65·66 Fahrenheit, under the pressure of 29·75 inches of mercury at the freezing-point of water ($t = 18°·7$ C., $b = 755^{mm}·64$), or in air for which $10 + \log \Delta = 7·07835$, in terms of the commercial pound W of the same density as the lost standard troy pound.

No.	Density.	Absolute values.	Commercial values.
		grain.	grain.
1	8·3613	I−0·00732	W+0·01963
2	8·3416	I−0·03582	W−0·01135
3	8·3046	I+0·00510	W+0·02510
4	8·3650	I+0 00425	W+0·03154
5	8·0612	I+0·01783	W+0·00730
6	8·2878	I−0·01714	W+0·00080
7	8·1216	I+0·01933	W+0·01654
8	8·1632	I+0·01428	W+0·01679
9	7·3761	I+0·11611	W+0·00422
10	8·2838	I−0·03910	W−0·02165
11	8·3630	I−0·04208	W−0·01503
12	8·3192	I−0·02060	W+0·00115
13	8·4318	I−0·03331	W+0·00191
14	8·3496	I−0·02844	W−0·00301
15	8·3611	I−0·02022	W+0·00667
16	8·0735	I−0·02747	W−0·03640
17*a*	8·1172	I−0·02614	W−0·02948
17*b*	8·5589	I−0·04428	W+0·00542
18	8·3037	I−0·00129	W+0·01857
19	8·3397	I−0·01473	W+0·00950
21	7·9737	I+0·03971	W+0·01777
22	8·1986	I−0·01214	W−0·00523
23	8·1514	I+0·01557	W+0·01655
24	8·1429	I−0·03932	W−0·03941
25	8·1016	I+0·00180	W−0·00354
26	8·1522	I−0·00112	W−0·00001
27	8·1619	I+0·01405	W+0·01635
28	8·1260	I−0·00416	W−0·00638
29	8·1845	I−0·00222	W+0·00293
30	8·1529	I−0·00170	W−0·00050

II. "On the Determination of Unknown Functions which are involved under Definite Integrals." By J. Gomes de Souza, Professor of Mathematics in the Military Academy of Rio Janeiro. Communicated by Professor Stokes, Sec. R.S.

The author, after referring to a previous memoir on the same subject, presented by him to the French Academy, proposes to himself

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the problem of determining the function ϕ which (f, F being given functions, and the limits α, β of the integration being also given) satisfies the equation

$$\int_\alpha^\beta f(x,\theta)\ \phi(x+\theta)\ d\theta = \mathrm{F}x.$$

He observes, that, unlike the methods employed in his former memoir, and the solutions there employed, which are quite rigorous, the methods of the present memoir depend upon developments into series, the strictness of which has been contested by some mathematicians; but that passing over these difficulties, he has solved the famous problem, the solution of which has been vainly sought after for the last two hundred years, because on the above-mentioned equation depends the integration of the generally linear equation of any order whatever of two variables, and consequently the whole Integral Calculus. The solution first obtained by the author, and which he afterwards exhibits under a variety of different forms, is as follows :—

Theorem I.—The equation being given,

$$\int_\alpha^\beta f(x,\theta)\,\phi(x+\theta)\,d\theta = \frac{\mathrm{F}_2(x)}{\mathrm{F}_1(x)} = \mathrm{F}(x),$$

where $f(x,\theta)$ is a given function of x and θ; $\mathrm{F}_2(x)$ is a given function of x such that the equation $\mathrm{F}_2(x)=\infty$ cannot hold good for any finite value of x; $\mathrm{F}_1(x)$ a given function of x containing all the factors which render $\mathrm{F}(x)$ infinite, and the function $\mathrm{F}(x)$ being absolutely arbitrary; and α and β being given constants (independent therefore of x and θ), the expression for ϕx which satisfies the preceding equation is

$$\phi x = \frac{\mathrm{F}_2(a_1)}{f_1(a_1)\mathrm{F}_1{}'(a_1)}\,e^{x\Phi(a_1)} + \frac{\mathrm{F}_2(a_2)}{f_2(a_2)\mathrm{F}'{}_1(a_2)}\,e^{x\Phi(a_2)} + \&\text{c.},$$

where $f_r(x)$ is determined by

$$f_r(x) = (x-a_r)\,e^{x\Phi(a_r)}\int_\alpha^\beta f(x,\theta)\,e^{m_r\theta}d\theta.$$

$\Phi(a_r)$ is a root of the equation

$$\frac{1}{\int_\alpha^\beta e^{m_r\theta}\,f(a_r,\theta)\,d\theta}=0$$

solved relatively to m_r, and a_1, a_2, a_3, &c. are the roots of

$$F_1(x) = 0.$$

The author afterwards considers the equation

$$\int_\alpha^\beta f(x, \theta)\,\varphi(\theta)\,d\theta = F(x),$$

and the solution of a linear equation is at once made to depend upon this as follows: viz. given for the determination of the function $\varphi\, x$ the equation

$$f(x, 0)\,\varphi(x) + f(x, 1)\,\frac{d\phi(x)}{dx} + \&c. = F(x).$$

Assume

$$\varphi(x) = \int_\alpha^\beta e^{\theta x}\,\psi(\theta)\,.\,d\theta,$$

α, β being constants, and $\psi(\theta)$ a function of θ to be determined. It is always permitted to assume this equation.

By this means, writing for shortness

$$f_1(x, \theta) = f(x, 0) + f(x, 1)\,\theta + \&c.,$$

the equation becomes

$$\int_\alpha^\beta e^{\theta x} f_1(x, \theta)\,\psi(\theta)\,d\theta = F(x),$$

which is of the desired form.

A solution which occurred to the author after the memoir was drawn up, is as follows: viz. given, as before, the equation

$$\int_\alpha^\beta f(x, \theta)\,\varphi(x+\theta)\,d\theta = F(x),$$

then $\Phi(\omega\sqrt{-1}$ being determined by the equation

$$\frac{1}{\int_\alpha^\beta e^{\theta\Phi(\omega\sqrt{-1})} f(\omega\sqrt{-1}, \theta)\,d\theta} = 0,)$$

and putting, for abbreviation,

$$e^{x(\Phi\omega\sqrt{-1})}\,(x - \omega\sqrt{-1}) \int_\alpha^\beta e^{\theta\Phi(\omega\sqrt{-1})} f(x, \theta)\,d\theta = f(x, \omega\sqrt{-1}),$$

the equation

$$\varphi x = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{x\Phi(\omega\sqrt{-1})}\,\mathrm{F}(\omega\sqrt{-1})}{f(\omega\sqrt{-1},\omega\sqrt{-1})}\,d\omega$$

gives the solution of the problem.

The above-mentioned formulæ are selected out of a great number of very general results contained in the memoir.

III. Letter from Dr. W. Bird Herapath to Professor Stokes, "On the Detection of Strychnia by the formation of Iodo-strychnia." Communicated by Professor Stokes, Sec. R.S. Received June 12, 1856.

Bristol, June 7, 1856.

My dear Sir,—Will you do me the favour to announce to the Royal Society, that I have been engaged during some time past in the application of my discovery of the optical properties of iodo-strychnia to the detection of this alkaloid in medico-legal inquiries? I find it is perfectly possible to recognize the 10,000th part of a grain of strychnia in pure solutions by this method, even when experimenting on very minute quantities. In one experiment I took $\frac{1}{1000}$th of a grain only, and having produced ten crystals of nearly equal size, of course each one, possessing distinct and decided optical properties, could not represent *more* than the $\frac{1}{10000}$th part of a grain; in fact, it really represents much less, inasmuch as one portion of the strychnia is converted by substitution into a soluble hydriodate, and of course remains dissolved in the liquid.

I had hoped to have been able to complete this matter during this summer, but I now find it impossible to do so in time for this session of the Royal Society. I trust to be able to do so before Christmas, however. Will you oblige me by getting this notice inserted in the 'Proceedings,' as a new test for strychnia at this juncture possesses considerable interest, the colour-tests having been so dubiously spoken of recently by toxicologists?

In order to operate in this experiment, it is merely necessary to use diluted spirit of wine, about in the proportions of one part of spirit