VI. "On Quantitative Measurement in Statical Electricity, and on some new Phenomena of Electrical Force." By Sir William Snow Harris, F.R.S. Received June 12, 1856.

(Abstract.)

The author observes, that number, weight, and measure are the foundation of all exact science, and that, as expressed by an eminent and learned writer (M. Quetelet), no branch of human knowledge can be held as being out of its infancy which does not in some way or the other frame its theories or correct its practice by reference to those elements; he was hence led to seek and establish such rigorous and exact quantitative processes in common electricity as would measure the quantity of electricity in operation; its attractive or repulsive force under given conditions, and its dynamical or current force when traversing bodies under the form of electrical discharge. The instruments which he has invented for this purpose have been all honoured by a place in the 'Philosophical Transactions of the Royal Society.' They amount to five in number, viz. the Unit Measure, the Balance and Hydrostatic Electrometers, the Thermo-electrometer, and the Bifilar Balance. In referring to such of these instruments as are employed in the present research, the author briefly adverts to their general construction, including the latest and best form under which they have been placed.

In the measurement of quantity, he considers the unit measure as being the best and most accurate means of estimating quantity as yet arrived at; and he describes a series of crucial experiments, the object of which is to show that the unit explosions are rigorously exact. If an electrical jar exposing about 5 square feet of coated surface be insulated, and a second equal and similar jar be so placed as to charge from its outer coating, and if the first jar be charged from the conductor of the machine through the unit measure, it is found by a Lane's discharging electrometer attached to each jar, that an equal number of measures are given off from the outer coating of the insulated jar at all periods of the progress of the charge. Thus, whether the first jar be charged with 20, or 40, or 60 measures,

it still evolves from its outer coating the same number of measures for each unit of quantity as it did at first; and conversely, the second jar receives as easily the unit of quantity taken in terms of the unit explosions when charged with 20, 40, or 60 measures, as it did at first. The author concludes, that the charging of an electrical jar is by a rough analogy rather to be associated with the pouring of an inelastic fluid such as water into an open vessel in measured quantities, which is done up to the point of overflow as easily at last as at the first.

Having given experimental illustrations of the nature of the several instruments just adverted to, and shown their accuracy as instruments of research, the paper proceeds to consider the phenomena of what the author, after the learned Mr. Cavendish, denominates electrical charge. By the term electrical charge of a given conducting substance, the author understands the quantity of electricity which the body can sustain under a given degree of the electrometer. In pursuing this interesting question, he commences with an examination of the charges of hollow spheres or globes of different diameters. The method of experiment is to place the given sphere in communication with the electrometer, and find by a transfer of measured quantities of electricity the precise number of measures required to bring the index to a given degree of the arc. These measures are obtained by insulated balls or plates of given dimensions, brought into contact with the ball of an insulated charged jar carefully prepared and screened from the external air. The author shows how this method of measuring quantity by means of what he terms a quantity-jar may be perfected, so as to be relied on as a means of estimating small quantities of electricity.

The results of a series of experiments with spheres and plates of equal area led to the deduction, that the charges of these bodies are as the square roots of the surfaces multiplied into the circumferences, and that the charge of a sphere is to the charge of a circular plate of equal surface as $1 : \sqrt{2}$, and the charge of a great circle of a sphere is to the charge of the sphere as $1 : \sqrt{4}$, or $1 : 2$.

Taking a given surface of 100 square inches, and placing it under various forms, viz. a sphere, circular plate, square plate, rectangular plates of variable extension, a hollow open cylinder, a cube, &c., and subjecting these to the same process of experiment by which is

measured the quantity of electricity which each can sustain under a given degree of the electrometer or what the author calls intensity, he deduces the following:—

1. The charges of spheres and circular planes, as also of plane rectangular plates, are as the square roots of these surfaces multiplied into their circumferences or perimeters.

2. The charge of a cylinder is as the square root of its surface multiplied into the sum of its length and circumference.

3. The charge of a cube is as the square root of its surface multiplied into twice its side.

4. The charge of a sphere is to the charge of one of its great circles as 2 : 1.

5. The charge of a sphere is to the charge of a plane circle of equal surface as 1 : $\sqrt{2}$, or as 1 : 1·41.

6. The charge of a cube is to the charge of a sphere of equal surface as 1 : 1·47 nearly.

7. The charge of a square plate is to the charge of a cube whose side equals the side of the square as 1 : 1·6 nearly.

8. The charge of a circular plate is to the charge of a square plate whose side equals the diameter of the circle as 1 : 1·28.

The author examines the charges of cylindrical rods or tubes of small diameter, and finds their capacity to be nearly as the length, the surface being constant; being quite in accordance with the result arrived at by Volta, who found that an insulated conductor composed of gilded rods could receive under the same intensity as much electricity as would produce a shock equal to a given extent of coated glass.

In referring to the beautiful experiments of Coulomb, the author conceives that the sharing of electricity between a circular plate and sphere of equal area, in proportion to the two surfaces of the plate to the one exterior surface of the sphere, is a different thing from the absolute charging of the plate on two surfaces, and adduces experiments to show that when a circular plate charges on both its surfaces, it takes up twice the quantity of electricity under the same intensity, which a plane circular plate in respect of a sphere of equal area does not; he conceives the sharing of electricity between a sphere and circular plate of equal area to be a pure result of the inductive susceptibility of the plate in consequence of the free exposure of its

entire surface; he further gives some new experiments on induction, with a view of proving that when one surface is opposed as it were to itself, as in the case of the interior surface of a sphere, the inductive susceptibility of one-half the surface is reduced to zero. The phenomena of what the author calls, after Cavendish, electrical charge, he refers to some peculiar arrangement or disposition of the electrical particles on the surfaces of the several conductors, by which they exhibit a greater or less degree of excitement, as observable by the electrometer.

The remaining portion of this paper is devoted to the laws and phenomena of electrical attractive force. The attractive force of a given surface under a given charge does not depend on the quantity of electricity, but on the number of attracting points called into operation by what is usually considered as the attracted body. Two circular discs of very light wood, of 5 inches diameter, being carefully prepared in a lathe, were divided into six concentric rings, including a central plate of about an inch in diameter. The attractive force on each pair of rings was dètermined by means of the electrical balance and carefully noted; the force was as the several opposed areas; and when the series was combined into one plate, the force was the sum of the forces of the respective rings; when the attractive forces of circular plates equal in area to the several rings respectively were examined, the force was the same as that exhibited by the two rings whose area was the same; hence it is inferred that whether the charge operates from the circumference or near the centre, the attractive force is the same. Two rings combined exhibit forces equal to the sum of the forces taken separately; and when the force is examined between the plates or the several rings and a plane circular area of large and continuous surface, the forces are no greater than that between two plates or rings of equal area. When the distances between the attracting surfaces or the quantities of the electrical accumulation varied, then the force was as the square of the accumulation directly, and as the square of the distances inversely.

The author extends these experiments to spheres of different diameters. He had shown in a former paper, that, taking the attractive force to be as the areas directly, and as the squares of the distances inversely, according to the expression $F \propto \frac{A}{D^2}$, two points

might be determined within the hemispheres in which all the force may be conceived to be collected, and to be the same as if proceeding from every point of the hemisphere. If Z=the distance of either point, $q\,q'$ taken within the hemisphere, r=radius, and a=distance between the near or what may be termed the touching points of the spheres, then we have $Z=\frac{(2ar+a^2)^{\frac{1}{2}}-a}{2}$; and if A=the surface, we have $F \propto \frac{A}{(a+2z)^2}$. When both hemispheres are equal, and distance $=a$ variable, then we have also $F \propto \frac{1}{a(a+2r)}$. The author in a former paper had applied these formulæ to the limited induction of a sphere of an inch radius; he now extends the inquiry to spheres varying from an inch to 5 inches or more in diameter, and finds the results conformable to the formulæ. He gives a table containing the results of a series of experiments with four spheres whose areas regularly increased, and the radii of which were from 1 to 2 inches in diameter. These were examined by the electrical balance. They were first placed with the points $q\,q'$, or centres of force as calculated for each at a constant distance of 1·1 of an inch, in which case the weights requisite to balance the force with a given number of measures of electricity were as the opposed areas, thus confirming the preceding results deduced with plane surfaces; when the distances were varied, the force varied as the squares of the distances between the centres of force, or according to the formula $F \propto \frac{1}{a(a+2r)}$.

With the view of further verifying these results, a set of plane circular plates in pairs, each pair equal in area to the areas of the respective hemispheres of the spheres, were submitted to experiment at the same constant distance 1·1, so as to cut the points $q\,q'$, or centres of force of the spheres; the attractive forces were found precisely the same as that of the opposed spheres to which the particular plates belonged.

The author has examined at various times and with very rigid attention, the several conditions under which electrical attractive force conforms to the law of force as deduced by Cavendish and Coulomb, and other eminent philosophers, and he finds this law true only for charged and neutral conductors of large inductive capacity; if either of the attracting surfaces have a narrow or limited susceptibility of

inductive charge, then this law of force no longer obtains. If, for example, the attracting plates be taken as mere planes of small thickness, and even although they be charged with opposite electricity, still in changing the distances between them we do not obtain at all distances a law of force in the inverse duplicate ratio of the distance such as we have found to obtain in other circumstances. The force will be commonly in an inverse simple ratio of the distance. If the neutral or suspended plane be taken very thin and insulated, then little or no attractive force is observable under any circumstances. If we continue to increase its thickness, then, as the author has shown in former papers*, attractive force begins to display itself, and will approach a law of change in the inverse duplicate ratio of the distance as we extend its dimensions. When we give it unlimited electrical extension by placing it in communication with the ground, then the force is as the square of the distance inversely; but it is not always so, until we effect this extension perfectly. When all these sources of disturbance are duly considered, it will not be difficult to reconcile the many conflicting results arrived at by several eminent philosophers in past times, in their endeavours to investigate the law of electrical force, and explain how, without any defect in their experimental processes, such conflicting results might arise. Volta, for example, found electrical force to vary in a simple inverse ratio of the distance. M. Simon, of Berlin, an eminent philosopher, and eulogized by Gilbert as being "remarkable for his dexterity and careful manipulation" in this branch of physics, failed to verify Coulomb's result, although he employed a new and very delicate apparatus, by which the repulsive force between two spheres was very accurately and beautifully measured. In these experiments he found the force to vary as the distance inversely†.

* Phil. Trans. for 1834.

† Poggend. Annal. for 1808, cap. 3, p. 277, and Ann. de Chim. vol. lxix. First Series.