cast-iron bars, under the direction of Professor Hodgkinson. The bars consisted of seventeen different kinds of iron, each set of bars being of the like quality and manufacture; and in several of these sets, which might have been expected to yield the same results, the difference is fully as great as in the cases here exhibited. From this fact an inference may be drawn in favour of the general applicability of the principles developed in the foregoing pages to cast-iron beams and girders of every variety of section.

II. "On the Theory of the Gyroscope." By the Rev. WILLIAM COOK, M.A. Communicated by Professor A. WILLIAMSON, F.R.S. Received February 13, 1857.

(Abstract.)

The explanation of the movements of the Gyroscope (as well as its mathematical theory) is founded on the principle enunciated in the two following verbal formulæ.

I. When a particle is made to move $\left\{\begin{array}{l}\text{towards}\\ \text{from}\end{array}\right\}$ a plane by any applied force, but in consequence of its connexion with some rigid body on the same side of the plane, loses some of its momentum in a direction perpendicular to the plane; all the momentum so lost is imparted to the rigid body, which is consequently impelled $\left\{\begin{array}{l}\text{towards}\\ \text{from}\end{array}\right\}$ the plane.

II. When a particle is made to move $\left\{\begin{array}{l}\text{towards}\\ \text{from}\end{array}\right\}$ a plane by any applied force, but in consequence of its connexion with some rigid body on the same side of the plane, receives an extra momentum in a direction perpendicular to the plane; all the momentum so gained is taken from the rigid body, which is consequently impelled $\left\{\begin{array}{l}\text{from}\\ \text{towards}\end{array}\right\}$ the plane.

The mass of the disc of the gyroscope is supposed to be compressed uniformly into the circumference of a circle of given radius (r), and to revolve round an axis with a given uniform angular velocity (ω). To facilitate the arithmetical computation of the for-

mulæ, masses are represented by weights; so that any effective accelerating force f is supposed to be due to a pressure P acting on a mass W, and their relation expressed thus, $f=\frac{Pg}{W}$.

The mass of any arc of the circle is denoted by $\frac{cr\theta}{l}$; θ being the angle at the centre, and c the mass of a given length l of the circumference. The terms of all the formulæ are thus made homogeneous.

The centre of gravity of the disc, axle, and the ring which carries the pivots of the axle is fixed, and the whole is moveable about that centre in any manner, subject to the condition that the line of the pivots of the ring is always horizontal, unless when detached from the stand. Let this straight line of the pivots be denoted by AB, the common centre of the disc and ring by O, the extremities of the axle by N and S; and ON$=a$.

Let M denote the place of a particle of the disc, its position being determined by the angle AOM (θ), and let M$'$ be another point in the disc indefinitely near to M, but more remote from A, the direction in which the disc will presently be supposed to revolve being AMM$'$B.

A given force F is applied at N perpendicular to the plane ANBS, so that the disc may describe an angle ϕ round AB in the time t; whereby the points M and M$'$ describe the two arcs MP$=y$ and M$'$P$'=y'$ simultaneously. Suppose the circumference of the circle AMB to be divided into four quadrants, commencing at A, where $y=0$, and corresponding with the four ranges of value of θ through each of four right angles; suppose M and M$'$ to be in the first quadrant, so that y' is greater than y; then if the disc be supposed to revolve, a particle at M is carried from the line MP to the line M$'$P$'$, so as to acquire an increase of velocity from the plane AMM$'$ independently of the force F, and consequently (by the first of the two verbal formulæ) all the momentum so acquired by the particle is lost to the disc, ring, &c., which are thus impelled as by a force in the direction PM or P$'$M$'$, so as to oppose the rotation imparted by F, but to impart another round O in the direction ANB in the plane of the ring; *i. e.* in a plane perpendicular to that in which F acts. A force having the same tendency is found, by

means of one or the other of the two verbal formulæ, in the other three quadrants, and thus every particle (dm) of the disc contributes to the same effect. This effect is due to the difference of the velocities $\frac{dy}{dt}$ and $\frac{dy'}{dt}$ at P and P′, or to the momentum $\left(\frac{dy'}{dt}-\frac{dy}{dt}\right)dm$ lost or gained by the particle dm in the time dt.

The value of $\frac{dy}{dt}$ is obtained from the equation $y=r\phi\sin\theta$, making both ϕ and θ to vary; but the value of $\frac{dy'}{dt}$ is obtained from that of $\frac{dy}{dt}$ by making θ only to vary. It is thus shown that

$$\left(\frac{dy'}{dt}-\frac{dy}{dt}\right)dm=\left(\cos\theta\,.\,\frac{d\phi}{dt}-\omega\,.\,\phi\sin\theta\right)r\omega dt dm.$$

It is thence shown, by taking the moments about AB, and applying D'Alembert's principle, that

$$\left(\frac{d^2\phi}{dt^2}+\omega^2\phi\right)\int\sin^2\theta d\theta-\omega\frac{d\phi}{dt}\int\sin\theta\,.\,\cos\theta d\theta=\frac{\mathrm{F}ag}{cr^3},$$

the integrals applying to θ only, and between the limits O and 2π; *i. e.* to all the particles of the disc simultaneously and independently of ϕ or t. From this is obtained the result

$$\phi=\frac{4\mathrm{F}ag}{\mathrm{W}r^2\omega^2}\,.\,\sin^2\left(\frac{\omega t}{2}\right);$$

W being the weight of the disc.

This value being periodical, and ranging between the limits O and the maximum $\frac{4\mathrm{F}ag}{\mathrm{W}r^2\omega^2}$, shows that the disc makes oscillations which are of less extent and duration, as the spinning of the disc is more rapid; *i. e.* as ω^2 is made greater compared with $\frac{\mathrm{F}}{\mathrm{W}}$; and thus if F denotes a small weight (such as is usually supplied with the apparatus by the makers), the extent of the oscillation becomes insensible. This formula, applied to the apparatus with which the experiments were made, gives the theoretical maximum of ϕ about 18 minutes of a degree. It is evident that when F represents a weight, it should be replaced in the differential equation by F $\cos\phi$, but the result practically coincides with that actually obtained when F is not excessive.

That these oscillations must exist will be evident, when it is considered that the gyroscope, with the weight attached and the disc not spinning, becomes an ordinary pendulum: the effect of the spinning being to disturb its oscillations, and to lessen their extent to an unlimited amount, whenever the spinning of the disc is sufficiently rapid.

The preceding investigations, as well as the experiments, show that whenever a force is applied to the axis of a revolving disc, more or less of the momentum due to this force is converted into a momentum of rotation parallel to a plane which is perpendicular to that in which the force acts.

April 2, 1857.

The LORD WROTTESLEY, President, in the Chair.

The following communications were read:—

I. "Researches on Silica." By Colonel PHILIP YORKE, F.R.S. Received March 25, 1857.

(Abstract.)

This communication is principally devoted to an attempt to determine the formula of silica, and to the relation of some remarkable results obtained in this research. After giving some account of the grounds on which the three different formulas now in use among chemists (viz. SiO_3, SiO_2, and SiO) had been advocated, the author proceeds to state, that it appeared to him that the direct method which had been followed by Rose deserved the preference. This method consists in determining the quantity of carbonic acid which is displaced from excess of an alkaline carbonate in fusion, by a given