binary and ternary quantics. But the theory of binary quantics is considered for its own sake; the geometry of one dimension is so immediate an interpretation of the theory of binary quantics, that for its own sake there is no necessity to consider it at all; it is considered with a view to the geometry of two dimensions. A chief object of the present memoir is the establishment upon purely descriptive principles of the notion of distance.

III. "On the Mathematical Theory of Sound." By the Rev. S. EARNSHAW. Communicated by Professor W. H. MILLER, For. Sec. R.S. Received November 20, 1858.

(Abstract.)

The principal feature of this communication is the discovery of an integral of a certain class of differential equations. This class includes, as a particular case, the differential equation of motion when a disturbance is transmitted through a uniform elastic medium confined in a horizontal tube. If the equation $\frac{dy}{dt}=F\left(\frac{dy}{dx}\right)$ be differentiated with regard to t, it produces the equation

$$\frac{d^2y}{dt^2}=\left\{F'\left(\frac{dy}{dx}\right)\right\}^2\cdot\frac{d^2y}{dx^2};$$

which, by means of the general function F′, can be made to coincide with any proposed differential equation in which the ratio between $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ is dependent on $\frac{dy}{dx}$ only. The integral obtained in this manner is that which arises from the elimination of (a) between the two following equations,—

$$y=ax+F(a)\cdot t+\phi(a),$$
$$0=x+F'(a)\cdot t+\phi'(a).$$

This integral, though not found by the direct integration of the differential equation, and though evidently not the general symbolical integral of it, is proved to be the general integral for wave-motion, from its affording the means of satisfying all the necessary equations of initial disturbance and wave-motion.

The author first discusses wave-motion when temperature is supposed to be unaffected by the passage of a wave; and then when the

change of temperature is allowed for. The most important result in the former case is the relation between pressure and velocity, which is shown to be that which is expressed by the equation

$$p=p_0\epsilon\frac{\nu}{\sqrt{\mu}};$$

from which several new results are obtained.

With respect to the velocity of sound, which has hitherto been found experimentally to exceed the velocity obtained by theory, it is shown that the value obtained by approximative methods is the *minimum limit* of sound-velocity, so that the actual velocity will always be greater; the excess depending upon the intensity and genesis of the sound. It is shown that all the parts of a wave do not travel at the same rate,—a circumstance which leads to the formation of a bore in the front of the wave. Several previously unexplained phenomena, which have been recorded by different experimentalists, such as double reports of fire-arms heard at a great distance, the outrunning of one sound by another observed by Capt. Parry, the comparative powers of different gases of transmitting sounds, and the laws of transmission of sound from one medium to another, are accounted for in this paper, and directly deduced from the integral of the equation of wave-motion.

IV. "Contributions towards the History of the Monamines." By A. W. Hofmann, LL.D., F.R.S. Received November 25, 1858.

2. *Action of Bisulphide of Carbon upon Amylamine.*

In a note on the alleged transformation of thialdine into leucine, addressed to the Royal Society about eighteen months ago*, I alluded to a crystalline substance observed by Wagner when submitting amylamine to the action of bisulphide of carbon. This substance was not analysed, but considering its mode of formation, Wagner suggested that it might possibly be thialdine.

$$\underbrace{C_{10}H_{13}N}_{\text{Amylamine.}}+C_2S_4=\underbrace{C_{12}H_{13}NS_4}_{\text{Thialdine.}}.$$

* Proceedings, vol. viii., Op. 4.