The following Table shows the progress and present state of the Society with respect to the number of Fellows :—

	Patron and Honorary.	Foreign.	Having compounded.	Paying £2 12s. annually.	Paying £4 annually.	Total.
December 1, 1858 ..	9	50	365	7	275	706
Since elected	....	......	+ 4	......	+13	+17
Since deceased	—2	—3	—16	......	—11	—32
November 30, 1859..	7	47	353	7	277	691

December 8, 1859.

Sir BENJAMIN C. BRODIE, Bart., President, in the Chair.

The President announced that, under the provisions of the Charter, he had appointed the following Members of the Council to be Vice-Presidents :—

Thomas Bell, Esq.
Sir Roderick I. Murchison.
Major-General Sabine.
The Rev. William Whewell, D.D.
Sir William Page Wood.
The Lord Wrottesley.

The following communications were read :—

I. "On the Analytical Theory of the Attraction of Solids bounded by surfaces of a Class including the Ellipsoid." By W. F. Donkin, Esq., M.A., F.R.S., F.R.A.S., Savilian Professor of Astronomy in the University of Oxford. Received September 2, 1859.

(Abstract.)

The surface of which the equation is

$$f(x, y, z, h, k)=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is called for convenience "the surface (h, k)." The space, or solid, included between the surfaces (h_1, k), (h_2, k), is called "the shell $\left(\begin{smallmatrix} h_2 \\ h_2 \end{smallmatrix}, k\right)$;" and that included between the surfaces (h, k_2), (h, k_2) is called "the shell $\left(h, \begin{smallmatrix} k_2 \\ k_2 \end{smallmatrix}\right)$." [This notation is borrowed, with a slight alteration, from Mr. Cayley.] It is assumed that the equation (1) represents closed surfaces for all values of the parameters h, k, within certain limits, and that (within these limits) the surface (h, k) is not cut by either of the surfaces $(h+dh, k)$, $(h, k+dk)$. It is also supposed that there exists a value h_∞ of h, for which the surface (h_∞, k) extends to infinity in every direction. Lastly, it is supposed that if k be considered a function of x, y, z, h, by virtue of (1), the two following partial differential equations are satisfied :

$$\frac{d^2k}{dx^2}+\frac{d^2k}{dy^2}+\frac{d^2k}{dz^2}=\phi(h),$$

$$\left(\frac{dk}{dx}\right)^2+\left(\frac{dk}{dy}\right)^2+\left(\frac{dk}{dz}\right)^2+n\,\frac{dk}{dh}=0;$$

in which $\phi(h)$ is any function of h (not involving k), and n is any constant independent of h and k. The following propositions are then demonstrated :—

The potential, on a given external point, of a homogeneous solid bounded by the surface (h, k), varies as the mass of the solid, if h vary while k remains constant.

The potentials, on a given external point, of the homogeneous shells $\left(h_2, \begin{smallmatrix} k_2 \\ k_2 \end{smallmatrix}\right)$, $\left(h_2, \begin{smallmatrix} k_2 \\ k_2 \end{smallmatrix}\right)$ are proportional to the masses of the shells.

The homogeneous shell $\left(h, \frac{k_2}{k_2}\right)$ exercises no attraction on an interior mass.

The external equipotential surfaces of the homogeneous infinitesimal shell $\left(h_2, \frac{k}{k}+dk\right)$, are the surfaces (h, k), in which h is arbitrary and k invariable *.

The potential of the homogeneous infinitesimal shell $\left(h_2, \frac{k}{k}+dk\right)$ upon an *exterior* point, is

$$\frac{4\pi}{n} dk\, \psi(h_2) \int_h^{h_\infty} \frac{dh}{\psi(h)},$$

and upon an *interior* point, is

$$\frac{4\pi}{n} dk\, \psi(h_2) \int_{h_2}^{h_\infty} \frac{dh}{\psi(h)}.$$

(In these expressions $\psi(h)$ is $\epsilon^{\frac{1}{n}\int \phi(h)dh}$, and h at the lower limit in the first, is the parameter of the surface (h, k) which passes through the attracted point. The density of the shell is supposed to be unity.)

The potential of the finite homogeneous shell $\left(h_2, \frac{k''}{k'}\right)$ (density $=1$) upon an exterior point (ξ, η, ζ), is

$$\frac{4\pi}{n}\psi(h_2)\left\{k''\int_{h''}^{h''\infty} \frac{dh}{\psi(h)} - k'\int_{h'}^{h_\infty} \frac{dh}{\psi(h)} + \int_{h'}^{h''} \frac{kdh}{\psi(h)}\right\}:$$

in this expression it has been assumed (for simplicity) that h_∞ is independent of k. Also h'', h' are the values of h corresponding to k'', k', when h and k vary subject to the relation $f(\xi, \eta, \zeta, h, k)=0$; and k, in the last integral, is the function of h, ξ, η, ζ determined by this relation.

The differential equations (2) are satisfied in the case of the ellipsoid. For if we put its equation in the form

$$\frac{x^2}{a^2+h}+\frac{y^2}{b^2+h}+\frac{z^2}{c^2+h}=k,$$

it is evident on inspection that

$$\frac{d^2k}{dx^2}+\frac{d^2k}{dy^2}+\frac{d^2k}{dz^2}=2\left(\frac{1}{a^2+h}+\frac{1}{b^2+h}+\frac{1}{c^2+h}\right),$$

* It is known that the last two propositions imply the first two (see Mr. Cayley's "Note on the Theory of Attraction," Quarterly Journal of Mathematics, vol. ii. p. 338) ; though this is not the order of proof in the present paper.

and

$$\left(\frac{dk}{dx}\right)^2+\left(\frac{dk}{dy}\right)^2+\left(\frac{dk}{dz}\right)^2+4\frac{dk}{dh}=0.$$

In this case we find $\psi(h)=\Big((a^2+h)(b^2+k)(c^2+h)\Big)^{\frac{1}{2}}$, and the above general expressions lead to the known results.

II. Supplement to a Paper, read January 27, 1859, "On the Thermodynamic Theory of Steam-engines with dry Saturated Steam, and its application to practice." By W. J. MACQUORN RANKINE, C.E., F.R.S. &c.*

(Abstract.)

This supplement gives the dimensions, tonnage, indicated horse-power, speed, and consumption of fuel, of the steam-ships whose engines were the subjects of the experiments referred to in the original paper. Results are arrived at respecting the available heat of combustion of the coal employed, and the efficiency of the furnaces and boilers, of which the following is a summary:—

No. of experiment.	Kind of boiler.	Total heat of combustion of 1 lb. of coal in ft.-lbs., estimated from chemical composition.	Available heat of combustion of 1 lb. of coal in ft.-lbs. computed from efficiency of steam and weight of coal burned per I.H.P.	*Available heat*, total heat, = efficiency of furnace and boiler.
I.	Improved Marine Boilers of ordinary proportions.	10,000,000	5,420,000	0·542
III.		10,000,000	5,300,000	0·53
II.	Boiler chiefly composed of small vertical water-tubes, with very great heating surface.	11,560,000	10,110,000	0·88

$$\text{Available Heat of Combustion of 1 lb. of coal} = \frac{1{,}980{,}000\ \text{ft.-lbs.}}{\text{Efficiency of steam} \times \text{lb. coal per I. H. P. per hour}}.$$

* Phil. Trans. 1859, p. 177; and Proceedings of the Royal Society, vol. ix. p. 626.