of the first, but with sign changed. The modulus is given for every 0·01 from 0·00 to 0·80. The corresponding deviations are given in degrees and minutes. For each modulus there is also given the mean of all the deviations in the semicircumference, for that modulus; by use of which, in comparison with the mean in any given instance, the modulus in that instance is determined.

XVIII. "On Axes of Elasticity and Crystalline Forms." By W. J. Macquorn Rankine, C.E., F.R.SS. L. & E. &c.

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An Axis of Elasticity is any direction with respect to which any kind of elastic force is symmetrical.

In this paper the deviation of a molecule of a solid from that condition as to volume and figure which it preserves when free from the action of external forces, is denoted by the word "*Strain*," and the corresponding effort of the molecule to recover its free volume and figure by the word "*Stress*."

In devising a nomenclature for quantities relating to the theory of elasticity, *strain* is denoted in composition by θλίψις, and *stress* by τάσις.

Every possible strain of a molecule, when referred to rectangular axes, may be resolved into six *elementary strains*; three elongations or linear compressions, and three distortions. Every possible stress, when referred to rectangular axes, may be resolved into six *elementary stresses*; three normal pressures, and three tangential pressures, which tend to diminish the corresponding elementary strains.

The elementary strains being in fact approximately linear functions of the elementary stresses, are treated in this investigation as exactly so.

The sum of the six integrals of the elementary stresses, each taken with respect to the corresponding elementary strain *from* its actual amount *to* zero, is the *Potential Energy of Elasticity*, and is a homogeneous function of the elementary strains of the second order, and of twenty-one terms, whose constant coefficients are here called the *Tasinomic Coefficients*, or coefficients of Elasticity.

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The principles of the Calculus of Forms, and in particular the *Umbral Notation* of Mr. Sylvester, are applied to the Orthogonal Transformations of the Tasinomic Coefficients.

Several functions of these coefficients are determined, called *Tasinomic Invariants,* which are equal for all systems of orthogonal axes in the same solid.

Certain functions of the Tasinomic Coefficients constitute the coefficients of two *Tasinomic Ellipsoids,* designated respectively as the *Orthotatic* and *Heterotatic* Ellipsoids, whose axes have the following properties.

ORTHOTATIC AXES.

At each point of an elastic solid there is one position in which a cubical molecule may be cut out, such, that a uniform dilatation or condensation of that molecule by equal elongations or compressions of its three axes, will produce no tangential stress at the faces of the molecule.

The existence of orthotatic axes in a solid constituted of mutually attracting and repelling physical points was first proved by Mr. Haughton; it is proved in this paper independently of any hypothesis as to molecular structure or action.

HETEROTATIC AXES.

At each point of an elastic solid there is one position in which a cubical molecule may be cut out, such, that if there be a distortion of that molecule round x *(*x *being any one of its axes) and an equal distortion round* y *(*y *being either of its other two axes), the normal stress on the faces normal to* x *arising from the distortion round* x*, will be equal to the tangential stress around* z *arising from the distortion round* y.

The six coefficients of the Heterotatic Ellipsoid represent parts of the elasticity of a solid which it is impossible to reduce to attractions and repulsions between points.

Fifteen constants called the *Homotatic Coefficients,* which are composed of Tasinomic Coefficients and their linear functions so constituted as to be independent of the *Heterotatic Coefficients,* are the coefficients of the fifteen terms of a homogeneous biquadratic function of the coordinates, which being equated to unity, characterizes the

Biquadratic Tasinomic Surface. This surface, for solids composed of physical points, was discovered by Mr. Haughton; it is here investigated independently of all hypothesis.

By rectangular linear transformations, three functions of the Homotatic Coefficients may be made to vanish. Three orthogonal axes are thus found, which are called the *Principal Metatatic Axes,* and have the following property: *if there be a linear elongation along any one of these axes, and an equal linear compression along any other, no tangential stress will result on planes normal to these two axes.*

In each of the three planes of the principal Metatatic Axes, there is a pair of Diagonal Metatatic Axes bisecting the right angles formed by the pair of principal axes in the same plane.

In each plane in an elastic solid, there is a system of two pairs of metatatic axes, making with each other eight equal angles of 45°.

Various kinds and degrees of symmetry are pointed out, which the tasinomic coefficients may have with respect to orthogonal axes.

The Potential Energy of Elasticity may be expressed as a homogeneous function of the second order of the Elementary Stresses. The twenty-one coefficients of this function are called Thlipsinomic Coefficients.

The Thlipsinomic and Tasinomic Coefficients are related to each other as Contragredient Systems.

The Orthogonal and Diagonal Tasinomic and Thlipsinomic Axes coincide.

For the complete determination of the properties of the Homotatic Coefficients, it is necessary to refer them to oblique axes of coordinates.

The application of oblique co-ordinates to this purpose is much facilitated by the employment along with them of three auxiliary variables called *Contra-ordinates.* The contra-ordinates of a point R are the projections of the radius-vector OR on the three axes. For rectangular axes, co-ordinates and contra-ordinates are identical. The co-ordinates x, y, z and contra-ordinates u, v, w of a point R are connected by the equation

$$ux+vy+wz=\overline{\mathrm{OR}}^2.$$

As there are six independent quantities in the directions of a system of three axes of indefinite obliquity, there is a system of

right or oblique axes in every solid for which six of the coefficients of the characteristic function of the Biquadratic surface disappear, reducing that function to its canonical form of nine terms. These three axes are called the

Principal Enthytatic Axes.

Every Enthytatic axis has this property, *that a direct linear elongation or compression along such an axis, produces a normal stress, and no oblique or tangential stress on a plane normal to the same axis.*

Every Enthytatic axis is normal to the Biquadratic Surface, and is a line along which the direct elasticity of the body is either a maximum or a minimum, or in that condition which combines the properties of maximum and minimum.

It is probable that the faces or edges of primitive crystalline forms are normal to Enthytatic axes, and that the planes of cleavage in crystals are normal to Enthytatic axes of minimum elasticity.

It is also probable that the symmetrical summits of crystals correspond to Enthytatic axes.

There are, in every solid, at least the three principal Enthytatic Axes, which are normal to the faces of a hexahedron, right or oblique as the case may be. In certain cases of symmetry of these axes and of the Homotatic Coefficients, there are *Secondary or Additional Enthytatic Axes,* which are determined by the following principles:

1. When the three principal axes and the Homotatic Coefficients are symmetrical round a central axis, that axis is an additional Enthytatic axis.

2. When there are a pair of orthogonal Enthytatic axes in a given plane, there may be, under certain conditions specified in the paper, a pair of *additional or secondary axes* in that plane, making with each other a pair of angles bisected by the orthogonal axes.

In the first column of a table, the possible systems of Enthytatic axes are arranged according to a classification and nomenclature of their degrees and kinds of symmetry; and in the second and third columns are stated the primitive crystalline forms to the faces and edges of which such systems of axes are respectively normal, and which embrace all the primitive forms known in crystallography.

The six Heterotatic Coefficients are independent of the fifteen Homotatic Coefficients which determine the Enthytatic axes.

The paper concludes with observations on some real and alleged differences between the laws of solid elasticity and those of the luminiferous force,—on some hypotheses in connexion with the wave-theory of light,—and on the refraction of light in crystals as connected with the symmetry of their Enthytatic axes.

"Report of a Committee appointed by the Council to examine the Calculating Machine of M. Scheutz." Inserted for the information of the Fellows by order of the President and Council*.

The various applications of mathematics to physical questions, or to the transactions of common life, continually require the computation of numerical results. At one time isolated results have to be calculated from particular formulæ; at another it is required to calculate a series of values of the same analytical formula; in other words, to tabulate a function. It is only in the latter case that different instances have so much in common as to permit of the application of general methods irrespective of the particular function to be calculated. But even in the tabulating of functions one or other of two objects may be kept in view. At one time a result may be arrived at expressed in a complicated, perhaps transcendental, formula, and the mathematician may desire to know merely the general progress of the function. In such a case it will be sufficient to calculate values at rather wide intervals, and the mode of calculation must depend upon the peculiar function. But at other times functions present themselves which are of such common occurrence, or of such practical importance, that it is desirable to tabulate them for values of the variable increasing by small steps. In these cases general methods of interpolation come into use: it is sufficient to perform the calculations directly for comparatively wide intervals of the variable, and the intervening values of the function can be supplied by the mere addition of differences.

* The Committee consisted of Prof. Stokes, Sec. R.S., Prof. W. H. Miller, Prof. Wheatstone, and the Rev. Prof. Willis.