Ex. 2.

2,2 7 0 8 9 7 0 2 2 4 7
2,7 2 1 9
7,2 3 2 7
2,3 3 3 0
3,3 4 3 2
3,4 1 2 2
4,1 0 2 4
1,0 5 0 7
4 7 4=6·79

Remark.—The principle which I have thus developed is touched upon in some manuals of arithmetic, when we are shown that the same remainder in the expressions

$$R\left(\frac{1000}{7}\right)=R\left(\frac{1000}{13}\right)=R\left(\frac{1000}{11}\right)=-1$$

leads to the identity

$$7\cdot 11\cdot 13=1001.$$

In the rules given in this Paper, I have shown the high *analytical* value of the principle; but the properties of numbers, to which we are led when we apply the same (principle) in a *synthetical* way, are not less remarkable. As an instance I may state that the symbol

$37\ldots\ldots\overset{d}{+a}$ leads to a curious relation among the members 7, 11, and 37.

V. "On the Structure of the *Chorda Dorsalis* of the Plagiostomes and some other Fishes, and on the relation of its proper Sheath to the development of the Vertebræ." By Professor Albert Kölliker, of Würzburg. Communicated by Dr. Sharpey, Sec. R.S. Received December 3, 1859.

I take the liberty to present to the Royal Society the results of an extended series of investigations into the development of the vertebræ of the plagiostomous and some other fishes.

I. *Chorda dorsalis.*

A. *Structure.*

The chorda dorsalis of the Plagiostomes, of *Chimæra*, *Acipenser*, *Scaphirhynchus*, *Toxodon*, and *Lepidosiren*, shows *four* distinct parts, viz.—

1st. The *outer elastic membrane*, a homogeneous elastic coat, which is not unfrequently perforated with holes of different sizes, of the same kind as those of the fenestrated membrane of Henle.

2nd. The *proper sheath*, formed of connective tissue of fibrous appearance, and generally provided with many plasm-cells.

3rd. The *inner elastic layer*, a reticulated elastic membrane; and

4th. The *gelatinous substance* of the chorda itself, made up of soft cartilage-cells, of different sizes and generally provided with nuclei.

Of these four layers it would seem that only the third and fourth are present in the higher animals, from the Amphibia (with the exception of the Batrachians) upwards; if, at least, my opinion be correct, that the structureless envelope of the chorda of these animals, generally called the sheath proper, corresponds to the third layer in the cartilaginous fishes. On the other hand, it seems that many of the osseous fishes present the same complications of structure as the Plagiostomes, if it is true that the bodies of their vertebræ are developed from the proper sheath of the chorda. So, for instance, there exists a beautiful elastic internal layer outside of the remnants of the gelatinous chorda in the genus *Orthagoriscus*.

B. *Form of the chorda proper.*

1st. The chorda retains in some instances its original cylindrical form, and this is the case when the vertebral column shows no indication of vertebral bodies (Cyclostomes, *Acipenser*, *Chimæra*, *Lepidosiren*, *Tilurus*, *Hyoprorus** (anterior vertebra)), as well as where vertebral divisions exist (*Leptocephalus*, *Helmichthys*, *Hyoprorus* (last vertebra)).

2nd. In other cases the chorda is contracted in the middle region of each vertebral body, which seldom happens where there is no trace of ossification (*Hexanchus*), but is very generally the case in

* Two genera belonging to the Leptocephalidæ, described by me (see Kaup, Apodal Fishes of the British Museum. London, 1856).

ossified vertebræ (Squali, osseous fishes, perennibranchiate amphibia, *Cæciliæ*).

3rd. Lastly, the chorda may be separated into as many parts as there are interstices between the vertebræ, which remaining parts in some cases are totally absorbed (*Raia* and most of the higher animals).

C. *Anterior end of the chorda.*

1st. In many full-grown fishes the chorda dorsalis reaches with its anterior attenuated end to the base of the cranium, and its cranial part is in some cases enveloped in its whole length by the cranial cartilage. This fact has been long known with regard to the Acipenseridæ, Cyclostomi, and Sirenoidei; but the same thing occurs amongst the Squali, and has been observed by Stannius in *Prionodon*, and by me in *Heptanchus*, *Centrophorus*, *Acanthias*, and *Squatina*. In these last fishes the chorda reaches as far as the region of the hypophysis, and is bent upwards at its termination, so that the end itself lies underneath the interior perichondrium of the cranium, or at least very near the surface of the cartilage. In other cases only the hinder part of the chorda is enclosed by the cranial cartilage, whilst the anterior half lies in a groove at the under part of it, as in *Leptocephalus* and *Helmichthys*. In one case (*Tilurus*) the whole cranial part of the chorda is free, and situated underneath the base of the cranium, between its cartilage and the perichondrium*.

2nd. In some genera of Squali and most of the osseous fishes, the cranial part of the chorda is reduced to the anterior half of the first ligamentum intervertebrale.

3rd. In the genus *Chimæra*, the chorda ends in the foremost part of the vertebral column. In this case the connexion between the cranium and the column is maintained by an articulation, which on the side of the column is formed by the cartilaginous vertebral arches.

4th. In the Raiidæ, finally, the chorda ends at a greater distance from the skull; and in this case also the anterior part of the column, which is formed only by the coalesced arches, is connected with the cranium by a real articulation.

* In all these fishes there exists rather a strong connexion between the vertebral column and the cranium; in *Squatina* besides this there are two lateral articulations between the cartilaginous arches of the first vertebra and the lateral parts of the cranial cartilage.

II. *Ossification and Development of the Bodies of the Vertebræ.*

A. *General remarks on the part which the chorda takes in the formation of the vertebræ.*

1st. In all cases where the chorda ossifies, it is only its second layer, or the *sheath proper*, which undergoes changes. At the same time the elastica externa disappears totally, or is at least dissolved in such a manner that its remnants are scarcely distinguishable, whilst the elastica interna and the chorda proper generally remain unaltered. In one case only, viz. in *Scymnus lichia*, ossification is to be seen even in the gelatinous substance of the chorda.

2nd. The ossification of the sheath of the chorda has been observed as yet only in the Plagiostomes and in certain genera of the osseous fishes; but it very probably will be found in all osseous fishes. On the contrary, it is absent in all higher Vertebrata—according to my observations, even amongst the Batrachia.

B. *Changes of the sheath of the chorda during ossification.*

1. *Vertebral column.*

1st. In the Plagiostomes the sheath of the chorda in the first place assumes a greater hardness in certain parts, these parts being transformed into fibro-cartilage or real cartilage, whilst the intervening parts retain their primitive softness. In this manner the first indications appear of the vertebral bodies and intervertebral ligaments, the interior parts of which are formed by the chorda itself and the elastica interna. The histological changes going on during this formation of the vertebral bodies, viz. the transformation of the primitive plasm-cells of the sheath into cartilage-cells, and the development of the homogeneous interstitial substance of the cartilage out of the fibrous substance of the sheath, speak strongly in favour of the view that both kinds of cells and intervening substances are closely allied, whatever may have been the development of the elements of the primitive sheath.

In the *Leptocephali* the sheath of the chorda ossifies without having been transformed into cartilage; and the same seems to hold good for the other osseous fishes.

2nd. Whilst this transformation of certain parts of the sheath of the chorda into cartilaginous vertebral bodies is going on, there are also formed in the interior of each of these bodies peculiar vertical dis-

sepiments. These dissepiments, developed by an interior growth of the sheath of the chorda, whereby the chorda proper becomes constricted, occur in some cases in vertebræ without any or with very slight traces of ossification, as in *Hexanchus* and the anterior vertebra of *Heptanchus*, whilst they may be almost wanting in others pretty well ossified (*Leptocephalus, Helmichthys, Centrophorus*).

3rd. The ossification of the cartilaginous vertebral bodies formed out of the sheath of the chorda never begins at the surface, but always in their interior, and also in their middle region, and is, as far as I know, without exception, in the first instance a calcified fibro-cartilage, or what I call a fibrous bone.

4th. The first osseous parts have the form of thin *rings* (*Heptanchus*, anterior vertebra), which afterwards assume that of hollow and thin *double cones* (*Heptanchus*, posterior vertebra, *Centrophorus*).

5th. The growth of these double cones, which are the real osseous vertebral bodies, when once they have assumed their whole length, takes place especially at their *outer side*, through the addition of *calcified cartilage* (chondriform bone, Williamson; *Knorpel-Knochen* in German), which is formed from the outer chordal cartilage of the vertebral body. In addition to this, the osseous double cone thickens also at the expense of the cartilage inside of it, but in a much smaller degree.

6th. In some cases the outer growth is everywhere the same, and in this manner stronger double-coned vertebral bodies of uniform thickness are formed. In other cases the growth is in some parts more active than in others, and vertebral bodies then originate with outer ridges and lamellæ (*Heptanchus, Raia, Carcharias, Mustelus, Galeus*). In one single instance the ossification of the outer cartilage takes place in such a way that the exterior parts of the vertebral bodies are formed by alternating circles of chondriform bone and cartilage (*Squatina*).

7th. With regard to the extension of this growth of the vertebral bodies formed by the ossification of the sheath of the chorda, it is to be remarked, that in some cases the whole, or nearly the whole sheath of the chorda ossifies, as in *Squatina* and the Raiidæ. In other cases greater or lesser parts of the primitive cartilage, inside and outside the vertebral body, remain in their primitive state (Squali).

2. *Skull.*

In some instances even *the sheath of the cranial part of the chorda ossifies* in its hindermost part, and *forms a true vertebral body for the occipital vertebra,* which entirely corresponds to those of the column. This has been observed by me as yet in *Leptocephalus* and several Squalidæ; but it is extremely probable that the *basilar occipital* of all osseous fishes, viz. that part of this bone which resembles a common vertebral body, is developed quite in the same way.

C. *On the manner in which the outer ossifying layer is concerned in the formation of the bodies of the vertebræ.*

1st. In those cases where the outer ossifying layer, viz. that layer in which the cartilaginous arches are developed, takes part in the formation of the vertebral bodies, there are to be distinguished two different processes,—one in which the crural cartilages themselves play a part in this formation, and a second, where only the periosteal layer between them is concerned.

2nd. Where the crural cartilages take a part, they form, in the first place, by their coalescence an *outer cartilaginous layer* around the body of the vertebra, which took its origin from the chorda, and which we shall henceforth call *the chordal vertebral body.*

3rd. This outer cartilaginous layer ossifies in many cases; and this ossification may take place in *two* places only, viz. on the right and left side of the vertebral body, as in *Heptanchus,* or in *four* places, in which case a superior point of ossification at the floor of the neural canal, and an inferior one at the roof of the hæmal canal, are added to the two lateral ones (*Acanthias, Scymnus*).

4th. These external ossifications of chondriform bone may retain their primitive form of plates, and may then be called the lateral, superior, and inferior osseous plates; or they acquire by additional growth, at the expense of the outer cartilaginous layer, the form of wedge-shaped or cuneiform bodies, and may be named the lateral, superior, and inferior wedges (*Zapfen, Keile,* Germ.).

5th. In both cases these external ossifications comport themselves in two different ways with regard to the chordal vertebral body, inasmuch as in some cases both coalesce at their ends (*Scymnus, Acanthias*), whilst in others they remain separated (*Heptanchus*).

6th. In some peculiar cases (Squali, possessing a nictitating eyelid, viz. *Mustelus, Carcharias, Galeus, Sphyrna*) the cartilaginous arches remain separated, and then the intermediate periosteal layer performs the part of an osteogenic stratum. The osseous parts produced in this way lie at the same places as the bony plates mentioned under 4 and 5; they always possess the form of wedges, and coalesce with the chordal vertebral body, in some cases only at their ends, in others in their whole length. Although these ossifications are not developed from cartilage and have a very peculiar structure—they consist of a calcified fibro-cartilage with peculiar ossified strong fibres running straight through their whole thickness,—it is clear enough that they exactly correspond to the above-mentioned plates and wedges of other Plagiostomes formed out of the coalesced crural cartilages.

From certain modes of transformation of the sheath of the chorda, combined with certain changes of the outer ossifying layer, the following types in the composition of the vertebral bodies may be established.

TYPE I.—*The vertebral body takes its origin entirely from the proper sheath of the chorda.*

A. *Sheath of the chorda thick.*

1st. Vertebral bodies soft (fibro-cartilaginous), incompletely separated from each other, and only distinguished by the interior septa of the chorda. *Hexanchus.*

2nd. Vertebral bodies partly cartilaginous, with annular ossifications of the form of short double cones. Ligamenta intervertebralia very strong. *Heptanchus* (anterior vertebra).

3rd. Vertebral bodies wholly cartilaginous, with thin osseous double cones of good length in the middle of the cartilaginous body. *Centrophorus.*

4th. Vertebral bodies well ossified, cylindrical and strong, formed inside by strong osseous double cones, and outside by alternating layers of cartilage and bone. *Squatina.*

B. *Sheath of the chorda thin.*

5th. Vertebral body a thin hollow osseous cylinder; chorda proper in its whole length cylindrical. *Leptocephalus, Helmichthys, Hyoprorus* (last vertebra).

6th. Vertebral bodies slightly constricted osseous double cones, with external longitudinal ridges. *Chauliodus, Stomias.*

TYPE II.—*The vertebral body is formed partly from the sheath of the chorda and partly from the outer ossifying layer.*

1st. Chordal vertebral body partly cartilaginous, with a stronger osseous double cone in its middle part. External part of the body formed by a thin layer of cartilage from the coalesced arches, with *two* lateral ossified plates. *Heptanchus* (posterior vertebræ).

2nd. The same with *four* external ossifications, whose ends coalesce with the internal double cone. *Acanthias, Scymnus.*

3rd. Chordal vertebral body nearly totally ossified, of the form of a strong double cone, with strong external longitudinal ridges. External part of the body a strong layer of cartilage with superficial ossifications continuous with those of the arches. *Raia, Torpedo.*

4th. Chordal vertebral body nearly wholly ossified, of the form of a thick double cone. External part of the body formed by cartilage, with four strong wedge-shaped ossifications uniting with the ends of the inner double cone. *Scyllium.*

5th. Chordal vertebral body a strong osseous double cone, partly with external ridges. External part of the body formed by four strong, wedge-shaped ossifications, derived from the periosteal layer between the cartilaginous arches, which in some genera totally coalesce with the inner double cone, whilst in others this happens only at the ends of the latter. *Mustelus, Carcharias, Sphyrna, Galeus.*

TYPE III.—*The vertebral bodies are wholly developed from the external ossifying layer.*

1st. The vertebral bodies are developed from four cartilaginous parts, viz. the superior and inferior arches. Anterior vertebræ of the Raiidæ.

2nd. The vertebral bodies are developed only from two cartilaginous or osseous parts.

a. From the two neural arches, which in uniting do not enclose the chorda, which lies underneath them. *Cultripes provincialis*, J. Müller, *Rana paradoxa*, Dugès.

b. From two lateral plates of ossified connective tissue, which

in uniting totally enclose the chorda. Acaudate Batrachia, according to my own observations.

c. From two lateral cartilages which enclose the chorda, and also develope the arches from themselves. Higher Vertebrata.

In terminating this Note, I take the liberty of adding that the only information heretofore existing on the subject to which it refers, is that contained in the very valuable memoirs by J. Müller* and Williamson†. The part which each of these has contributed to the elucidation of this subject, will be stated in a paper which will appear in the next Number of the Würzburg Transactions, to which I refer those who take a more special interest in this matter, and desire to know on what data the results here given are founded.

VI. "Remarks on the late Storms of October 25–26 and November 1, 1859." By Rear-Admiral FitzRoy, F.R.S. Received December 22, 1859.

As many of our Society must doubtless be interested in the nature and character of that storm in which the 'Royal Charter' went to pieces on Anglesea Island, and as abundant information has been obtained from Lighthouses, Observatories, and numerous private observers, I would take this earliest opportunity of stating that the combined results of observations prove the storm of October 25th and 26th to have been a complete horizontal cyclone.

Travelling bodily northward, the area of its sweep being scarcely 300 miles in diameter, its influence affected only the breadth of our own Islands (exclusive of the west of Ireland) and the coast of France.

While the central portion was advancing northward, not uniformly but at an *average* rate of about twenty miles an hour, the actual velocity of the wind—circling (as against watch-hands) around a small central "lull"—was from forty to nearly eighty miles an hour.

At places north-westward of its centre, the wind appeared to "back" or "retrograde," shifting from east through north-east, and north to north-west; while at places eastward of its central passage, the apparent change, or veering, was from east, through south-east, south, south-west, and west.

* Vergleichende Anatomie der Myxinoiden. † Phil. Trans. 1850.