the remainder may be written $13(a+b+c)$, which is divisible by 13, whatever be the value of a, b and c.

In the same way it may be shown that a number of this kind is divisible by 11.

When the first figure of the period is 0, and the second any whatever i and j, the number is $0ij0ij=ij0ij$; or any number of five places, the first two and the last two being the same, while the middle place is 0, is divisible by 7, 11 and 13. Thus 34034, 14014 are so divisible.

When the first two places are 0, the number may be written $00i00i=i00i$, or any number of four places, the first and last figures being the same, while the two middle places are 0, is divisible by 7, 11 and 13. Thus 5005, 8008 are so divisible.

Like properties may be found for 17, 19, 23, but the periods are longer. The prime divisor being $2n+1$, it is manifest the number of places in the period cannot exceed, however it may fall short of n.

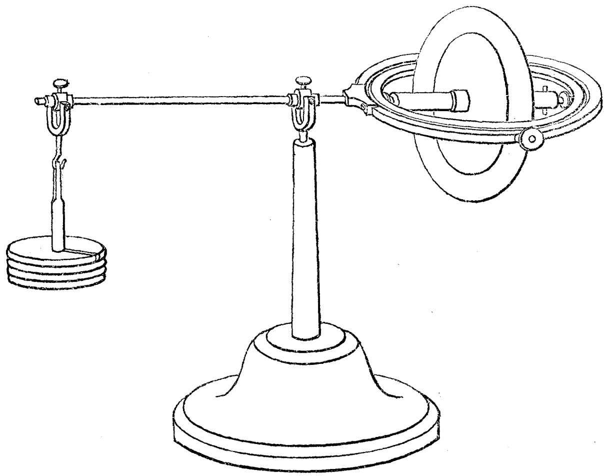
Thus when the divisor is 17, the number of places in the period is eight.

III. "On Fessel's Gyroscope." By C. Wheatstone, Esq., F.R.S. Received April 6, 1854.

Since the announcement of M. Foucault's beautiful experiment which has afforded us a new mechanical proof of the rotation of the earth on its axis, the phenomena of rotary motion have received renewed attention, and many ingenious instruments have been contrived to exhibit and to explain them. One of the most instructive of these is the Gyroscope invented by M. Fessel of Cologne, described in its earlier form in Poggendorff's Annalen for September 1853, and which, with some improvements by Prof. Plücker and some further modifications suggested by myself, I take the present opportunity of bringing before the Royal Society.

It is thus constructed: a beam is capable of moving freely round a horizontal axis which is itself moveable round a vertical axis, so that the beam may move in any direction round a fixed point; at one end of the beam is fixed a horizontal ring which carries a heavy

disc, the axis of rotation of which is in a line with the beam ; at the opposite extremity is a shifting weight by means of which the equilibrium of the beam may be established or disturbed at pleasure.



The Gyroscope.

If the beam be brought into equilibrium, and the disc be rapidly rotated, by means of a thread unrolled from its axis, it will be seen that the beam has no tendency to displace itself in any direction. Not so, however, if the equilibrium be in any way disturbed; on moving the weight towards the centre of the beam, thus causing the *disc* to preponderate, it will be observed that if the disc rotates from right to left the beam will move round the vertical axis also from right to left; and if the motion of the disc be reversed the rotation of the beam will be reversed also. On causing the *equipoise* to preponderate contrary effects will take place. The velocity of the rotation of the beam round the vertical axis increases in proportion to the disturbance of the equilibrium. It will also be observed that, notwithstanding the increased or diminished action of gravity on the disc, its axis of rotation always preserves the same inclination to the vertical axis at which it has been originally placed. The effect produced is a seeming paradox. When the equilibrium is disturbed while the disc is at rest, the beam being placed in any other position than the vertical, gravity acts so as to turn it round a horizontal axis; but when the

disc is in motion the usual effect of gravity disappears, and there is substituted for it a continued rotation round a vertical axis, that is, round an axis perpendicular to the plane which contains the axes of the two original rotations.

A similar composition of forces takes place when the disc is caused to rotate while the equilibrium of the beam is maintained, by impressing on the beam a rotation round the vertical axis. When the disc rotates from right to left, the slightest pressure tending to produce rotation round the vertical axis in the same direction, causes the end of the beam carrying the disc to ascend, and a pressure in the opposite direction causes it to descend, that is, the beam is constrained to move round a horizontal axis perpendicular to the vertical plane which contains the two axes of impressed rotation, a case exactly analogous to the preceding. The beam ascends and descends in like manner, after rotation has spontaneously taken place round the vertical axis in consequence of the equilibrium being disturbed, whenever this rotation is any how accelerated or retarded; the disc rotating from right to left and its weight predominating, the rotation round the vertical axis is from left to right; accelerating the latter motion will cause the disc to descend, and retarding it will occasion it to ascend.

As the centre of gravity of the beam is below its point of suspension, even when equipoised it is in perfect equilibrium only when it is horizontal, consequently, if it be elevated above or depressed below this position it will endeavour to resume it, tending to produce in the two cases rotations in opposite directions round a horizontal axis; the rotation of the disc combined with this tendency gives rise, as in the other cases I have mentioned, to a continued rotation round the vertical axis. If the disc rotate from right to left, and the end of the beam carrying it be elevated above the horizontal position, the rotation round the vertical axis will be from right to left; if, on the contrary, the same end of the beam be depressed below the horizontal position, that rotation will be from left to right.

In all the experiments above mentioned the axis of the rotating disc has remained in the prolongation of the beam, but, by means of an internal ring moveable round a line perpendicular thereto, this axis may be placed at any inclination and at any azimuth with respect to it. Very obvious considerations show that the

inclination of this axis should produce no difference in the character of the effects but merely in their intensity, since in any inclined position of the disc its rotation is resolvable into two others, one perpendicular to the beam, and the other, which is incapable of producing any effect, in a plane containing it. When the axis of the rotating disc is vertical and at right angles to the beam, no rotation on the vertical axis ought to take place in any case; but, contrary to this expectation, although the beam be horizontal and in perfect equilibrium, a motion round the vertical axis results, which is in opposite directions according as one or the other end of the axis of the disc is uppermost. It is, however, easy to see that this rotation is not owing to the same cause which gives rise to the phenomena hitherto considered, for whether it be accelerated or retarded no change is produced in the horizontal position of the beam; it is, in fact, occasioned by the friction of the pivots of rotation dragging the beam into a corresponding motion. Attention to this extraneous cause of rotation will explain numerous anomalies which present themselves in many of the instruments contrived to exemplify the phenomena of combined rotary motions. It is one of the advantages of Fessel's apparatus that the phenomena may be exhibited in their more important phases without being affected by this source of error.

We may form a clearer conception of these phenomena by first considering some simpler facts which do not appear to me to have been hitherto sufficiently attended to. For this purpose let the system of rings carrying the disc be removed from the rest of the apparatus, and by unfastening the tightening screw let the inner ring be allowed to move freely within the outer. Having set the disc in rapid rotation, hold the outer ring at the extremities of the diameter which is in the plane in which the axis of motion of the disc is free to move, then giving to the outer ring a tendency to rotation round that diameter, it will be observed that, in whatever position the axis is, it will fly to place itself in the fixed axis thus determined, and rotation will take place round it in the same direction. Considerable resistance is felt so long as the moveable axis is changing its position, but when once it coincides with the fixed axis the rotation of the external ring round its diameter is effected with facility. A slight alternate motion of the outer ring, tending to give to it rotations in opposite directions, will occasion a continued

rotation of the moveable axis. The same result takes place when an endeavour is made to rotate the outer ring round an axis perpendicular to its plane. In all cases when the axis of the rotating disc is free to move in a plane, and the outer ring is constrained to rotate round a line in this plane, the moveable axis will place itself so as to coincide with that line, and so that the disc shall rotate in the same direction as the ring; if the fixed axis be in a different plane the moveable axis will assume permanently that position in its plane which approaches nearest to the former. The moveable axis is thus apparently attracted towards the fixed axis if the rotations are in the same direction, and repelled from it if the rotations are in opposite directions.

In the experiments just described the free and constrained axes of rotation intersect, but in Fessel's apparatus they are distant from each other. In the latter case the rule must be thus modified, that the free axis of rotation tends to place itself *parallel* to the constrained axis of rotation, or to as near a position thereto as possible. By this principle all the results manifested are easily explained. The beam being in equilibrium, a motion impressed on it round the vertical axis causes it to ascend or descend, because the axis of the rotating disc tends to place itself parallel to the vertical axis of rotation and so that the disc rotates in the impressed direction. When the equilibrium of the beam is destroyed, gravity tends to make it rotate round a horizontal axis; the axis of the disc endeavours to place itself parallel with that axis, but both being unchangeably at right angles to each other, the tendency to place itself there gives rise to a continued rotation. Other results with this apparatus, to which I have not yet adverted, are similarly explained. Fix the outer ring horizontally and loosen the inner ring, keeping them both however in the same plane; then, on moving the beam round the vertical axis, the axis of the rotating disc will immediately fly to place itself parallel thereto, with rotation of the disc in the impressed direction. The rings being placed in the vertical plane, the same result will take place if the beam be moved in a vertical plane, *i. e.* round a horizontal axis.

The following additional experiments may be made with the rings detached from the apparatus. The results are necessary consequences of what has been previously explained:—

1. Suspend, by means of a string, the outer ring at the extremity

of a diameter perpendicular to the axis of the inner ring; and, having loosened the latter, place it at right angles to the former. On causing the disc to rotate, its axis will retain its original position; but if the slightest effort be made to turn the outer ring round the vertical line, the axis of the rotating disc will instantly fly into this position, and the disc will move in the same direction as that of the impressed rotation.

2. The horizontality of the loose inner ring being restored, if a weight be suspended from either end of the axis of the disc, that axis will, while it preserves its horizontal or any inclined position, revolve round the vertical line; the direction of the motion will change if either the weight be applied to the opposite end of the axis or the disc rotate in the opposite direction. If this rotation be arrested, gravity will immediately cause the weighted end of the axis to descend.

3. Clamp the rings together either in the same plane or at right angles to each other, and fasten a string, in the first case, at the extremity of a diameter coinciding with the axis of the inner ring, and in the latter case at the extremity of a diameter perpendicular thereto. Having set the disc spinning, if a rotation round the vertical line be given to the system the axis of the disc will ascend, carrying with it the disc and rings notwithstanding their weight, and, even when the impressed rotation has ceased to act, will continue to rotate in the same direction until the motion of the disc ceases.

In this note I have purposely avoided entering into the mathematical theory of the phenomena, my intention having been solely to describe the apparatus exhibited and to give an intelligible account of its effects. Those who wish to investigate the subject more profoundly, will find the best guide in the Astronomer Royal's essay on Precession and Nutation published in his Mathematical Tracts.