V. "On the Effect of Local Attraction upon the Plumb-line at stations on the English Arc of the Meridian, between Dunnose and Burleigh Moor; and a Method of computing its Amount." By the Venerable Archdeacon PRATT. Communicated by the Rev. J. CHALLIS, F.R.S. Received June 5, 1855.

The author states that in a former communication he had pointed out a method for calculating the deflection of the plumb-line at stations on the Indian arc, caused by the attraction of the Himalayas and of the vast regions beyond, with a view to the correction of the astronomical amplitudes of the measured subdivisions of the arc before they are applied to the determination of the ellipticity of the earth.

The same subject is taken up in the present paper, but in reference to the English arc between Dunnose and Burleigh Moor; and a different method of calculating the attraction is given.

The paper consists of three parts. In the first, the ellipticity of the English arc is calculated without taking account of attraction. The arc is divided into five parts, and the lengths and amplitudes assigned to them in Mudge's Trigonometrical Survey of England, vols. ii. and iii., are made the basis of the calculation. These portions of the arc are compared two and two, and ten values of the ellipticity thence deduced; the mean of which is $-\frac{1}{43{\cdot}8687}$. The ten values, of which this is the mean, differ considerably among each other, indicating that there is some disturbing cause, like local attraction, affecting the plumb-line, and therefore the apparent latitudes. The variations of the observed amplitudes are then discussed; and the necessity of calculating the local attraction pointed out.

In the second part a formula is obtained for calculating the attraction. The method is different from that given by the author in his first communication. The curvature of the earth is neglected, as this would have no sensible effect on the results in the British Isles. The attracting mass is divided into a number of smaller masses standing on rectangular bases at the sea-level, and the height of each

is taken equal to the average height of the surface above the sea-level. The dimensions of the bases may differ from each other, and are determined by the contour of the surface in such a way that the average height in each mass may not depart materially from the height of any part of it. The investigation leads to the following Rule for determining the horizontal attraction deflection of the plumb-line caused by any one of these Tabular Masses (as the author calls them) :—

RULE.—*Take the origin of coordinates at the station where the plumb-line is. Let the plane of* xy *be horizontal, and the axis of* x *in the vertical plane in which the amount of deflection is to be found.*

Write down the coordinates XY xy *of the furthest and nearest angles of the Tabular Mass from the origin;* Y *is always to be considered* $+^{ve}$, *and* y $+^{ve}$ *or* $-^{ve}$ *accordingly.*

Form four ratios, by first dividing each ordinate by the abscissa not belonging to it, and then by dividing each ordinate by its own abscissa, viz.

$$\frac{Y}{x}, \frac{y}{X}, \frac{Y}{X}, \frac{y}{x}.$$

Look in a Table of Tangents for the four angles of which the tangents equal the above ratios.

Form four more angles by adding (subtracting if they be negative) half of each of these angles just found to (or from) 45°.

From the sum of the log-tangents of the first two of these angles subtract the sum of the log-tangents of the second two.

This result, multiplied by H *feet and by* $\frac{1''}{369}$, *will give the required deflection of the plumb-line in seconds of a degree*—H being the height of the Tabular Mass above the sea-level, and its density being taken equal to half the mean density of the earth, which is that of granite.

The only restriction to be attended to in the application of this Rule is, that the ratio of the height of the attracted station above the sea to each of the horizontal coordinates of the nearest angle of the Tabular Mass, must be so small that its square may be neglected.

If any part of the attracting mass is nearer to the station than this, it must be divided into vertical prisms and the attraction of each found; for which the author gives a formula in a note.

In the third part the Rule is applied, for the purpose of illustration, to obtain an approximate value of the deflection of the plumb-line at Burleigh Moor, the north station of the arc under consideration, situated on the north coast of Yorkshire. The deflection is found to be 3″·644 to the south. The data upon which this calculation is based are gathered by the author from the Map which accompanies General Mudge's account of the English Survey, and the heights marked down on that map.

The deflection at several other stations is deduced from this result of calculation, by using the amplitudes given in Mudge's work, and also in Captain Yolland's 'Astronomical Observations made with Airy's Zenith Sector,' published in 1852, and by supposing the curvature of the meridian to be the mean curvature of the whole globe as laid down by Mr. Airy in his article on the Figure of the Earth. Thus the deflection at Black Down on the Dorsetshire coast (one of the places mentioned in Captain Yolland's volume), the author finds to be 5″·886 to the north, a quantity which tallies well with the deflection assigned to Burleigh Moor on the Yorkshire coast, if the relative heights of the two coasts are compared. This affords a satisfactory evidence of the correctness of the principles laid down in the paper; and, as the author thinks, makes the subject well worthy the attention of those who are interested in the English Survey, and who have it in their power to furnish the most accurate data for the application of the Rule he lays down. The subject is also no less important to the mathematician in his investigation of the figure of the earth.

In a Postscript the author makes the following remarks upon the Astronomer Royal's method of accounting for the large amount of deflection on the Indian arc deduced by the author in his former communication :—

"Since the above was written, I have had the opportunity of seeing a notice of the communication of the Astronomer Royal on the Density of Table-lands supposed to be supported by a dense fluid or semi-fluid mass; and the use he makes of his suggestions to remove the discrepancy, pointed out in my first communication, between the values of the deflection of the plumb-line in India, as determined by calculating the attraction of the Himalayas, and as indicated by the results of the Great Trigonometrical Survey. The following diffi-

culties occur to me in the way of this highly ingenious and philosophical method of removing the discrepancy :—

" 1. It assumes that the hard crust of the earth is sensibly lighter than the fluid or semi-fluid mass, imagined to be a few miles below the surface. But I know of no law, except the unique law of water and ice, which would lead us to suppose that the fluid mass in consolidating would expand and become lighter. One would rather expect it to become denser, by loss of heat and mutual approximation of its particles.

" 2. There is, moreover, every reason to suppose, that the crust of the earth has long been so thick, that the position of its parts relatively to a mean level cannot be any longer subject to the laws of floatation. If the elevations and depressions of the earth's surface have always remained exactly what they were at the time when the laws of floatation ceased to have an uncontrolled effect, then the same reasoning would no doubt apply in our case, as if they still had their full sway. But geology shows that other laws are in constant operation (arising most probably, as Mr. Babbage has suggested, from the expansion and contraction of the solid materials of the crust), which change the relative levels of the various parts of the earth's surface, quite irrespectively of the laws of floatation. If Mr. Hopkins's estimate of the thickness of the crust be correct, viz. at least 1000 miles, these laws of change in the surface must have been in operation for such an enormous interval of time, as quite to obliterate any traces of the form of surface which the simple principles of hydrostatics would occasion. Indeed, it seems to me highly probable that the elevation of the Himalayas and the vast regions beyond may have arisen altogether from the slow upheaving force arising from this cause.

" I am inclined to think that the only explanation of the discrepancy between my calculations and the results of the Indian Survey, is to be found in the greater curvature of the Indian Arc."