Hydrogen and nitrogen undergo no change of volume when exposed to the action of either form of discharge. Cyanogen is readily decomposed by the spark, but presents so great a resistance to the passage of electricity, that the action of the silent discharge can scarcely be observed. Protoxide of nitrogen is readily attacked by both forms of discharge, with increase of volume and formation of nitrogen and hyponitric acid. Deutoxide of nitrogen exhibits the remarkable example of a gas which, under the action either of the silent discharge or of the spark, undergoes, like oxygen, a diminution of volume. It also is resolved into nitrogen and hyponitric acid. Carbonic oxide has given results of great interest; but the nature of the reaction has been only partially investigated. The silent discharge decomposes this gas with production of a substance of a bronze colour on the positive wire. The spark acts differently, destroying, as in the case of oxygen, the greater part of the contraction produced by the silent discharge. The authors are engaged in the further prosecution of this inquiry.

II. "On the Equation of Differences for an Equation of any Order, and in particular for the Equations of the Orders Two, Three, Four, and Five." By Arthur Cayley, Esq., F.R.S. Received March 2, 1860.

(Abstract.)

The term *equation of differences*, denotes the equation for the squared differences of the roots of a given equation; the equation of differences afforded a means of determining the number of real roots, and also limits for the real roots of a given numerical equation, and was upon this account long ago sought for by geometers. In the Philosophical Transactions for 1763, Waring gives, but without demonstration or indication of the mode of obtaining it, the equation of differences for an equation of the fifth order wanting the second term: the result was probably obtained by the method of symmetric functions. This method is employed in the 'Meditationes Algebraicæ' (1782), where the equation of differences is given for the equations of the third and fourth orders wanting the second terms; and in p. 85 the before-mentioned result for the equation of the fifth order wanting the second term, is reproduced. The formulæ for

obtaining by this method the equation of differences, are fully developed by Lagrange in the 'Traité des Equations Numériques' (1808); and he finds by means of them the equation of differences for the equations of the orders two and three, and for the equation of the fourth order wanting the second term; and in Note III. he gives, after Waring, the result for the equation of the fifth order wanting the second term. It occurred to me that the equation of differences could be most easily calculated by the following method. The coefficients of the equation of differences, *quà* functions of the differences of the roots of the given equation, are leading coefficients of covariants, or (to use a shorter expression) they are "Seminvariants"*, that is, each of them is a function of the coefficients which is reduced to zero by one of the two operators which reduce a covariant to zero. In virtue of this property they can be calculated, when their values are known, for the particular case in which one of the coefficients of the given equation is zero. To fix the ideas, let the given equation be $(*\between v, 1)^n=0$; then, when the last coefficient or constant term vanishes, the equation breaks up into $v=0$ and into an equation of the degree $(n-1)$, which I call the reduced equation; the equation of differences will break up into two equations, one of which is the equation of differences for the reduced equation, the other is the equation for the squares of the roots of the same reduced equation. This hardly requires a proof; let the roots of the given equation be $\alpha, \beta, \gamma, \delta$, &c., those of the equation of differences are $(\alpha-\beta)^2$, $(\alpha-\gamma)^2$, $(\alpha-\delta)^2$, &c., $(\beta-\gamma)^2$, $(\beta-\delta)^2$, $(\gamma-\delta)^2$, &c.; but in putting the constant term equal to zero, we in effect put one of the roots, say α, equal to zero; the roots of the equation of differences thus become $\beta^2, \gamma^2, \delta^2$, &c., $(\beta-\gamma)^2$, $(\beta-\delta)^2$, $(\gamma-\delta)^2$, &c. The equation for the squares of the roots can be found without the slightest difficulty; hence if the equation of differences for the reduced equation of the order $(n-1)$ is known, we can, by combining it with the equation for the squares of the roots, form the equation of differences for the given equation with the constant term put equal to zero, and thence by the above-mentioned property of the Seminvariancy of the coefficients, find the equation of differences for the given equation. The present memoir shows the application of the process to equations of the orders two, three, four, and five: part of the calculation for the

* The term "Seminvariant" seems to me preferable to M. Brioschi's term Peninvariant.

equation of the fifth order was kindly performed for me by the Rev. R. Harley. It is to be noticed that the best course is to apply the method in the first instance to the forms $(a, b, \ldots \between v, 1)^n=0$, without numerical coefficients (or, as they may be termed, the *denumerate forms*), and to pass from the results so obtained to those which belong to the forms $(a, b, \ldots \between v, 1)^n=0$, or *standard forms*. The equation of differences, for $(\alpha-\beta)^2$, &c., the coefficients of which are seminvariants, naturally leads to the consideration of a more general equation for $(\alpha-\beta)^2\ (x-\gamma y)^2\ (x-\delta y)^2$, &c., the coefficients of which are covariants; and in fact, when, as for equations of the orders two, three, and four, all the covariants are known, such covariant equation can be at once formed from the equation of differences; for equations of the fifth order, however, where the covariants are not calculated beyond a certain degree, only a few of the coefficients of the covariant equation are given. At the conclusion of the memoir, I show how the equation of differences for an equation of the order n can be obtained by the elimination of a single quantity from two equations each of the order $n-1$; and by applying to these two equations the simplification which I have made in Bezout's abridged method of elimination, I exhibit the equation of differences for the given equation of the order n, in a compendious form by means of a determinant; the method just employed is, however, that which is best adapted for the actual development of the equation of differences for the equation of a given order.

III. "On the Theory of Elliptic Motion." By Arthur Cayley, Esq., F.R.S. Received March 9, 1860.

The present Note is intended to give an account of the results which, by means of a grant from the Donation Fund of the Royal Society, I have procured to be calculated for me by Messrs. Creedy and Davis, and which are contained in a memoir presented to the Royal Astronomical Society, entitled "Tables of the Developments of Functions in the Theory of Elliptic Motion." The notation employed is

r, the radius vector;
f, the true anomaly;
a, the mean distance;
e, the excentricity;
g, the mean anomaly;