perpendicular, the deviation will be about three times the inclination of the surfaces; and if the angles of incidence be  $50^\circ$ ,  $60^\circ$ ,  $70^\circ$ ,  $80^\circ$  or  $85^\circ$ , the deviations of the reflected rays will be about 4, 5, 7, 13 or 26 times the inclination of the surfaces, respectively. Had the deviation been the same at all incidences of the rays on the index-glass, no error would have been produced in the observation; because the course of the ray would have been equally affected in the adjustment of the instrument, as in the observation. But, from what has been just laid down, this is far from being the case, the deviation increasing according to the obliquity with which the rays fall upon the index-glass; so that in very oblique incidences of the rays, such as happen in measuring a large angle by the fore-observation or a small angle by the back-observation, the least defect in the parallelism of the planes of the two surfaces of the index-glass may produce a sensible error in the observation.

What is here said only takes place in the fullest extent, if the thickest or thinnest edge of the index-glass, or, to express the same thing in other words, the common section of the planes of the surfaces of the index-glass stands perpendicular to the plane of the quadrant; but, if the common section of the planes is inclined to the plane of the quadrant, the error arising from the defect of the parallelism of the

surfaces will be lessened in the proportion of the sine of the inclination to the radius; so that at last, when the common section becomes parallel to the plane of the quadrant, the error intirely vanishes. For this reason, Mr. Hadley very properly directed the thickest and thinnest edges of the index-glass to be placed parallel to the plane of the quadrant. But, as it may well be questioned whether this care is always taken by the instrument-maker, and it cannot be supposed that the glasses can be ground perfect parallel planes, it would certainly be an advantage acquired to the instrument, could the error arising from a want of parallelism of the planes be removed in whatever position the common section of the planes should be placed with respect to the plane of the quadrant. This will be effected for celestial observations, if the upper part of the index-glass be left unsilvered on the back, and made rough and blacked, the lower part of the glass being silvered as usual, which must be covered whenever any celestial observations are made. Then, if the telescope be sufficiently raised above the plane of the quadrant, it is evident that the observations will be made by the rays reflected from the fore-surface of the upper part of the index-glass, and consequently, if the quadrant be adjusted by making use of the same part of the index-glass, the observations will be true whether the two surfaces of the index-glass are parallel planes or not. The Sun or Moon may be thus observed by reflection from the unsilvered parts of the index-glass and horizon-glass, so that a paler darkening glass will suffice, and they will appear much distincter than from an index-glass wholly silvered with a deeper darkening

ing glass; for although the surfaces of a glass may be parallel, yet there always arises some little confusion from the double reflection. Neither will the Moon appear too weak by two unsilvered reflections, even when her crescent is very small, except she should be hazy or clouded; and then the light may be increased by lowering the telescope so as to take in part of the silvered reflection of the index-glass, which in this case must be uncovered: the same is also to be understood with respect to the Sun, should his light be too much weakened by haziness or thin clouds. The horizon-glasses should be adjusted, or the error of adjustment found by the Sun or Moon; the first will be in general the best object for the purpose; and, as the Sun or Moon seen directly through the unsilvered part of the horizon-glass will be much brighter than the image of the same seen by two unsilvered reflections, it must be weakened by a darkening glass placed beyond the horizon-glass, the reflected image being farther weakened, if necessary, by a paler darkening glass placed in the usual manner between the index-glass and the horizon-glass.

If a quadrant was designed principally for taking the distance of the Moon from the Sun and fixed stars, and was not wanted for observing terrestrial angles, it would be the best way to have none of the glasses silvered, but to leave the horizon glasses intirely transparent, and to put a red glass for an index-glass of the same matter with the darkening glasses, which would reflect light from the fore-surface only.

The Sun's altitude might also be observed with this instrument, either by the fore or back-observation; and

and the altitude of the Moon might be taken with it in the night. But the altitudes of stars could not be observed with it, nor the Moon's altitude in the day time, which would however be no great inconvenience, as these observations might be well enough supplied by common quadrants.

The following rules for the size of the glasses and the silvering them, and the height of the telescope may be of use. The index glass and two horizon-glasses should be all of equal height, and even with one another in height both at top and bottom. The telescope should be moveable parallel to itself nearer to or farther from the plane of the quadrant, and the range of its motion should be such that its axis when at the lowest station should point about  $\frac{1}{10}$ th of an inch lower than the top of the silvering of the horizon-glasses, and when at the highest station should point to the height of the middle of the unsilvered part of the index-glass. The height of the glasses, and the quantity of parts silvered and parts unsilvered, should vary according to the aperture of the object-glass, as in the following table; where the first column of figures shews the dimensions in parts of an inch answering to an aperture of the object-glass of  $\frac{5}{16}$ ths of an inch in diameter; the second column what answer to an aperture of the object-glass of  $\frac{4}{16}$ ths of an inch in diameter; and the third, what are suitable to an aperture of the object-glass of  $\frac{3}{16}$ ths of an inch in diameter.

Diameter

	Parts of an Inch		
Diameter of aperture of object-glass	,30	0,40	0,50
Height of glasses	,90	1,13	1,37
Height of silvered part of index-glass	,50	0,63	0,77
Height of unsilvered part of ditto	,40	0,50	0,60
Height of silvered part of horizon-glasses	,25	0,33	0,42
Height of unsilvered part of ditto	,65	0,80	0,95

If the telescope has a common object-glass, the first aperture of  $\frac{1}{4}$ ths of an inch will be most convenient ; but if it has an achromatic object-glass, one of the other apertures of  $\frac{4}{10}$ ths or  $\frac{5}{10}$ ths of an inch, will be most proper. The field of view of the telescope should be 5 or 6 degrees, and the objects should be rendered as distinct as possible throughout the whole field, by applying two eye-glasses to the telescope. The breadth of the glasses should be determined as usual, according to the obliquity with which the rays fall on them and the aperture of the object-glass.

I shall conclude this paper with some easy rules for finding the apparent angular distance between any two near land objects by the Hadley's quadrant.

To find the angular distance between two near objects by the fore-observation. Adjust the fore-horizon-glass by the object intended to be taken as the direct object ; and the angle measured by the fore-observation on the arch of the quadrant between this object and any other object seen by reflection will be the true angle between them as seen from the centre of the index-glass. But, if the quadrant be  
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already well adjusted by a distant object, and you do not chuse to alter it by adjusting it by a near one, move the index, and bring the image of the near direct object to coincide with the same seen directly, and the number of minutes by which (o) of the index stands to the right hand of (o) of the quadrant upon the arch of the excess is the correction, which added to the angle measured by the arch of the quadrant between this direct object and any other object seen by reflection will give the true angular distance between them reduced to the centre of the index-glass.

To find the angular distance between two near objects by the back-observation.

It is supposed that the-horizon-glass is truly adjusted; if it is not, let it be so. Observe the distance of the objects by the back-observation, and take the supplement of the degrees and minutes standing upon the arch to 180 degrees, which call the instrumental angular distance of the objects; this is to be corrected as follows. Keep the centre of the quadrant or index-glass in the same place as it had in the foregoing observation, and observe the distance between the near object, which has been just taken as the direct object, and some distant object, twice; by making both objects to be the direct and reflected ones alternately, holding the divided arch upwards in one case and downwards in the other, still preserving the place of the centre of the quadrant. The difference of these two observations will be the correction, which added to the instrumental angular distance,

distance, found as above in the first observation between the first object and any other object seen by reflection, will give the true angular distance between them reduced to the centre of the index-glass.

But if you should happen to be in a place where you cannot command a convenient distant object, the following method may be used.

The back-horizon-glass being adjusted, find the instrumental angular distance between the objects; this is to be corrected by means of the following operations. Set up a mark at any convenient distance opposite or nearly so to the object which has been taken as the direct object; and looking at the direct object move the index of the quadrant, and bring the image of the mark to coincide with the direct object, and read off the degrees and minutes standing on the arch of the quadrant, which subtract from 180 degrees, if (0) of the index falls upon the quadrantal arch; but add to 180 degrees, if it falls upon the arch of excess; and you will have the instrumental angular distance of the object and mark. Invert the plane of the quadrant, taking care at the same time not to change the place of its centre, and looking at the same direct object as before, move the index of the quadrant, and bring the image of the mark to coincide again with the direct object, and read off the degrees and minutes standing on the arch, and thence also find the instrumental angular distance of the object and mark. Take the sum of this and the former instrumental angular distance; half of its difference from  $360^\circ$  will be the correction, which added to the instrumental angular distance first found between the same direct object and the other object seen

by reflection will give the true angular distance between them reduced to the centre of the index-glass.

It is to be observed, that if the mark be set up at the same distance from the quadrant as the direct object is, there will be no occasion to invert the plane of the quadrant, but the observer need only make the image of the mark coincide with the direct object, then turn himself half round, and now taking the mark for the direct object cause the image of the former direct object to coincide with the mark, the divided arch of the quadrant being kept upwards, and the place of the centre of the quadrant remaining also the same in both cases: half the difference of the sum of the two instrumental angles from 360 degrees will be the correction of the adjustment as before.

Should only one of the objects be near, and the other remote (that is to say, half a mile distant or more) let the distant object be taken for the direct one, and the near object for the reflected one; and the true distance of the objects as seen from the centre of the index-glass will be obtained without requiring any correction, whether it be the back or fore observation that is made use of; only observing, as usual, to take the supplement of what is shewn upon the arch to 180 degrees in the back-observation.