

fractional) of a matrix, and thence to arrive at the notion of a rational and integral function, or generally of any algebraical function of a matrix. I obtain the remarkable theorem that any matrix whatever satisfies an algebraical equation of its own order, the coefficient of the highest power being unity, and those of the other powers functions of the terms of the matrix, the last coefficient being in fact the determinant. The rule for the formation of this equation may be stated in the following condensed form, which will be intelligible after a perusal of the memoir, viz. the determinant, formed out of the matrix diminished by the matrix considered as a single quantity involving the matrix unity, will be equal to zero. The theorem shows that every rational and integral function (or indeed every rational function) of a matrix may be considered as a rational and integral function, the degree of which is at most equal to that of the matrix, less unity; it even shows that in a sense, the same is true with respect to any algebraical function whatever of a matrix. One of the applications of the theorem is the finding of the general expression of the matrices which are convertible with a given matrix. The theory of rectangular matrices appears much less important than that of square matrices, and I have not entered into it further than by showing how some of the notions applicable to these may be extended to rectangular matrices.

IV. "A Memoir on the Automorphic Linear Transformation of a Bipartite Quadric Function." By ARTHUR CAYLEY, Esq., F.R.S. Received December 10, 1857.

[Abstract.]

The question of the automorphic linear transformation of the function  $x^2 + y^2 + z^2$ , that is the transformation by linear substitutions, of this function into a function  $x_i^2 + y_i^2 + z_i^2$  of the same form, is in effect solved by some formulæ of Euler's for the transformation of coordinates, and it was by these formulæ that I was led to the solution in the case of the sum of  $n$  squares, given in my paper "Sur quelques propriétés des déterminants gauches," Crelle, t. xxxii. pp. 119-123 (1846). A solution grounded upon an *à-priori*

investigation and for the case of any quadric function of  $n$  variables, was first obtained by M. Hermite in the memoir "Remarques sur une Mémoire de M. Cayley relatif aux déterminants gauches," Cambridge and Dublin Mathematical Journal, t. ix. pp. 63-67 (1854). This solution is in my Memoir "Sur la transformation d'une fonction quadratique en elle-même par des substitutions linéaires," Crelle, t. l. pp. 288-299 (1855), presented under a somewhat different form involving the notation of matrices. I have since found that there is a like transformation of a bipartite quadric function, that is a lineo-linear function of two distinct sets, each of the same number of variables, and the development of the transformation is the subject of the present memoir.

- V. "On some of the Products of the Destructive Distillation of Boghead Coal."—Part II. By C. GREVILLE WILLIAMS, Esq., Lecturer on Chemistry in the Normal College, Swansea. Communicated by Professor STOKES, Sec. R.S. Received December 17, 1857.

[Abstract.]

In this paper the author describes the method adopted by him for the separation of the three classes of hydrocarbons forming the more volatile portion of the distillate. On treatment with bromine in presence of water, the naphtha is entirely converted into a heavy oil, containing the  $C^n H^n$  series chemically, and propyle and benzole mechanically combined. The two latter may be removed by mere distillation on the water-bath. They are easily separable by fuming nitric acid, the benzole being dissolved while the propyle is untouched. The nitro-benzole obtained in this manner, on treatment by Béchamp's process, yields aniline mixed with a little toluidine, but no bases belonging to any other class.

The bromine compound (in consequence of its preparation in presence of water) could not be obtained free from oxygen. When kept for some time it separates into three layers, the upper being water faintly acidulated with hydrobromic acid, the middle bromine compound, and the lower, hydrobromic acid of 37 per cent., and the