the author, with the additional correction for ocean attraction introduced.

	a.	*b.*	*b* : *a*−*b*.
I.	20926500	20855400	294
II. . . .	20920328	20846522	283·7
III. . . .	20919988	20846981	286·55

The residual errors of latitude at Damargida, Kaliana, and Kalianpur, which in Captain Clarke's ellipse were $+1''\cdot05$, $-0''\cdot95$, $+1''\cdot20$, are now reduced to $+0''\cdot93$, $-0''\cdot37$, $+0''\cdot74$.

In conclusion, the author calculates the distance of a point in the latitude of Kaliana from the centre of the earth in the three ellipses, and finds it to be near 7000 feet greater in the ellipses II. and III. than in the mean ellipse I. That deviations to such an extent as this from the mean ellipse should actually occur he thinks likely enough, and he is not disposed to have recourse to some yet undiscovered cause to reconcile the Indian Arc with the mean ellipse. The occurrence of marine fossils in mountains and elevated regions, shows that great changes of level of the land relatively to the water have actually taken place; and it seems unlikely that an extensive internal change in the state of the earth would cause an upheaval or depression of the land or the water alone; it might rather be expected that both would be affected, though unequally. Hence the absolute change of distance of the land from the centre of the earth may have been much greater than the elevation relatively to the water, while the phenomena adduced indicate that even the latter must have been very great.

IV. "Comparison of some recently determined Refractive Indices with Theory." By the Rev. BADEN POWELL, M.A., F.R.S., F.G.S., F.R.A.S., Savilian Professor of Geometry in the University of Oxford. Received November 17, 1859.

In a series of papers inserted in the Philosophical Transactions (1835, 1836, 1837), and afterwards, in a more correct and complete form, in my Treatise 'On the Undulatory Theory applied to the Dispersion of Light' (1841), I endeavoured to investigate the great problem of the explanation of the unequal refrangibility of light on

the principles of the undulatory theory, as proposed by M. Cauchy about 1830, by numerical comparison with the indices observed, more especially in cases of the most highly dispersive media then examined.

The general result then arrived at was, that while the theory applied perfectly through an extensive range of media of low and moderate dispersive power, it did not apply well to those of higher; and to the highest in the scale (which of course formed the true test of the theory) it did not apply within any allowable limits of accuracy. Since that time little has been done towards prosecuting the subject.

In the *experimental* part of the inquiry, about 1849, I had observed the indices for a few new media*; but these were not high in the scale; yet though perhaps thus of little importance, I have now thought it as well to go through the calculation for them: the results are of the same general character as just described.

Soon after, finding that my friend, the Rev. T. P. Dale, F.R.A.S., was desirous to carry on some researches of this kind, I placed at his disposal the apparatus with which I had determined all my indices†.

In 1850 that gentleman communicated to the Royal Astronomical Society a short general account of his observations ‡ relative to some substances not very high in the scale.

In 1858, Mr. Dale, in conjunction with Dr. J. H. Gladstone, F.R.S., presented to the Royal Society § a valuable series of determinations, evincing highly interesting results relative to the change of refractive power in various substances under different temperatures.

None of these media being high in the scale, they have little bearing on the main object of my inquiries. In two cases (viz. water and alcohol) the indices agree so closely with mine, that it was not worth while to recalculate them. In two other cases I have carried out the numerical comparison, which affords a good agreement with the theory.

Very recently the same gentlemen have, however, published some

* See British Association Reports, 1850, Sect. Proc. p. 14.

† Described and figured, British Association Reports, 1839.

‡ Notices, vol. xi. p. 47.

§ Phil. Trans. 1858.

observations on several other media, especially phosphorus, a substance at the very summit of the scale, for which I had long been extremely desirous to obtain some determinations of indices*.

Among these results only two sets are in a form in which they can be made available for comparison with theory. These are the indices for the standard rays in bisulphide of carbon, and for solution of phosphorus in that medium, which I have now calculated theoretically.

The results (given in the sequel) in both cases indicate discrepancies between theory and observation too great to be due to any reasonable allowance for error; and we are confirmed in the conclusion before arrived at, that, *for highly dispersive substances, the theory,* IN ITS PRESENT STATE, *is defective.*

But these comparisons are all made by means of the same formula employed in my former researches, viz. that derived from Cauchy's theory by Sir W. R. Hamilton, which he communicated to me, and which I explained in a paper in the Philosophical Magazine†.

Considering the unsatisfactory condition in which the question was left when tried by the test of the higher media in my former inquiries, it is a matter of some surprise that in the long interval since the publication of those results no mathematician has been induced to *revise the theory.* Some criticisms indeed were advanced by Mr. Earnshaw‡, and others by Prof. Mosotti and the Abbé Moigno§, bearing on the general principle. Sir W. R. Hamilton's formula in particular was founded on certain assumptions confessedly but *approximate.* It remains then a promising field for inquiry to analysts, whether a better formula might not be deduced, or other improvements made in the general theory, by which a method applying so well to lower cases might be made equally successful for the higher.

Results of calculation, for Ether, Hydrate of Phenyl, Oils of Spikenard, Lavender and Sandal-wood, Benzole, Bisulphide of Carbon, and Solution of Phosphorus in that medium.

Three indices assumed from observation, viz. μ_B, μ_F, and μ_H, give the medium constants, viz.

* See Phil. Mag. July 1859.

† Vol. viii. N. S. March 1836.

‡ See Phil. Mag. April 1842 and August 1842.

§ See British Association Reports, 1849, Sect. Proc. p. 8.

$$D = \mu_F - \mu_B,$$
$$D' = \mu_B + \mu_H - 2\mu_F.$$

The values of the wave-length constants A and B for each ray, independent of the medium, are taken from my Treatise (Undulatory Theory applied to Dispersion, &c., Art. 270). Combining these, we obtain AD and BD′ for each ray in the medium.

Thence Sir W. R. Hamilton's formula (*ib.* Art. 237) gives for any ray,

$$\mu = \mu_F \pm (AD + BD');$$

the upper sign being used for rays above F, the lower for those below.

Ether.—Dale and Gladstone.

Ray.	μ. Observation.	μ. Theory.	Difference.
B	1·3545		
C	1·3554	1·3544	—·0010
D	1·3566	1·3566	—·0000
E	1·3590	1·3586	—·0004
F	1·3606		
G	1·3646	1·3646	·0000
H	1·3683		

Hydrate of Phenyl.—Dale and Gladstone.

Ray.	μ. Observation.	μ. Theory.	Difference.
B	1·5416		
C	1·5433	1·5428	—·0005
D	1·5488	1·5495	+·0007
E	1·5564	1·5567	+·0003
F	1·5639		
G	1·5763	1·5772	+·0009
H	1·5886		

In both these media, of low dispersive and refractive power, the accordances of theory and observation are sufficiently close.

Oil of Lavender.—Powell.

Ray.	μ.		Difference.
	Observation.	Theory.	
B	1·4641		
C	1·4658	1·4632	—·0026
D	1·4660	1·4678	+·0018
E	1·4728	1·4726	—·0002
F	1·4760		
G	1·4837	1·4848	+·0011
H	1·4930 ?		

Oil of Sandal-wood.—Powell.

Ray.	Observation.	Theory.	Difference.
B	1·5034		
C	1·5058	1·4988	—·0070
D	1·5091	1·5062	—·0029
E	1·5117	1·5102	—·0015
F	1·5151		
G	1·5231	1·5271	+·0040
H	1·5398 ?		

Oil of Spikenard.—Powell.

Ray.	Observation.	Theory.	Difference.
B	1·4732		
C	1·4746	1·4744	—·0002
D	1·4783	1·4082	—·0001
E	1·4829	1·4826	—·0003
F	1·4868		
G	1·4944	1·4945	+·0001
H	1·5009		

Benzole.—Powell.

Ray.	Observation.	Theory.	Difference.
B	1·4895		
C	1·4961	1·4907	—·0054
D	1·4978	1·4965	—·0013
E	1·5041	1·5029	—·0012
F	1·5093		
G	1·5206	1·5210	+·0004
H	1·5310		

In oil of lavender and of sandal-wood there was some indistinctness in the line H which renders its index a little uncertain. It may be owing to this circumstance that the assumption of that index may have occasioned the discrepancy between theory and observation.

In oil of spikenard the accordance is good. In benzole the discrepancies are too great.

Bisulphide of Carbon.—Dale and Gladstone.

Ray.	μ.		Difference.
	Observation.	Theory.	
B	1·6177		
C	1·6209	1·6169	—·0040
D	1·6303	1·6251	—·0052
E	1·6434	1·6425	—·0009
F	1·6554		
G	1·6799	1·6807	+·0108
H	1·7035		

Phosphorus dissolved in Bisulphide of Carbon.—
Dale and Gladstone.

Ray.	Observation.	Theory.	Difference.
B	1·9314		
C	......	1·9298	
D	1·9527	1·9522	—·0005
E	1·9744	1·9726	—·0018
F	1·9941		
G	2·0361	2·0363	+·0002
H	2·0746		

In the first of these media the differences are greater than can be fairly allowed to errors of observation.

In the second case it is yet more clearly apparent that the theory is defective. The ray C was not observed; but the theoretical index is evidently in error to a large amount, as it is even lower than that of B. The indices for D and C are perhaps within the limits of error; but that of E is too much in defect to be allowed.