XX. *M.* De Luc's *Rule for meaſuring Heights by the Barometer, reduced to the* Engliſh *Meaſure of Length, and adapted to* Fahrenheit's *Thermometer, and other Scales of Heat, and reduced to a more convenient Expreſſion. By the Aſtronomer Royal.*

Redde, Jan. 13, 1774.

M. DE LUC, F. R. S. in a large and valuable Treatiſe upon the Barometer and Thermometer, lately publiſhed at Geneva, the reſult of many years labour and ſtudy, has given a rule for the meaſurement of heights by the barometer, deduced from his experiments, and far more accurate, than any publiſhed before; ſince it appears that he could determine heights by it generally to 10 or 15 feet, and that the error ſeldom, if ever, amounted to double that quantity. This valuable degree of exactneſs he has obtained principally by detecting the faults of the common barometer, and, in conſequence, improving the conſtruction of it; and by introducing the uſe of the mercurial thermometer, to accompany that of the barometer. The principal faults, which he found in the common barometers, aroſe from

the repulſion of the quickſilver by the glaſs tube, from air and moiſture admitted into the tube, and from the variations of the denſity of quickſilver by heat and cold; another very conſiderable error aroſe, in calculating heights from the barometer, by not allowing for the changes of the denſity of the air, whoſe gravity affords us this meaſure of heights, owing to heat and cold. The firſt cauſe of error, that of the repulſion of the tubes, he remedied, by ſubſtituting a ſyphon-barometer inſtead of the ſimple upright tube, the repulſion of the two legs of the ſyphon, counteracting itſelf; the error ariſing from air and moiſture in the tube he cured by boiling the quickſilver, after it was put into the tube, and other precautions; the errors, in the eſtimation of the heights, ariſing from the changes of the denſity of the quickſilver, and denſity of the air by heat and cold, he ſhews how to correct by allowances depending on two thermometers, one attached to the frame of the barometer itſelf, and the other made to be expoſed to the open air, to ſhew its degree of heat; which thermometers are to be noted both at the top and bottom of the hill. Laſtly, by a great number of experiments made with accurate barometers and thermometers of his own conſtruction, he has deduced a rule for calculating heights of places; the exactneſs of which he has ſufficiently proved by a large table of experiments. But this rule is expreſſed in French meaſure, and is adapted either to a thermometer, whoſe freezing point is 0, and that of boiling water 80, or to thermometers of particular ſcales. It

may

may be therefore useful, to reduce M. DE LUC's rule to English measure, and to adapt it to the thermometer of Fahrenheit's scale, which is generally used in this country.

M. DE LUC, in the winter season, heated the air of his room to as great degree as he could, and noted the rise of the barometer, owing to the diminution of its density, or specific gravity, by heat; he also noted the height of the thermometer, both before and after the room was heated. Hence he deduced a rule, that when the barometer is at 27 French inches, which was the case in this experiment, an increase of heat, from freezing to that of boiling water, will raise the barometer 6 lines, or $\frac{1}{54}$th part of the whole. It is easy to see, that when the barometer is higher than 27 French inches, this variation will increase in the same proportion; or will be always $\frac{1}{54}$th of the heighth of the barometer; therefore, if the height of the barometer be called B, the rise of the barometer, for an increase of heat from freezing to boiling water, will be $\frac{B}{54}$; and, as it will be less for a less difference of heat, therefore, if the number of degrees, marked on the thermometer, between freezing and boiling water, be called K, and the rise of the thermometer from any given point be called H, the correspondent rise of the barometer will be $\frac{B}{54} \times \frac{H}{K}$, by the increase of heat from the given point by the number of degrees H. If the heat, instead of increasing, was to decrease, then H would

would signify so many degrees decrease of heat, and the barometer would sink by $\frac{B}{54} \times \frac{H}{K}$. The fixt temperature of heat, to which M. DE LUC thought best to reduce his observations of the barometer, is $\frac{1}{8}$th of the interval from freezing to boiling water above the former point: and if the thermometer was higher than this degree, he substracted $\frac{B}{54} \times \frac{H}{K}$; if it was lower, he added it to the observed height of the barometer; and thus he obtained the exact height of the barometer, such as it would have been, i[illegible] [illegible] density of its quicksilver had been the same as answers to the fixt degree of temperature. He thus corrected the height of both his barometers (that at the bottom, and that at the top of the hill) for the particular degree of heat, indicated by a thermometer attached to the barometer, at each station; for it might and would commonly happen, that the degree of heat would be different at the two stations. The heights of the barometers, thus corrected, were what he made use of in his subsequent calculations. Calling these two altitudes of the barometer B and *b*, putting log. B and log. *b*, for the logarithms of B, and *b*, taking only the four first places of figures, after the characteristic, or considering the remaining figures as decimals, and putting C for the mean height of a thermometer, exposed to the air at top and bottom of the hill, the freezing point being 0, and the point of boiling water at 80, he finds, by his experiments, that the height of the hill will be given in French toises, when C is $16\frac{3}{4}$, by

 simply

ſimply taking the difference of the logarithms of the heights of the barometer, or will be equal to log. B — log. *b*; and in any other degree of heat, will be greater or leſs, in proportion as the rarity of the air is greater or leſs, than in the fixt temperature; or greater or leſs by $\frac{1}{215}$th part of the whole, for every degree of the thermometer reckoned from the fixt temperature 16¾; and conſequently the height of the place will be expreſſed generally in French toiſes, by this *formula*

$$\log. B - \log. b + \overline{\log. B - \log. b} \times \frac{C - 16\frac{3}{4}}{215}$$

$$= \overline{\log. B - \log. b} \times \overline{1 + \frac{C - 16\frac{3}{4}}{215}}.$$

To reduce this *formula* to Engliſh meaſure, and to the ſcale of Fahrenheit's thermometer, we ſhould firſt premiſe ſome particulars. The French foot is to the Engliſh foot, as 1,06575 to 1, as was found by a very accurate experiment: ſee Phil. Tranſ. Vol. LVIII. for 1768, p. 326; and it is well known, that the point of freezing, on Fahrenheit's thermometer, is at 32, and that of boiling water at 212, or the interval between them 180 degrees. But M. DE LUC's point of boiling water 80, was marked when the barometer was at 27 French inches; and it is the cuſtom of our principal Engliſh workmen to mark the point of boiling water, 212, on Fahrenheit's thermometer, when the barometer ſtands at 30 inches, which is equal to 28 inches, 1,8 lines French meaſure; or 13,8 lines higher than M. DE LUC's barometer, when he ſet off the point of boiling water on his thermometers; and it is well known, that the heat of boiling water varies with the weight of the atmoſphere: M. DE LUC finds, by his experiments,

ments, this rule, that an increase of 1 line in the height of the barometer, raises the quicksilver of the thermometer, placed in boiling water, by $\frac{1}{1134}$th part of the interval between the freezing point and that of boiling water: he afterwards indeed found, that this rule would not answer for such large variations of the barometer, as take place in ascending to very great heights above the earth's surface; *vid. Essai sur les variations du chal: de l'eau bouill:* but it is accurate enough for any small variation of the barometer, on one side or other of its mean height in these lowest regions of the atmosphere. The change therefore of the boiling point on Fahrenheit's scale, for a change of 1 line in the barometer, will be $\frac{180}{1134} = 0,16$; therefore 13,8 lines will cause $0,16 \times 13,8 = 2,2$ degrees of Fahrenheit's scale; and a thermometer, whose point of boiling water was marked 212, when the barometer stood at 30 English inches = 28 inches, 1,8 lines French measure, will, when the barometer descends to 27 French inches, sink 2,2 degrees in boiling water, or to 209,8, or in round numbers to 210 degrees, which is distant only 178 from 32, the point of freezing. Hence an extent of 80° of M. DE LUC's thermometer answers to an extent of 178 of our Fahrenheit's thermometer; and putting F for the degrees of this thermometer, corresponding to C of M. DE LUC's, we shall have C : F — 32 :: 80 : 178, and $C = \overline{F - 32} \times \frac{80}{178}$ which substituted in M. DE LUC's *formula*, gives

$$\overline{\log. B - \log. b} \times \overline{1 + \frac{C - 16\frac{3}{4}}{215}} =$$

$$= \overline{\log. B - \log. b} \times \overline{1 + \frac{\overline{F - 32} \times \frac{80}{178} - 16\frac{3}{4}}{215}} =$$

=

$$=\overline{\log.B-\log.b}\times\overline{1+\frac{80}{178\times 215}\,\overline{F-32-\tfrac{178}{80}\,16{,}75}}$$

$$=\overline{\log.\,B-\log.\,b}\times\overline{1+\frac{F-32-37{,}27}{478{,}38}}=$$

$$=\overline{\log.\,B-\log.\,b}\times\overline{1+\frac{F-69{,}27}{478{,}38}}.$$ Where the answer will still come out in French toises, though adapted to Fahrenheit's thermometer. To bring it out in English fathom (or measure of 6 feet) multiply the above expression by 1,06575, and we shall have $\overline{\log.\,B-\log.\,b}\times\overline{1+\frac{F-69{,}27}{478{,}38}}\times 1{,}06575$

$$=\overline{\log.\,B-\log.\,b}\times\overline{409{,}11+F}\times\frac{1{,}06575}{478{,}38}=$$

$$=\overline{\log.\,B-\log.\,b}\times\frac{409{,}11+F}{448{,}87},$$ or in round numbers

$$=\overline{\log.\,B-\log.\,b}\times\frac{409+F}{449}=$$

$$=\overline{\log.\,B-\log.\,b}\times\overline{1+\frac{F-40}{449}};$$ which will express the height between the two stations in English fathom.

In the foregoing expressions, B and b, as has been mentioned before, signify heights of the barometer, at the lower and higher stations, both corrected according to Mr. DE LUC's directions, for the difference of heat between a fixed temperature, (namely $\frac{1}{4}$th of the interval between freezing and boiling water), and the present heat, indicated by the thermometer attached to the barometer at each station; but it is not necessary, to correct *both* barometers for the effect of heat, but only one for the difference of heat of the two; which will be more convenient also on another account, because the difference of heat, at the two stations, will

will be generally ſmall, and the correction to reduce one barometer to the heat of the other will conſequently be ſmall alſo; whereas the difference of the preſent heat, and the fixt temperature, and conſequently the correction of both barometers, may be frequently very conſiderable: this is evident: becauſe if the heat of the barometers, at both ſtations, was the ſame, however different from the fixt temperature choſen by M. DE LUC, no correction would be neceſſary; the mercury in the barometer in both ſtations, being expanded in the ſame proportion, and conſequently the difference of the logarithms of its height, at both ſtations, being the ſame, as if the heat of both barometers had agreed with that of the fixt temperature. I ſhall now therefore ſuppoſe the upper barometer is to be corrected, to reduce it to the temperature of the lower one, and that b ſignifies the height of this barometer, as obſerved, and not yet corrected; the correction, from what has been ſaid above, calling D the difference of height of the thermometer attached to the barometer at the two ſtations, will be $\pm \frac{Db}{54K}$, according as the thermometer ſtands higheſt at the lower or upper ſtation; and the upper barometer corrected, inſtead of b, will be $b \pm \frac{Db}{54K}$, which ſubſtituted in the *formula*, gives $\overline{\log. B - \log.(b \pm \frac{Db}{54K})} \times \overline{1 + \frac{F-40}{449}}$. But the correction, on account of the difference of heat of the barometer at the two ſtations, may be reduced to a ſtill eaſier expreſſion, in which the variable quantity b, the height of the upper barometer, ſhall

ſhall not appear. The fluxion of a logarithm is to the fluxion of its natural number, as the modulus of the ſyſtem to the natural number; and 4343 is the modulus of the common logarithms, when the four places, next following the characteriſtic, are taken as whole numbers, inſtead of decimals, which is meant to be done in the uſe of the foregoing *formula.* Therefore $\frac{Db}{54K}$ being very ſmall with reſpect to b, we ſhall have, variation of log. b : variation of b $\left(= \frac{Db}{54K}\right)$ :: 4343 : b very nearly, and thence variation of log. $b = \pm \frac{Db}{54K} \times \frac{4343}{b} = \pm \frac{4343 D}{54K}$. Which (putting $K = 178$) $= \pm 0{,}452 D$. Hence log. $\left(b \pm \frac{Db}{54K}\right) =$ log. $b \pm 0{,}452 D$; which being ſubſtituted in the *formula* above, will give the difference of height, of the two ſtations, in Engliſh fathom, in a more convenient expreſſion, namely $\overline{\log. B - \log. b \mp 0{,}452 D} \times \overline{1 + \frac{F - 40}{449}}$; where the upper ſign, —, is to be uſed, when the thermometer of the barometer is higheſt at the lower ſtation, and the lower ſign, +, is to be uſed, when the ſaid thermometer is loweſt at the lower ſtation. The firſt caſe will be moſt common; eſpecially where the difference of height of the two ſtations is conſiderable. It ſhould alſo be obſerved, that when F, the height of Fahrenheit's thermometer, is leſs than 40°, $+ \frac{F - 40}{449}$ becoming negative or ſubſtractive, muſt be applied in the calculation accordingly.

It

It may perhaps be convenient to repeat here the meaning of the algebraic terms, used in the foregoing *formula*, that any person may make use of it, without having occasion to recur to the foregoing investigation. B signifies the observed altitude of the barometer at the lower station, and *b* that at the upper station; Log. B and Log. *b*, signify their logarithms taken out of the common tables, by assuming the four first figures next following the characteristic as whole numbers, and considering the three remaining figures, to the right hand, as decimals; D signifies the difference of height of Fahrenheit's thermometer, attached to the barometer at the top and bottom of the hill; and F signifies the mean of the two heights of Fahrenheit's thermometer, exposed freely for a few minutes to the open air in the shade, at the top and bottom of the hill.

The *formula*, for the measure of heights, may also be changed, and adapted to thermometers of particular scales, for the convenience of calculation, as M. DE LUC has done; but these scales will be different from his. The thermometer, attached to the barometer, had better be divided with the interval between freezing and boiling water, consisting of 81,4 degrees ($=180 \times ,452$) the freezing point may be marked o, and the point of boiling water will be 81,4; for then, if the difference of height of this thermometer, at the two stations, be called *d*, we shall have $d =$, $0{,}452 \times D$. for $d : D :: 81{,}4 : 180 :: 0{,}452 : 1$, and the number of degrees exprest by *d*, will shew immediately the correction for the difference of heat of the two barometers. If the thermometer, designed to shew the temperature of the air, be divided

divided with the interval between freezing and boiling water $=200$, and the freezing point be marked -9, and the boiling point $+191$, and the heights of this thermometer, at the two ſtations, be called G and I, we ſhall have $\frac{F-40}{449} = \frac{G+I}{2\times 500}$ $= \frac{G+I}{1000}$. For $F-40=F-32-8$, is the height of Fahrenheit's thermometer, reckoned from 8 degrees above freezing, and $449 : 500 :: 180 : 200 :: 8 : 9$, and the fraction $\frac{F-32-8}{449}$, if both the numerator and denominator be increaſed in the ratio of 449 to 500, will become $= \frac{\overline{F-32-8}\times\frac{500}{449}}{500}$ $= \frac{\overline{F-32}\,\frac{500}{449}-9}{500} = \frac{G+I}{2\times 500} = \frac{G+I}{1000}$, becauſe $\frac{G+I}{2}+9$ $= \overline{F-32}\times\frac{500}{449}$. Therefore, if the thermometer of the barometer has the freezing pointmarked 0, and the point of boiling water 81,4, and the difference of its height, at the two ſtations, be called d; and the thermometer for meaſuring the temperature of the air, be divided with the interval of 200 between the freezing point and that of boiling water, and the firſt be marked -9, and the latter $+191$, and the degrees, ſhewn by this, at the two ſtations, be called G and I; the *formula*, that will give the height of the upper ſtation above the lower one, in Engliſh fathoms, will be $\overline{\log.\ B - \log.\ b \mp d} \times \overline{1 + \frac{G+I}{1000}}$; which conſequently multiplied by 6, will give the height in Engliſh feet. It is to be obſerved, as before, that $-d$ or $+d$ is to be uſed, according as the thermometer, attached to the barometer, is higheſt

at the lower or upper ſtation; and if G and I ſhould happen to fall below o of the ſcale, or to be ſubſtractive, they muſt be applied accordingly in the calculation.

I ſhall now add nothing more, but to give the rule for finding heights by the barometer, according to the *formulæ* delivered above, in common language; firſt, as adapted to Fahrenheit's thermometer, and next, as adapted to the two thermometers of particular ſcales. Take the difference of the tabular logarithms of the obſerved heights of the barometer, at the two ſtations, conſidering the 4 firſt figures, excluſive of the index, as whole numbers, and the three remaining figures to the right as decimals, and ſubſtract or add $\frac{452}{1000}$th of the difference of the altitude of the Fahrenheit's thermometer, attached to the barometer at the two ſtations, according as it was higheſt at the lower or upper ſtation; thus you will have the height of the upper ſtation above the lower, in Engliſh fathom, nearly; to be corrected, as follows: make this proportion; as 449 is to the difference of the mean altitude of Fahrenheit's thermometer, expoſed to the air at the two ſtations, from 40°, ſo is the height of the upper ſtation found nearly, to the correction of the ſame; which added or ſubſtracted, according as the mean altitude of Fahrenheit's thermometer was higher or lower than 40°, will give the true height of the upper ſtation above the lower, in Engliſh fathoms; and multiplied by 6, will give it in Engliſh feet.

The ſame rule, adapted to the thermometers of particular ſcales, is this:

Take the difference of the tabular logarithms of the obſerved heights of the barometer, at the two ſtations, conſidering the 4 firſt figures (exclusive of the index) as whole numbers, and the three remaining figures to the right as decimals; and ſubſtract or add the difference of the thermometer, of a particular ſcale, attached to the barometer, at the two ſtations, according as it was higheſt at the lower or upper ſtation, and you will have the height of the upper ſtation above the lower one, in Engliſh fathom, nearly; to be corrected as follows: make this proportion; as 1000 is to the ſum of the altitudes of the thermometer of a particular ſcale, expoſed to the air at both ſtations, ſo is the height of the upper ſtation above the lower, found nearly, to the correction of the ſame; which added or ſubſtracted, according as the ſum of the altitudes of the thermometers, expoſed to the air, is poſitive or negative, will give the true height of the upper ſtation above the lower in Engliſh fathoms; and multiplied by 6, will give it in Engliſh feet.

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