May 20, 1858.

The LORD WROTTESLEY, President, in the Chair.

The following communications were read :—

I. "On the Resistance of Tubes to collapse." By WILLIAM FAIRBAIRN, Esq., C.E., F.R.S. &c. Received April 21, 1858.

(Abstract.)

The object kept in view by the author of these researches was to determine the law which governed the resistance of cylindrical tubes to external uniform pressure. The anomalous condition in which these constructions have been placed in reference to the internal flues of boilers, and the frequent fatal accidents from explosions produced by collapse, have imperatively called for inquiry into the causes which have led to these unfortunate results. Ever since the first introduction of the steam engine as improved by Watt, and especially since the increased demand for its construction, and its application to almost every branch of industry and every system of transit, the consideration of all circumstances which may affect its economy and security, has become of vast public importance.

During the more early period which followed its first introduction, the form of boiler and its powers of resistance to strain, were considerations of much less importance than at present. Then the force of steam, or the pressure under which it was generated, was only about one-eighth, and in some cases less than a sixteenth of what it now is. Besides, the fertile genius of Watt had provided against accident, by a self-acting apparatus, which regulated not only the pressure, but the supply of water to the boiler. Since that time a total change has taken place in the construction and working of the steam engine; and boilers which were perfectly safe at 7 lbs. upon the square inch, are absolutely inadequate for generating steam at 40 lbs. to 50 lbs. on the square inch. This being the case, it follows that every precaution becomes urgently necessary which may serve to increase the strength, and equalize the resisting power of vessels containing

an element of such potent influence, and yet so essential to the comforts and enjoyments of civilized life.

Entertaining these views, the author goes on to say, that hitherto it has been considered an axiom in boiler engineering, that a cylindrical tube, placed in the position of an internal flue, is equally strong in every part when subjected to uniform external pressure; the length not affecting the strength of a flue so placed. This rule is, however, only true when applied to tubes of infinitely great length, and it is very far from true when the length of the tube does not exceed certain limits, and when the ends are retained in form by being riveted to the boiler, and thus prevented from yielding to external pressure. These facts were fully demonstrated by the experiments related in the Paper, which, for obvious reasons were conducted under circumstances as nearly as possible analogous to those now in actual operation upon a larger scale. With this view, a large and powerful cylinder, 8 feet long, and 2 feet in diameter, was prepared for the reception of the tubes; and being acted upon by hydraulic pressure, collapse was produced, and the results recorded, as fully explained in the Paper. It will suffice here to state the more important conclusions derived from the investigation, which fell under the following heads: viz.—1st, the strength of tubes as affected by length; 2nd, the strength of tubes as affected by diameter; lastly, the strength of tubes as affected by thickness of metal.

1. On the first head, *the strength as affected by length*, the results are conclusive and interesting. Within the limits of from 1 foot 6 inches to about 10 feet in length, it is found that the strength of tubes similar in every other respect, and supported at the ends by rigid rings, varies inversely as the length, as may be seen from the following results obtained with 4-inch tubes.

Resistance of 4-inch tubes to collapse.

Diameter. Inches.	Thickness of Plates. Inches.	Length. Inches.	Collapsing Pressure. lbs. per square inch.
4	·043	19	137
4	·043	60	43
4	·043	40	65

The remarkable differences in the resisting pressure of the above similar tubes will be at once apparent, and it will be found by calcu-

lation that they follow the law of inverse proportion, the same as those of larger dimensions, the strengths diminishing as the lengths are increased.

The same law of resistance is maintained in 6-inch tubes, giving, for a tube 30 inches long, 55 lbs., and for one 59 inches long only 32 lbs. on the square inch, as the pressure of collapse. Again, in 8-inch tubes we have, in a long series of experiments, 32 lbs. per square inch in a tube 39 inches long, and 39 lbs. in one 30 inches long. In the same manner all the experiments on tubes of 10 and 12, and up to 18 inches in diameter may be compared, and the law of resistance is in like manner shown to hold true in every case. Discrepancies to a certain extent do certainly occur; but they are comparatively small, and, as they appear to follow no law, are evidently to be accounted for from defects in the construction of the tubes inseparable from such a mode of research.

2. *The strength as affected by the diameter.*

A precisely similar law is found to hold in relation to the diameter. Tubes similar in other respects vary in their resistance to collapse inversely as their diameters; and with a view of testing this law, we may place the calculated pressure beside that derived from experiment, as under:—

Resistance of tubes to collapse 5 feet long.

Diameter. Inches.	By Experiment. lbs. per square inch.	By Calculation. lbs. per square inch.	Variation. lbs.
4	43·0		
6	32·0	28·6	−3·4
8	20·8	21·3	+0·7
10	16·0	17·2	+1·2
12	12·5	14·3	+1·8

The above variations are slight when compared with the resisting powers of the tubes; they are doubtless caused by the varying rigidity of the iron, or by defects in the cylindrical form. Similar results follow in the experiments on tubes 2 feet 6 inches long; and although some slight variations occur, they are nevertheless not more than might have been anticipated within the ordinary limits of error.

3. *The strength of tubes as affected by thickness.*

In these experiments it is found that the tubes vary in strength according to a certain power in the thickness; the index of which,

taken from the mean of the experiments, is 2·19, or rather higher than the square.

Combining the above laws into a general expression, we have, as the formula for the strength of tubes subjected to a uniform external force,

$$P = C \times \frac{k^{2\cdot19}}{L \times D},$$

where P is the collapsing pressure, k the thickness of the plates, L the length of the tube, which should not be less than 1·5, or greater than 10 feet; D the diameter, and C a constant to be determined by the experiments. For tubes of greater length than those above specified, a variable quantity, dependent upon the length, must be introduced; and the value of this has yet to be determined. For ordinary practical calculations the following formula will probably, however, afford the needful accuracy:—

$$P = 806{,}300 \times \frac{k^2}{L \times D}.$$

Thus, for example, take a tube or boiler-flue 10 feet long, 2 feet diameter, and composed of plates $\frac{1}{4}$ inch thick; and the collapsing pressure will be

$$P = 806{,}300 \times \frac{\cdot25^2}{10 \times 24} = 210 \text{ lbs.}$$

per square inch or nearly so.

Some experiments have also been made upon elliptical tubes; and the results have been most conclusive as to the weakness of such forms in resisting external pressure. No tubes in use for boilers should ever be made of that form.

With regard to cylindrical internal flues, the experiments indicate the necessity of an important modification of the ordinary mode of construction, in order to render them secure at the high pressures to which they are now almost constantly subjected. If we take a boiler of the ordinary construction, 30 feet long, 7 feet in diameter, and with one or more flues 3 feet diameter, it will be found that the outer shell or envelope is from three to three and a half times as strong in resisting an internal force as the cylindrical flues which have to resist the same external force. This being the case, it is evident that the excess of strength in those parts of the vessel subjected to tension, *is*

actually of no use so long as the elements of weakness are present in the other parts subjected to compression.

To remedy these defects, it is proposed to rivet strong rings of angle iron at intervals along the flue—thus practically reducing its length, or in other words increasing its strength to a uniformity with that of the exterior shell. This alteration in the existing mode of construction is so simple, and yet so effective, that its adoption may be confidently recommended to the attention of all those interested in the construction of vessels so important to the success of our manufacturing system, and yet fraught with such potent elements of disaster when unscientifically constructed or improperly managed.

II. "On some Remarkable Relations which obtain among the Roots of the Four Squares into which a Number may be divided, as compared with the corresponding Roots of certain other Numbers." By the Rt. Hon. Sir FREDERICK POLLOCK, F.R.S., Lord Chief Baron. Received April 26, 1858.

(Abstract.)

The first property of numbers mentioned in this paper is best illustrated by an example—

$$13^2 = 169 \qquad 15^2 = 225.$$

These odd numbers may be divided into 4 squares, and the roots may be so arranged that they will have this relation to each other: the middle roots will be the same, and the exterior roots will be, the one 2 more, the other 2 less than the corresponding roots of the other. Putting the roots below the number and comparing them, the result is obvious.

169	225
0,3,4,12	−2,3,4,14
−2,4,7,10	−4,4,7,12
−4,5,8,8	−6,5,8,10
−6,4,9,6	−8,4,9,8

Each of the numbers may be divided into 4 squares in 4 different ways with this result, that the two middle roots of each are the same; and as to the exterior roots they differ by 2, the one being 2 more