such that if a strain of any one of those types be impressed on the body, the elastic reaction is balanced by a stress orthogonal to the five others of the same system.

It is next shown that there is necessarily one, and in general only one, such system of six types of strain for an elastic solid which are all mutually orthogonal; and the types belonging to this system are called the Six Principal Strain Types of the body.

The characteristic of a Principal Strain Type is, that the *stress* required to keep a body in a state of strain of such a type, *is of the same type as the strain.* The six Principal Elasticities of a body are the six coefficients by which strains of the six Principal Types must be multiplied to find the stress required to maintain them.

In conclusion, reasons are given for believing that natural crystals may exist for which there are six unequal Principal Elasticities, and consequently six different, and only six different, Principal Strain-types.

A corollary regarding the property which certain liquids and crystals possess of causing a rotation in the plane of polarization of light passing through them, and Faraday's optical property of transparent bodies under magnetic force, is inferred, and is more fully considered in a subsequent communication to the Royal Society.

II. "On the Construction of the Imperial Standard Pound, and its Copies of Platinum; and on the comparison of the Imperial Standard Pound with the Kilogramme des Archives." By W. H. Miller, M.A., F.R.S., Professor of Mineralogy in the University of Cambridge.—Part I. Received April 16, 1856.

(Abstract.)

The Commissioners appointed in 1838 to consider the steps to be taken for the restoration of the standards of weight and measure, to replace those which were destroyed by the burning of the Houses of Parliament, found provisions for the restoration of the lost standards prescribed to them by Sections 3 and 5 of the Act 5th George IV., whereby it is directed that, in case of the loss of the standards, the yard shall be restored by taking the length which shall bear a certain

relation to the length of the pendulum, vibrating seconds of mean time, in a vacuum, at the level of the sea; and that the pound shall be restored by taking the weight which bears a certain proportion to the weight of a cubic inch of water weighed in a certain manner. The Commissioners, however, in their Report dated December 21, 1841, decline to recommend the adoption of these provisions for the following reasons: "Since the passing of the said Act it has been ascertained that several elements of reduction of the pendulum experiment therein referred to are doubtful or erroneous. It is evident, therefore, that the course prescribed by the Act would not necessarily reproduce the length of the original yard. It appears also that the determination of the weight of a cubic inch of water is yet doubtful (the greatest difference between the best English, French, Austrian, Swedish and Russian determinations being about $\frac{1}{1200}$ of the whole weight, whereas the mere operation of weighing may be performed to the accuracy of $\frac{1}{1,000,000}$ of the whole weight). Several measures, however, exist, which were most carefully compared with the former standard yard; and several metallic weights exist which were most accurately compared with the former standard pound; and by the use of these the values of the original standards can be respectively restored without sensible error. And we are fully persuaded that, with reasonable precautions, it will always be possible to provide for the accurate restoration of standards by means of material copies which have been carefully compared with them, more securely than by experiments referring to natural constants."

At the end of the Travaux de la Commission pour fixer les Mesures et les Poids de l'Empire de Russie, Professor Kupffer has collected the results of observations made in France, England, Sweden, Austria and Russia for finding the weight of a given volume of water. The resulting values of the weight of an English cubic inch of water in a vacuum at 62° Fahr., expressed in doli, of which 22504·86 make a kilogramme, are as follows:—

French observations	368·365
English observations...............	368·542
Swedish observations	368·474
Austrian observations	368·237
Russian observations...............	368·361

Assuming the Russian observations to be the best, as they probably are, it will be seen that a troy pound deduced according to the method prescribed by the Act, would be 2·829 grains too heavy; while, if the Austrian observations had been accepted as the best, the troy pound would have been 4·707 grains too heavy. On the other hand, it was possible to recover the weight of the lost standard in air to within a fraction of 0·001 grain, by means of the troy pounds which had been compared with it, and could be easily brought together for recomparison. Seeing, then, that the error of one of these two methods of restoring the lost standard, is at least 2829 times as large as the error of the other method, the Committee could not hesitate to recommend the adoption of the latter.

A Committee was appointed by a Treasury Minute of June 20, 1843, to carry out the recommendations contained in the Report referred to above. The evidence for ascertaining the weight of the lost standard, placed at the service of this Committee, consisted of the following weights:—The brass troy pounds of the Exchequer Office; the brass troy pounds from the cities of London, Edinburgh and Dublin; the platinum troy pound and the two brass troy pounds then in the possession of Professor Schumacher; the platinum troy pound of the Royal Society; the troy pound used by the late Mr. Robinson of Devonshire Street, purchased by the Committee; four troy pounds made in 1758, two of which were formerly in the possession of Mr. Bingley of the Royal Mint, one the property of Messrs. Vandome and Titford, and one the property of the Bank of England.

The troy pounds of the Exchequer, and of the cities of London, Edinburgh and Dublin had been compared with the lost standard by Captain Kater in 1824. The three troy pounds in the custody of Professor Schumacher, and the troy pound of the Royal Society, were compared with the lost standard with extraordinary care in 1829 by the late Captain v. Nehus. The troy pounds bearing the date 1758 were constructed, along with the lost standard, by Mr. Harris, Assay Master of the Mint. These were referred to at the suggestion of Professor Schumacher, in the hope of arriving at a knowledge of the volume of the lost standard, which, unfortunately, had never been determined by weighing it in water. For, as long

as the volume of the lost standard remains unknown, the weight of the air displaced by it, and, consequently, its absolute weight, is uncertain within limits far exceeding the errors of weighing.

Let U denote the lost standard; Ex, L, Ed, D, RM the troy pounds of the Exchequer, the cities of London, Edinburgh, Dublin, and the Royal Mint, respectively; Sb, K two brass troy pounds, Sp a platinum troy pound, all in the custody of Professor Schumacher; RS the platinum troy pound of the Royal Society. Let Δ prefixed to the symbol by which any weight is designated denote the ratio of the density of the weight at the freezing-point to the maximum density of water; t the temperature of the air in degrees of Fahrenheit's scale; b the height of the mercury in the barometer in inches reduced to the freezing-point. The symbol $\triangleq$ placed between the symbols of two weights will be used to denote that they appear to be equal when weighed in air. The two weights in this case will not be equal unless their volumes are equal. When the weighings have been made in air of given density, or have been reduced to what they would have been in air of given density; or, when the volumes of the weights, the temperatures and pressures of the air being unknown, we are compelled to assume that their volumes are equal, the symbol = may be substituted for $\triangleq$.

By the observations of Captain Kater (Philosophical Transactions, 1826),—

$$\begin{aligned} Ex &= U + 0\cdot0010 \\ L &= U + 0\cdot0005 \\ Ed &= U + 0\cdot0015 \\ D &= U + 0\cdot0022 \\ RM &= U + 0\cdot0021 \end{aligned}$$

By the observations of Captain v. Nehus in 1829—

No. of obs.		b.	t.
300	Sp $\triangleq U - 0\cdot00857$	29·722	65·62
140	RS $\triangleq U - 0\cdot00205$	29·806	65·73
60	Sb $\triangleq U - 0\cdot01034$	29·965	64·50
92	K $\triangleq U + 0\cdot03389$	29·646	65·09
16	RM $\triangleq U + 0\cdot00887$	29·679	65·91

$$10 - \log \Delta\, Sp = 8\cdot67392, \quad 10 - \log \Delta\, RS = 8\cdot67392,$$
$$10 - \log \Delta\, Sb = 9\cdot08471, \quad 10 - \log \Delta\, K = 9\cdot09724.$$

These weights were afterwards compared with each other with a balance of extreme delicacy procured from Mr. Barrow. In its construction it nearly resembled the balances of the late Mr. T. C. Robinson. The beam is made sufficiently strong to carry a kilogramme in each pan. Instead of having an index pointing downwards, as is usual in balances of this description, a thin slip of ivory is affixed to one end of the beam, a little more than half an inch long, divided into spaces of about 0·01 inch each. This scale is viewed through a compound microscope having a single horizontal wire in the focus of the eye-piece. A screen was interposed between the observer and the front of the balance-case, having a very small opening opposite to the eye-piece of the microscope.

In making a large number of comparisons, the weights compared are exposed to the risk of being injured by wear. In order to obviate this danger, two light pans were used of very nearly equal weight, each of which has a loop of wire forming an arch, the ends of which are attached to the pan at opposite extremities of a diameter of the pan. To the upper point of the loop of wire is affixed an iron hook. Each pan is suspended by a wire of suitable length bent into a hook at either end, from the ring attached to the agate plane resting on the knife-edge at either end of the balance.

Calling the weights of the pans X and Y, and the weights to be compared P and Q, P was placed in X and Q in Y, and $P+X$ compared with $Q+Y$ n times; then P was placed in Y and Q in X, and $P+Y$ compared n times with $Q+X$. The weights were thus exposed to the wear of two ordinary comparisons only in the course of $2n$ comparisons. The mean of the $2n$ comparisons gives the difference between P and Q unaffected by the very small, but unknown difference between X and Y. This contrivance was found to be especially useful when either of the weights to be compared consisted of several parts.

In using the method of double-weighing, the counterpoise was placed in the left-hand pan of the balance, and the detached pan X containing the weight P, and the detached pan Y containing the weight Q, were alternately suspended from the right-hand end of the beam, and the positions of equilibrium deduced in each case from the extreme positions of the beam at the beginning of each of three consecutive oscillations (usually twenty times). The weights were then

interchanged, and the pan Y containing the weight P, and the pan X containing the weight Q, suspended alternately from the right-hand end of the beam the same number of times.

In weighing by Gauss's method, in which the two weights to be compared as suspended from the right and left-hand ends of the beam respectively, and are then interchanged, it was desirable to be able to transfer the pans and the weights contained in them from one end of the beam to the other, without opening the doors of the balance-case, and thus avoid sudden changes of temperature of air in the balance-case, and consequent production of currents of air. In order to effect this, a slender brass tube 38 inches long was made to pass freely through two holes in the ends of the balance-case, which is nearly 23 inches long, near the top of the case and half-way between the balance and the front of the case. To the middle of the tube is attached a depending loop of wire. Suppose that by sliding the tube the loop is brought near to the right-hand end of the beam, and the pan with a weight in it transferred from the end of the beam to the wire loop by a brass rod having a hook at the end, which is inserted through a hole in the right-hand end of the balance-case. By sliding the tube in the opposite direction, the loop with the pan and weight suspended from it, is brought near to the left-hand end of the beam, to which it is transferred by a brass rod having a hook at the end, passing through a hole in the left-hand end of the balance-case. A similar tube halfway between the balance and the back of the case, serves to transfer the other pan and weight from one end of the beam to the other. In this manner any number of comparisons may be made without opening the balance-case, except in the middle of the series, for the purpose of changing the pans.

A sufficient number of preliminary comparisons of Sp, RS, Sb, K, Ex, L, Ed having been made in 1844, the results were reduced, when the material of one weight was platinum and that of the other brass, to what they would have been in air ($t=65{\cdot}66$, $b=29{\cdot}75$), or, of the mean density of the air during the comparisons of Sp and RS with U in 1829. Using U, Sp, RS, &c. to denote the apparent weights of U, RS, &c. in air ($t=65{\cdot}66$, $b=29{\cdot}75$), it was found that—

In 1829.	In 1844.
gr.	gr.
RS=Sp +0·0051	RS=Sp +0·0057
Sp =Sb +0·0022	Sp =Sb +0·0030
RS=Sb +0·0073	RS=Sb +0·0032
K =Sp +0·0420	K =Sp +0·0362
K =RS+0·0369	K =RS+0·0304
K =Sb +0·0442	K =Sb +0·0317

In the interval between 1829 and 1844, the difference between the two platinum troy pounds Sp and RS had undergone no very sensible relative change. If, as appears probable, Sp and RS have undergone no sensible absolute change, Sb has gained 0·0046 grain, and K has lost 0·0061 grain. On the same supposition it appears that—

In 1824.	In 1844.	Increase of	Interval in years.
gr.	gr.	gr.	
Ex −U=+0·0010	Ex −U=+0·0099	Ex 0·0089	20
L −U=+0·0005	L −U=+0·0151	L 0·0146	20
Ed −U=−0·0015	Ed −U=+0·0206	Ed 0·0221	20
D −U=+0·0022	D −U=+0·0248	D 0·0226	20
RM−U=+0·0021	(1829) RM−U=+0·0089	RM 0·0068	5

With the single exception of K, all the brass weights have become heavier since they were compared with U, in consequence probably of the oxidation of their surfaces, while U, which was made in 1758, was protected from further change by the coat of oxide already formed. One of these weights, Sb, appeared to have been protected by gilding, though imperfectly, since parts of its surface were slightly tarnished. Ex and L were brighter than Ed and D. K, though it had become lighter, was much tarnished. The discordances presented by the different weighings of K appear to have greatly perplexed both Professor Schumacher and Captain Kater, and were probably the cause of the numerous and accurate comparisons of the several troy pounds placed at the disposal of the Committee with the lost standard, on which alone depends the possibility of restoring it with sufficient accuracy. Previous to the comparison of K in 1844, a small fragment of wood, like a grain of coarse sawdust, was found adhering so firmly to its under surface, that it was detached with some difficulty. It appears probable that the changes of the weight

of K were caused by this bit of wood being weighed with it after the first comparison of K by Captain Kater, and by the gradual oxidation of the surface of K. The discrepancies presented by the weighings of the brass troy pounds at different times, due to the effect of oxidation or other causes, are so large, that I resolved, with the consent of the Astronomer Royal, to rest for the evidence of the weight of the lost standard entirely on the 300 comparisons of Sp and the 140 comparisons of RS with U.

If we consider the discordances presented by the weighings of the brass troy pounds simply as errors of observation, without paying any regard to their probable causes, the resulting value of U will not be very different from that given by the platinum troy pounds alone.

By the observations of 1824 and 1829,

	gr.	weight.
U=Sp	+0·0081	30
U=RS	+0·0030	14
U=Sb	+0·0103	6
U=K	−0·0339	9
U=$\frac{1}{4}$(Ex+L+Ed+D)	−0·0022	6

By the observations of 1844,

		gr.
RS	=Sp	+0·0057
Sb	=Sp	+0·0030
K	=Sp	+0·0363
Ex+L+Ed+D	=2(Sb+K)	+0·0260

Whence, supposing the errors of weighing in 1844 to be insensible, compared with the discordances of the brass troy pounds,

	gr.	weight.
(1)	U=Sp+0·0081	30
(2)	U=Sp+0·0087	14
(3)	U=Sp+0·0133	6
(4)	U=Sp+0·0024	9
(5)	U=Sp+0·0261	6

The mean of all the equations gives U=Sp+0·0096 grain.

Excluding the last, which depends upon the weighings in 1824, U=Sp+0·0079 grain.

Excluding all except the results of the comparisons of U with the two platinum troy pounds, U=Sp+0·0083 grain.

The temperatures were determined by means of three thermometers by Bunten, having centesimal scales etched upon the tube, and two thermometers having arbitrary scales traced upon the tubes with a diamond point. The zero-points of these were determined at distant intervals. They were often compared with each other, and, lastly, with an excellent standard thermometer constructed at Kew under the directions of Mr. Welsh, in order to form tables of the errors at any point of their scales, and to determine the position of their zeros at any given time. The barometer employed was a portable cistern barometer by Ernst of Paris, the scale of which was divided into millimetres. It was compared first with the standard barometer of the Paris Observatory, and afterwards with a standard barometer, having a tube of very large bore, belonging to the Taylor Library of Sidney Sussex College, Cambridge.

According to Ritter (Mémoires de la Société de Physique de Genève, t. iii. p. 361), the observations of Regnault show that in Paris, lat. 48° 50′ 14″, 60 metres above the mean level of the sea, a litre of dry atmospheric air, containing the average amount, 0·0004 of its volume, of carbonic acid, the density of which is 1·529 of that of atmospheric air at 0° Cent., under the pressure of 760 mm. of mercury at 0° Cent., weighs 1·2934963 gramme. If G be taken to denote the force of gravity at the mean level of the sea in lat. 45°, the force of gravity in lat. λ, at the mean level of the sea, $=G(1-0{\cdot}0025659\cos 2\lambda)$ (Baily, Mem. Ast. Soc. vol. vii. p. 94). The force of gravity in a given latitude at a place on the surface of the earth at the height z above the mean level of the sea $=\left\{1-\left(2-\frac{3}{2}\frac{\rho'}{\rho}\right)\frac{z}{r}\right\}\times$ force of gravity at the level of the sea in the same latitude, where r is the radius of the earth, ρ its mean density, and ρ' the density of that part of the earth which is above the mean level of the sea (Poisson, Traité de Mécanique, t. ii. p. 629).

According to Regnault, the expansion of air under constant pressure from 0° to 100 Cent., is 0·36706 of its volume at 0° Cent.; also at 50° Cent., the mercurial thermometer is about 0°·2 in advance of the air thermometer (Mémoires de l'Institut, t. xxi. p. 91. p. 238, Annales de Chimie, 3 série, t. v. p. 99). Hence, density air at

0° : density air at $t = 1 + 0{\cdot}003656t$. The density of the vapour of water is 0·622 of that of air. Hence, if t be the temperature of the air in centesimal degrees, b its barometric pressure, v the pressure of vapour, both in millimetres of mercury at 0° Cent., the weight in grammes of a litre of air at a place on the surface of the earth at a height z above the mean level of the sea in lat. λ, will be

$$\frac{1{\cdot}2930693}{1+0{\cdot}003656t}\;\frac{b-0{\cdot}378v}{760}\left(1-1{\cdot}32\frac{z}{r}\right)(1-0{\cdot}0025659\cos 2\lambda).$$

Regnault finds that in rooms not heated artificially, the pressure of vapour is two-thirds of the maximum pressure corresponding to the temperature (Memorie della Società Italiana della Scienze in Modena, t. xxv. p. 1).

The weight of air used in reducing the weighings was calculated from the above expression.

The mean rate of expansion of brass, for 1° Cent., from 0° Cent. to 100° Cent., usually assumed 0·0000187 of its length at 0° Cent., is considerably larger than the rate of expansion at ordinary atmospheric temperatures, according to the observations of Mr. Sheepshanks, who found that at about 17° Cent. the coefficient of the linear expansion of brass $= 0{\cdot}00001722$ for 1° Cent. This value of the expansion has been accordingly adopted.

The linear expansion of platinum is assumed to be 0·00000900 for 1° Cent., as given by Schumacher in his first table (Phil. Trans. 1836). The expansion of water is calculated from a mean of the experiments of Despretz, I. Pierre and Kopp, corrected for the error of the assumed expansion of mercury by Regnault's observations, and assuming the temperature of maximum density to be 3°·945 Cent., in accordance with the result obtained by Messrs. Playfair and Joule. The logarithms of the expansion to 7 places considered as integers, are given with sufficient accuracy, between 4° Cent. and 25° Cent., by $32{\cdot}72(t-3{\cdot}945)^2 - 0{\cdot}215(t-3{\cdot}945)^3$.

Though it appears that only two of the nine weights with which U was compared in 1826 and 1829 are in a state of unexceptionable preservation, and that the number of trustworthy comparisons is reduced from 669 to 440, these are amply sufficient for the purpose of ascertaining the weight of U in air ($t = 65^\circ{\cdot}66$ Fahr., $b = 29{\cdot}75$ inches). But in order to find the absolute weight of U, or indeed

its apparent weight in air of a density different from that which it has when $t=65°·66$ Fahr., $b=29·75$ inches, a knowledge of the volume of the lost standard is requisite. An indirect way of arriving at it was suggested by Professor Schumacher, by an examination of certain Parliamentary Reports, presented May 26, 1758, April 11, 1759, March 2, 1824. It appears from the first of these, that Mr. Harris, then Assay Master of the Mint, presented to the Committee three troy pounds made under his direction, one of which was the lost Imperial standard; and from the third, that one of the two remaining pounds came into the possession of Mr. Vandome, and the second into the possession of Mr. Bingley of the Mint. Professor Schumacher then observes that we can still either determine, with the highest degree of probability, the density of the lost Imperial standard, or know with certainty that all hope to arrive at this knowledge is lost. It will be only requisite to ascertain with the greatest care the densities of both pounds, the one in the possession of Mr. Bingley, the other in the possession of Mr. Vandome. If the density of both is found the *same*, we might from that circumstance draw the highly probable conclusion, that the three single pounds of Mr. Harris, according to my hypothesis, were really made of the same identical metal; and the density of the two remaining pounds might with safety be considered as that of the lost standard. If, on the contrary, the two remaining pounds prove to be of *different* densities, the hypothesis that all three were made of the same metal is evidently erroneous; and nothing can be inferred from the density of either of the two remaining.

Mr. Vandome readily consented to allow his troy pound to be experimented upon by the Committee. Denoting this weight by the letter V, by weighing in air and in water it was found that $\Delta V=8·15084$, and that it was about 0·309 grain lighter than U.

Mr. Bingley had in his possession two troy pounds, both dated 1758. One of these, O, said to be the original weight from which the standard was made for the House of Commons in 1758, has since been purchased by the Committee; the other, M, has been presented to the Mint by Mr. Bingley. As Mr. Bingley was unwilling to permit either of these weights to be weighed in water, Messrs. Troughton and Simms were commissioned to construct an instrument on the principle of the Stereometer invented by M. Say for the

purpose of finding the density of gunpowder (Ann. de Chimie, 1797, t. xxiii. p. 1), but with some improvements which I had described in the Philosophical Magazine for July and December, 1834, vol. v. p. 203. Let v prefixed to the symbol of any weight denote the volume of that weight at 0° Cent., the unit of volume being the volume of a grain of water at its maximum density. Then, by means of the Stereometer, it was found that $vV - vO = 22{\cdot}68$, $vV - vM = 17{\cdot}38$. These differences show that the volume of lost standard cannot be inferred with any high degree of probability from a comparison of the remaining pounds. The only resource now remaining was indicated by Professor Schumacher's remarks on the figure of the lost standard:—"As soon as the Imperial standard troy pound was brought to Somerset House, Captain Nehus's first care was to make an accurate drawing of its shape and marks, measuring all its dimensions with the greatest care. The annexed drawing represents this pound in its actual dimensions; and is now, since the original has been destroyed by the calamitous fire that consumed the two Houses of Parliament in 1834, the only thing remaining which can preserve an idea of it." By a comparison of the figure of U in the Philosophical Transactions for 1836, with a profile of V traced mechanically, the axis and the extreme diameter of the knob and cylindrical portion of U, appeared to be a very little greater than the corresponding dimensions of V. On comparing the profiles of U and V, it did not seem possible to suppose that the volume of U was less than that of V. But the volume of O, as well as that of M, being less than that of V, it appeared that of the three weights V, O, M, V approximated most nearly to U in volume. As the existing data were utterly insufficient to determine how much, if at all, U exceeded V in volume, it appeared safest to assume the volumes of U and V to have been equal. This course was also recommended by Professor Schumacher.

It was afterwards found that O was 0·144 grain lighter than U, $\Delta O = 8{\cdot}4004$; and that M was 0·047 grain lighter than U, $\Delta M = 8{\cdot}3491$.

In a letter from William Miller, Esq., of the Bank of England, dated August 22, 1855, I was apprised of the existence of a fourth troy pound of 1758. This weight was 0·249 grain heavier than U; its density $= 8{\cdot}3175$.

If U, the lost standard, be supposed to have the same density as V, the comparisons of Sp and RS with U by Captain v. Nehus in 1829, give,—

$$Sp = U - 0{\cdot}52959$$
$$RS = U - 0{\cdot}52444.$$

The Commissioners for the Restoration of the Standards of Weight and Measure, in their Report, dated December 21, 1841, recommended that the avoirdupois pound of 7000 grains be adopted instead of the troy pound of 5760 grains, as the New Parliamentary Standard of Weight, and that the new standard and four copies of it be constructed of platinum.

In accordance with this recommendation, five weights were made by Mr. Barrow, a little in excess of 7000 grains, of platinum prepared by Messrs. Johnson and Cock. The form of these pounds is that of a cylinder, nearly 1·32 inch in height and 1·15 inch in diameter, with a groove round it, the middle of which is about 0·34 inch below the top of the cylinder, for insertion of the prongs of a forked lifter of ivory. They are marked PS 1844 1 lb.; PC No. 1 1844 1 lb.; P CNo. 2 1844 1 lb.; PC No. 3 1844 1 lb.; PC No. 4 1844 1 lb., respectively.

The weights of 7000 grains might have been derived from that of 5760 grains, by the use of either a decimal or a binary system of weights. In either case, however, the number of weights to be compared with one another and with the weights of 7000 and 5760 grains would have been large, and the errors of their comparisons among themselves might, by their accumulation, sensibly affect the resulting weight of 7000 grains. Also, the repeated comparison of weights made up of the sum of several others, was a very troublesome process, previous to the use of the detached pans, already described, which had not been thought of when the weights were ordered.

These two evils were in a great measure avoided by the use of a platinum weight T of about 5760 grains, or more correctly very nearly equal to Sp or RS, and of the following auxiliary weights, also of platinum, and all constructed by Mr. Barrow: A, B, C, D each of 1240 grains; F of 800 grains; G of 440 grains; H of 360 grains; K, L, M, N each of 80 grains; R, S each of 40 grains, nearly. The numbers of the weights of each denomination, and their values, are given by the quotients and divisors obtained in the conversion of

$\frac{7000}{5760}$ into a continued fraction. The errors of these weights are found by the following comparisons:—Sp and RS with T; T with A+B+C+D+F; each of the weights A, B, C, D with F+G; F with G+H; G with each of the weights H+K, H+L, H+M, H+N; H with K+L+M+N+R and K+L+M+N+S; each of the weights K, L, M, N with R+S.

Sp and RS, instead of being true troy pounds, and, consequently, equal to U in a vacuum, had been adjusted so as to appear nearly as heavy as U when weighed in air of ordinary density, and are therefore lighter than U by about 0·53 grain, the weight of the air contained in the space equal to the difference between the volume of U and that of Sp or RS. A space equal to the difference between the volume of 7000 grains of metal of the assumed density of U, and 7000 grains of platinum, contains about 0·645 grain of air. Calling this Q, PS may be compared with each of the weights T+A+Q, T+B+Q, T+C+Q, T+D+Q. In order to determine Q with the greatest precision, Mr. Barrow supplied ten weights Q of about 0·645 grain each, so accurately adjusted that no appreciable difference could be detected between them; a weight V of 6·451 grains, and a weight W of 12·901 grains, all of platinum. Then Y and Z being two platinum weights of 20 grains each, the following comparisons became possible:—each of the weights R and S with Y+Z; each of the weights Y and Z with W+V+ each of the weights Q in turn; W with V+ sum of ten weights Q; V with the sum of the ten weights Q. In comparing PS with each of the weights T+A+Q, T+B+Q, T+C+Q, T+D+Q, the weight Q was changed at the end of every four comparisons, and thus each of the ten weights Q used in turn in a series of forty comparisons.

By numerous weighings in air and in water the densities of the several weights were found to be as follows:—

T	21·1661
PS	21·1572
PC No. 1	21·1671
PC No. 2	21·1640
PC No. 3	21·1615
PC No. 4	21·1556

By 286 comparisons of T with Sp and 122 comparisons of T with

RS, assuming the density of U to have been the same as that of V, T=5759·47141 grains, of which U contained 5760. By numerous comparisons of the auxiliary weights with each other and with T, A=1239·88621, B=1239·88604, C=1239·88596, D=1239·88579, Q=0·64509.

By 80 comparisons of PS with each of the weights T+Q+A, T+Q+B, T+Q+C, and 100 of PS with T+Q+D,

	gr.	*t.*	*b.*
PS≏	T+Q+A−0·002936	19·47	758·38
PS≏	T+Q+B−0·001731	19·19	759·31
PS≏	T+Q+C−0·001621	18·83	754·38
PS≏	T+Q+D−0·000774	19·63	764·43
Mean PS≏	T+Q+$\frac{1}{4}$(A+B+C+D)−0·00177	19·28	759·12

whence, supposing U to have the same density as V,

PS=7000·00090 grains, of which U contained 5760.

Results of comparisons of PC No. 1, PC No. 2, PC No. 3, PC No. 4 with PS:—

gr.	No. of Comparisons.
PC No. 1=PS+0·00051	200
PC No. 2=PS−0·00089	216
PC No. 3=PS−0·00178	204
PC No. 4=PS−0·00316	204

The weights Sp, Sb, K were returned to Professor Schumacher accompanied by a weight V such that, by a mean of 200 comparisons, Sp+V≏PS−0·00071 grain in air (t=13·1, b=759·09).

By the good offices of M. Arago, permission was obtained from the French Government to compare the new English weights with the standard kilogramme of platinum, known as the *kilogramme des Archives,* and which will be denoted by the letter 𝔄. The comparison was made by two perfectly independent methods. In one of these 𝔄 was compared sixty times with PC No. 1 + PC No. 2 + auxiliary weight B+ a platinum weight V of nearly 192·436 grains. In the other, 𝔄 was first compared 200 times with the platinum kilogramme ℭ, purchased for the British Government. ℭ was afterwards compared with PS+ each of the four platinum copies of the

pound in succession, together with a platinum weight of about 1432·324 grains, the weight of which was found with great precision by a process to be described presently.

𝔄 had never been weighed in water. By observations made with the stereometer, it was found that at 0° C. the volume of 𝔄 exceeded that of 𝔈 by a quantity equal to the volume of 21·119 grains of water at its maximum density. By weighing 𝔈 in air and in water, it was found that Δ𝔈=20·54877. Some time after these observations were made, the Committee received from Professor Schumacher some observations of his own in manuscript, and a copy of Professor Steinheil's paper, entiled 'Das Bergkrystall-kilogramm,' from the fourth volume of the Transactions of the Bavarian Academy of Sciences, containing the determination of the volume of 𝔄, by comparing its linear dimensions with those of a platinum kilogramme of his own 𝔖, the density of which had been found by weighing it in air and in water. The two weights being cylinders, and the linear dimensions measured with an extremely delicate instrument constructed by Gambey, this kind of observation admitted of being made with great accuracy. The resulting difference between the volume of 𝔄 and that of 𝔈, was found to be equal to the volume of 20·933 grains of water at its maximum density. On account of the large number of observations, and the extreme care with which they were made, this value of the volume 𝔄— volume 𝔈 is to be preferred to that which was obtained by the stereometer, and has accordingly been used in reducing the observations for comparing the weights of 𝔄 and 𝔈.

𝔈 was compared with PS by the method which had proved so satisfactory in deducing the avoirdupois pound from the troy pound. Let I, K, L, M, N denote PS and its four platinum copies, A, B, Γ, Δ, platinum weights of about 1432·322 grains each, Z a weight of about 1270·708 grains, Θ a weight of 161·629 grains, made up of weights the values of which had been carefully determined. 𝔈 was compared with each of the weights $I+K+A$, $I+L+B$, $I+M+\Gamma$, $I+N+\Delta$, each of the lbs. K, L, M, N having been previously compared with I; I with $A+B+\Gamma+\Delta+Z$; each of the weights A, B, Γ, Δ with $Z+\Theta$. In this manner it was found that the kilogramme des Archives weighed 15432·34874 grains, of which the new Imperial Standard pound contains 7000, or kilogramme =2·20462125 lb. This is pro-

bably the best determination of the weight of 𝔄 in terms of the English standard of weight.

The value of 𝔄, as deduced from the direct comparison of 𝔄 with K + L + B + V, is subject to some uncertainty, arising from the circumstance that the platinum, of which A, B, C, F were made, had been very badly prepared and contained cavities filled with some hygroscopic substance which rendered the weight of B slightly variable, according to the greater or less amount of moisture present in the atmosphere. According to these observations, the kilogramme des Archives = 15432·34816 grains.

By the observations of Schumacher and Steinheil on the ratio of the weight of 𝔄 to that of Sp, subject to an uncertainty of 0·00139 grain, on account of an error of the press, and the comparison of Sp with PS, the weight of 𝔄 is either 15432·34873 or 15432·35012 grains, of which PS contains 7000.

The French standard of commercial weight is a brass kilogramme 𝔏, known as the *kilogramme type laiton.* It is deposited at the Ministère de l'Intérieure. According to a comparison of 𝔏 with 𝔄, the result of which is published in the 25th volume of the Modena Transactions, the apparent weight of 𝔏, when weighed in air at Somerset House, the mercury in the barometer, reduced to the freezing-point, standing at 29·75 inches, and the thermometer at 65·66 F. (b = 755·64 mm., t = 18·7 C.), is 15432·344 grains, of which the English commercial standard contains 7000.

The Society then adjourned to Thursday, May 8.