VIII. "On the Application of the Calculus of Probabilities to the results of measures of the Position and Distance of Double Stars." By THE LORD WROTTESLEY, V.P.R.S., &c. Received May 27, 1859.

In a communication addressed to the Royal Society "On the results of Periodical Observations of the Positions and Distances of certain Double Stars," printed in the Philosophical Transactions for 1851, I took occasion to remark that the differences between mean results obtained on different evenings were greater in proportion than those of the separate or partial measures obtained on the same evening, which arise from chance errors of observation, and that this circumstance rendered the application of the Formulæ of the Calculus of Probabilities to the reduction of the observations embarrassing and difficult. In other words, the differences between the mean positions and distances obtained on different nights were greater than would have been anticipated by one who had merely computed the probable error of a single measure in the usual manner from the data furnished by the sums of the squares of the partial differences from the mean.

The observations made since 1851 fully confirm the anomaly in question. It is probable, therefore, that there is some cause which modifies sensibly and in some unknown manner the results obtained. It may be temperature acting on the micrometer screw; it may be the state of the atmosphere or the method of making the observation; but whatever it be, the observations show conclusively that such causes are sometimes in operation.

For the purpose of obtaining some numerical expression, however imperfect, of the effect produced, I adopted the following method:—I took the difference between two mean results of position obtained on two different nights, where not more than about two months intervened between the observations; and I ascertained also the mean of the probable errors of such positions as computed in the ordinary method. In order that each star might be subjected to exactly the same treatment, I selected always the observations of the first two nights on which it was observed, except when the two consecutive means were obtained at too long an interval apart. Now as the number of partial measures of angle obtained on each separate night very often did not exceed six, these probable errors are certainly not

theoretically correct, or to be depended upon absolutely as a test of the accuracy of the observation; but it may perhaps be assumed that any errors arising from this cause will not materially affect the mean of a very great number of results.

It appeared then that the mean of 218 differences taken at hazard from among such as were most accessible, and from observations made by different observers, was $37'\cdot67$, and that the corresponding mean of the means of all the probable errors was $13'\cdot29$; that is to say, the latter is 35·27 per cent. only of the value of the former. As some proof that the cause, whatever it may be, is not very variable in its operation, I may add that the first 110 differences, which were all obtained before the end of 1854, give these numbers, $37'\cdot79$, $11'\cdot86$, and 31·37 per cent. respectively. Again, the last 108 differences, which were all derived from the observations of one observer only, give $37'\cdot56$, $14'\cdot75$, and 39·27 per cent. Of these 108 last differences, the first 50, taken from the middle epoch of all the observations, give $40'\cdot06$, $14'\cdot60$, and 36·44 per cent., and the last 58 of the 108 give $35'\cdot40$, $14'\cdot88$, and 42·03 per cent. There is, however, a circumstance which must be taken into account in making a comparison between the first 110 differences and mean probable errors, and the last 108. During the course of the observations from which the former were derived, it was the practice to take always 10 measures of each star on each night when possible; during the observations from which the latter were derived, 6 measures only were taken. This would tend to make the differences less in the former case; and with respect to its effect on the probable errors, if we put F for the error of a single measure in each case, the probable errors in the former case should be less by a quantity $=0\cdot0768\times F$; for if we put C for the constant and P for the probable error, then we have $P=\frac{3\sqrt{C}}{\sqrt{45}}\times F$, where 6 measures are obtained; and $P=\frac{\sqrt{5C}}{\sqrt{45}}\times F$, where 10 are taken; and the difference between these values $=\cdot0768$ F.

The facts above disclosed create the difficulties in applying the Calculus of Probabilities which have been before referred to.

In the first place, the partial measures obtained on each separate night are generally too few in number to eliminate the effects of one-sided chance errors.

In the second place, it seems probable that some cause remains in action during a whole night, modifying the result, whose origin and law remain to be discovered, but which seems tolerably constant in its operation.

The observations of my Catalogue of double stars are drawing to a close, and it became extremely desirable that if there were any fault in the reductions or method of computing hitherto employed it should be speedily remedied, and the necessary corrections made; I therefore applied to the Astronomer Royal, stating the embarrassments arising from the above-mentioned causes, and requesting his opinion as to the best mode of proceeding. The Astronomer Royal exhibited on this, as on all other occasions, where his aid has been solicited, the greatest readiness to give me the benefit of his extensive knowledge of all that appertains to Astronomical science.

Mr. Airy observed that if there were a constant cause of error on any night, no multiplication of observations on that night would tend to remove it, and in that case he knew of no mode of proceeding which would *quite* meet the difficulty but the adoption of the following formulæ.

Assuming that all the observations are equally good, or can be made so by grouping discordant measures, let f be the probable error of a single observation, and e the probable value of the error of each night. Let S_1, S_2, S_3, &c. represent the sums of the squares of the errors obtained in the usual manner from the observations on the first, second, third, &c. nights respectively, and put n_1, n_2, &c. for the number of observations obtained on each of those nights; then the observations of the first night give,

$$(n_1-1)f^2=\cdot4549\times S_1\,;$$

those of the second,

$$(n_2-1)f^2=\cdot4549\times S_2,$$

and so on: and from the sum of all these equations f may be accurately determined.

Then to find e, compare the mean result obtained from the observations on all the nights, *i. e.* the mean of all the means, with the separate means for separate evenings; then putting S for the sum of

the squares of the errors found by such comparison, and m for the whole number of nights, we have

$$(m-1)\, e^2 = \cdot 4549 \times \mathrm{S},$$

which gives the value of e.

If A be taken to represent any convenient constant, the combining weight for *each* of the first night's observations will be $\frac{\mathrm{A}}{n_1 e^2 + f^2}$, for each of the second night's $\frac{\mathrm{A}}{n_2 e^2 + f^2}$, and so on.

Let P stand for the probable error of the final result, then

$$\mathrm{P} = \frac{1}{\sqrt{\left(\frac{n_1}{n_1 e^2 + f^2} + \frac{n_2}{n_2 e^2 + f^2} + \frac{n_3}{n_3 e^2 + f^2} + \&\mathrm{c.}\right)}}.$$

The probable error of the mean of the first night's observations $= \sqrt{\left(e^2 + \frac{f^2}{n_1}\right)}$, of the second $= \sqrt{\left(e^2 + \frac{f^2}{n_2}\right)}$, &c.

Mr. Airy, however, while remarking that the mode of proceeding above described is the only one which really meets the difficulties of the case, admits at the same time that it would not be expedient to use so elaborate a process in dealing with observations like those in question, in which the ordinary errors of observation are large in amount, and in which such extreme accuracy in the results is not obtainable as in some other cases to which the principles of the Calculus are applicable.

He suggests therefore that all the observations of all the several nights should be combined together for the purpose of obtaining the probable error and weight of the final result; and this may be done in two different ways:—First, by treating all the single measures of all the nights, as if they had been made on one and the same night, and obtaining the final result and its probable error and weight accordingly in the usual manner: Secondly, by treating each group or set of 6 or 10 as a single observation.

The only other method of proceeding is that above described as the correct one, but which has not been adopted, as being too cumbrous for the occasion. This will be designated as the Third Method.

For the purpose of ascertaining the result of employing each of

these three methods, I requested my assistant, Mr. Morton, to observe three stars, selected from among those which present only average difficulties, a very great number of times, so that the measures should be sufficiently numerous to eliminate all one-sided errors.

The observations of Position only have been used; but these have been dealt with in the three different methods above described, that is, the final result and its probable error and weight have been obtained by each of the three modes. The results are here subjoined, and the errors and their squares are given in full as to two of the stars, together with the whole computation; and it is to be hoped that this may not only prove interesting to observers of double stars, but may throw some light on the curious mathematical question involved in the inquiry which is the subject of the above remarks.

Among the stars selected as above mentioned for the trial of the three methods, was 2 Comæ Berenices, or Σ 1596. Now this star had been very frequently observed during the six years from 1843 to 1848, at the time of the Parallax investigation, to which reference has been already made. The comparison then made between the mean of all the measures of position obtained and the value of the angle of position given by Struve, gave reason to believe either that the angle had not altered during a period of sixteen years, or at least that it had altered very little. The observations of 1859 fully confirmed this opinion. Rejecting from the observations of 1843–8 those made on two nights, when less than 6 measures were obtained, the result of 1859 differs only 8′ from that of 1843–8. I was thus enabled, for the purposes of this inquiry, to treat these observations of 1843–8, 156 in number, as if they had all been made within an interval of time not greater than about two months. Now these observations had been made by three different observers, and while the results of separate nights were very discordant, the probable errors derived from the partial values of nights, the results of which differed greatly from the general mean, were as remarkably small; on the other hand, the observations of the same star in 1859, 215 in number, were by one observer only, and the results of different nights agree very closely. The applications of the three methods to the early and late observations of this star therefore illustrate very strikingly the effect produced by discordancy in the values obtained on different nights, when the peculiarities of the object observed

are eliminated. Thus the good observations of 1859 give the e^2 equal to 89′ only, while in those of 1843–8 the e^2 attains the great value of 8731′.

The values of f^2 given by the observations of the three stars accord very well, considering the different circumstances under which they were obtained. It will be seen also that little effect is produced on the mean result by using these different methods of reduction.

In the account of the American Coast Survey of 1856, and at pages 307–8, will be found a formula by which the probable error is deduced from the differences from the mean alone, the probable error or $P = 0 \cdot 845347 \frac{\Sigma \epsilon}{n \sqrt{n-1}}$, where ϵ represents the error of a single observation.

I have tested this in the case of three stars in which n was equal to 6, 10, and 156, respectively, and the probable error deduced was a little greater in the first two cases, and a very little smaller in the last.

Computation of P by Method 1 for Σ 1596, 2 Com. Ber.

Epoch.	Sum of Measures.	No. of Meas.
	° ′	
1859·238	22 29	10
·241	9 53	6
·244	21 43	10
·244	15 44	10
·244	17 25	10
·244	20 10	10
·260	13 31	10
·260	13 42	10
·260	14 27	10
·260	26 1	10
·260	20 48	10
·263	13 28	9
·288	16 45	10
·288	22 24	10
·290	15 15	10
·293	11 40	10
·293	18 4	10
·293	21 49	10
·296	12 1	10
·296	16 26	10
·296	24 30	10
·296	21 35	10
22)1·547	389 50	÷215
	= 1 49	
1859·270	+ 30 0	
	31 49	
Zero	=450 19	
	418 30	
	238° 30′ =Mean Position.	

Errors.	ϵ^2.	Errors.	ϵ^2.	Errors.	ϵ^2.	Errors.	ϵ^2.
36	1296	82	6724	32	1024	53	2809
75	5625	26	676	40	1600	137	18769
39	1521	115	13225	21	441	22	484
42	1764	59	3481	48	2304	18	324
46	2116	15	225	63	3969	19	361
8	64	19	361	23	529	20	400
42	1764	12	144	6	36	2	4
9	81	29	841	29	841	65	4225
76	5776	167	27889	61	3721	58	3360
42	1764	34	1156	83	6889	3	9
58	3364	33	1089	14	196	20	400
23	529	53	2809	33	1089	9	81
54	2916	68	4624	14	196	33	1089
35	1225	94	8836	31	961	86	7396
12	144	68	4624	71	5041	79	6241
5	25	25	625	88	7744	33	1089
151	22801	35	1225	102	10404	100	10000
89	7921	3	9	69	4761	104	10816
61	3721	13	169	63	3969	35	1225
32	1024	13	169	21	441	60	3600
47	2209	78	6084	40	1600	10	100
34	1156	13	169	29	841	8	64
47	2209	39	1521	5	25	61	3721
67	4489	126	15876	17	289	57	3249
84	7056	78	6084	10	100	17	289
5	25	58	3364	32	1024	58	3364
19	361	79	6241	102	10404	7	49
3	9	64	4096	48	2304	17	289
49	2401	100	10000	48	2304	20	400
83	6889	37	1369	5	25	95	9025
41	1681	35	1225	134	17956	23	529
117	13689	94	8836	1	1	74	5476
5	25	39	1521	14	196	134	17956
35	1225	34	1156	21	441	91	8281
0	0	25	625	54	2916	82	6724
68	4624	85	7225	35	1225	41	1681
56	3136	113	12769	27	729	61	3721
14	196	26	676	86	7396	24	576
61	3721	48	2304	54	2916	51	2601
109	11881	105	11025	65	4225	27	729
128	16384	20	400	45	2025	127	16129
32	1024	89	7921	52	2704	8	64
181	32761	54	2916	31	961	92	8464
1	1	121	14641	28	784	56	3136
12	144	44	1936	85	7225	16	256
77	5929	131	17161	34	1156	109	11881
16	256	31	961	98	9604	204	41616
94	8836	87	7569	17	289	149	22201
10	100	73	5329	0	0	103	10609
149	22201	74	5476	94	8836	145	21025
76	5776	58	3364	34	1156		
68	4624	69	4761	47	2209		276891
92	8464	58	3364	25	625		152076
23	529	93	8649	23	529		266740
50	2500	35	1225	30	900		241952
	241952		266740		152076	S ϵ^2=	937659

Method 1 (*continued*).

Log. of 937659 = $\bar{5}$·9720449
Constant = $\bar{6}$·9950980

$P^2 = 9''·271$ = 0·9671429
$P = 3''·045$ = 0·4835715

W = ·1079

Log. of 215 = 2·3324385
Log. of 214 = 2·3304138

4·6628523

Log. of ·454936 = $\bar{1}$·6579503
4·6628523

Constant = $\bar{6}$·9950980

Method 2.

Mean Position = 238° 30′.

Errors.	ϵ^2.	Errors.	ϵ^2.
26	676	19	361
10	100	8	64
21	441	25	625
15	225	17	289
4	16	39	1521
12	144	1	1
28	784	22	484
27	729	37	1369
22	484	10	100
47	2209	38	1444
16	256	21	441
	6064		6699
			6064
		$\Sigma\,\epsilon^2=$	12763

Log. of 12763 = $\underline{4}$·10595
n = 22, Constant = $\bar{4}$·99331

1·09926 = λ of P^2
0·54963 = λ of P

$P^2 = 12''·568$
$P = 3''·545$
W = ·0796

Method 3.

$15063 = S_1$ — $9f^2 = CS_1$
$7583 = S_2$ — $5f^2 = CS_2$
$218206 = S_3$ — $39f^2 = CS_3$
$241621 = S_4$ — $49f^2 = CS_4$
$24012 = S_5$ — $8f^2 = CS_5$
$48249 = S_6$ — $19f^2 = CS_6$
$34511 = S_7$ — $9f^2 = CS_7$
$85199 = S_8$ — $29f^2 = CS_8$
$245830 = S_9$ — $39f^2 = CS_9$

920274 — $206f^2 = C \times (S_1 + S_2 + \&c.)$

Log. of 920274 = $\underline{5}$·9639172
Log. of C = ·4549 &c. = $\bar{1}$·6579503

5·6218675
Log. of 206 = 2·3138672

$f^2 = 2032''·4$ = 3·3080003

Method 3 (*continued*).

$b_1 = 32^\circ\ 15'$	$b_1 - b = 26$ & $(b_1 - b)^2 =$	676
$b_2 = 31\ \ 39$	$b_2 - b = 10$	100
$b_3 = 31\ \ 53$	$b_3 - b = \ \ 4$	16
$b_4 = 31\ \ 46$	$b_4 - b = \ \ 3$	9
$b_5 = 31\ \ 30$	$b_5 - b = 19$	361
$b_6 = 31\ \ 57$	$b_6 - b = \ \ 8$	64
$b_7 = 31\ \ 32$	$b_7 - b = 17$	289
$b_8 = 31\ \ 43$	$b_8 - b = \ \ 6$	36
$b_9 = 31\ \ 52$	$b_9 - b = \ \ 3$	9
$b = 31\ \ 49$		$\Sigma = 1560$

See Method 1.

Log. of 1560	$= 3\cdot1931246$
Constant Log.	$= \bar{1}\cdot6579503$
	$2\cdot8510749$
Log. of $8 = m - 1$	$= 0\cdot9030900$
$e^2 = 88'\cdot71$	$= 1\cdot9479849$

$$P = \frac{1}{\sqrt{\left\{\left(\frac{10}{10\times88\cdot7+2032}\right)+\left(\frac{6}{6\times88\cdot7+2032}\right)+\left(\frac{40}{40\times88\cdot7+2032}\right)+\left(\frac{50}{50\times88\cdot7+2032}\right)+\left(\frac{9}{9\times88\cdot7+2032}\right)+\left(\frac{20}{20\times88\cdot7+2032}\right)+\left(\frac{10}{10\times88\cdot7+2032}\right)+\left(\frac{30}{30\times88\cdot7+2032}\right)+\left(\frac{40}{40\times88\cdot7+2032}\right)\right\}}}$$

$10\times88\cdot7 =$	887		
	$+\,2032$	$\lambda\ 10 =$	$1\cdot$
	2919	& λ	$= 3\cdot46523$
1st W	$= \cdot003426$		$= \bar{3}\cdot53477$
$6\times88\cdot7 =$	$532\cdot2$		
	$+\,2032\cdot$	$\lambda\ 6 =$	$0\cdot77815$
	$2564\cdot2$	& λ	$= 3\cdot40895$
2nd W	$= \cdot002340$		$= \bar{3}\cdot36920$
$40\times88\cdot7 =$	3548		
	$+\,2032$	$\lambda\ 40 =$	$1\cdot60206$
	5580	& λ	$= 3\cdot74663$
3rd W	$= \cdot007169$		$= \bar{3}\cdot85543$

$50\times88\cdot7 =$	4435		
	$+\,2032$	$\lambda\ 50 =$	$1\cdot69897$
	6467	& λ	$= 3\cdot81070$
4th W	$= \cdot007732$		$= \bar{3}\cdot88827$
$9\times88\cdot7 =$	$798\cdot3$		
	$+\,2032\cdot$	$\lambda\ 9 =$	$0\cdot95424$
	$2830\cdot3$	& λ	$= 3\cdot45184$
5th W	$= \cdot003180$		$= \bar{3}\cdot50240$
$20\times88\cdot7 =$	1774		
	$+\,2032$	$\lambda\ 20 =$	$1\cdot30103$
	3806	& λ	$= 3\cdot58047$
6th W	$= \cdot005255$		$= \bar{3}\cdot72056$

Method 3 (*continued*).

7th W = ·003426

9th W = ·007169

$30 \times 88{\cdot}7 =$ 2661
+2032
4693

8th W = ·006393

λ 30 = 1·47712
& λ = 3·67145
= $\bar{3}$·80567

1st = ·003426
2nd = ·002340
3rd = ·007169
4th = ·007732
5th = ·003180
6th = ·005255
7th = ·003426
8th = ·006393
9th = ·007169

W = ·046090

λ of 1 = 0·
λ of ·04609 = $\bar{2}$·66361

1·33639 = λ of P^2
0·66820 = λ of P

$P^2 = 21'{\cdot}70$
$P = 4'{\cdot}658$
W = ·0461

$a_1=75$, $a_2=39$, $a_3=53$, $a_4=46$, $a_5=30$, $a_6=57$, $a_7=32$, $a_8=43$, $a_9=52$

$W_1=34$, $W_2=23$, $W_3=72$, $W_4=77$, $W_5=32$, $W_6=53$, $W_7=34$, $W_8=64$, $W_9=72$

461

31°+

75	39	53	46	30	57
34	23	72	77	32	53
300	117	106	322	60	171
225	78	371	322	90	285
2550	897	3816	3542	960	3021

32	43	52
34	64	72
128	172	104
96	258	364
1088	2752	3744

2550
897
3816
3542
960
3021
1088
2752
3744

461)22370(48·5
1844

3930
3688

2420
2305

∴ Mean result = 31° 49′
Zero = 450 19

418 30

Mean Angle = 238° 30′

Σ 3049, σ Cassiopeæ. Computation of P. Method 1.

Epoch.	Sum.	No. of Meas.
	° ′	
1858·572	32 48	6
·712	32 54	6
·821	64 27	10
·857	64 13	10
·857	60 25	10
·876	56 9	10
·876	50 43	10
·876	63 0	10
·887	53 33	10
·887	53 0	10
·890	56 58	10
·890	63 2	10
12)10·001	651 12	÷112
	= 5 49	
1858·833	+120 0	
	125 49	
Zero	=450 15	
	324° 26′ =Mean Position.	

Errors.	ϵ^2.	Errors.	ϵ^2.	Errors.	ϵ^2.
132	17424	145	21025	14	196
18	324	31	961	159	25281
35	1225	16	256	10	100
30	900	9	81	1	1
66	4356	11	121	77	5929
47	2209	22	484	21	441
21	441	38	1444	32	1024
77	5929	56	3136	41	1681
163	26569	28	784	143	20449
89	7921	17	289	33	1089
35	1225	55	3025	5	25
17	289	45	2025	42	1764
7	49	39	1521	84	7056
21	441	26	676	85	7225
171	29241	97	9409	89	7921
63	3969	24	576	152	23104
54	2916	54	2916	54	2916
18	324	120	14400	115	13225
201	40401	95	9025	61	3721
75	5625	103	10609	153	23409
46	2116	78	6084	66	4356
97	9409	32	1024	67	4489
119	14161	89	7921	53	2809
64	4096	35	1225	6	36
56	3136	49	2401	61	3721
41	1681	72	5184	37	1369
14	196	86	7396	19	361
73	5329	15	225	30	900
105	11025	36	1296	4	16
41	1681	81	6561	28	784
32	1024	69	4761	106	11236
148	21904	5	25	75	5625
110	12100	97	9409	11	121
101	10201	9	81	37	1369
40	1600	46	2116	95	9025
73	5329	131	17161	52	2704
103	10609			65	4225
				87	7569
				16	256
	267375		155633		207528
					155633
					267375
				S ϵ^2=	630536

Log. of 112 = 2·0492180
Log. of 111 = 2·0453230

4·0945410

Log. of ·454936 = $\bar{1}$·6579503
4·0945410

Constant = $\bar{5}$·5634093

Log. of 630536 = 5·7997099
Constant = $\bar{5}$·5634093

1·3631192
0·6815596

$P^2 = 23''·074$
$P = 4''·8035$
$W =$ ·0433

Method 2.

° ′
324 47
324 46
323 48
323 50
324 11
324 38
325 11
323 57
324 54
324 57
324 33
323 57
12) 53 29
Mean = 324 27

Errors.	e^2.
20	400
19	361
39	1521
37	1369
16	256
11	121
44	1936
30	900
27	729
30	900
6	36
30	900
Σe^2	= 9429

Log. of 9429 = 3·97447
Constant = $\bar{3}$·53738

1·51185
0·75593

$P^2 = 32''\cdot50$
$P = 5''\cdot701$
$W = \cdot0308$

Log. 12 = 1·07918
Log. 11 = 1·04139

2·12057

Log. ·4549 &c. = $\bar{1}$·65795
2·12057

Constant = $\bar{3}$·53738

Method 3.

$23792 = S_1$
$39974 = S_2$
$80279 = S_3$
$114116 = S_4$
$113454 = S_5$
$123083 = S_6$
$81960 = S_7$

576658

$5f^2 = CS_1$
$5f^2 = CS_2$
$9f^2 = CS_3$
$19f^2 = CS_4$
$29f^2 = CS_5$
$19f^2 = CS_6$
$19f^2 = CS_7$

$105f^2 = C \times (S_1 + S_2 + \&c.)$

Log. of 576658 = 5·7609183
Log. of ·4549 &c. = $\bar{1}$·6579503

5·4188686
Log. of 105 = 2·0211893

$f^2 = 2498''\cdot5$ = 3·3976793

Method 3 (*continued*).

	° ′		
$b_1=125$	28	$b_1-b=21$ & $(b_1-b)^2=$	441
$b_2=125$	29	$b_2-b=20$	400
$b_3=126$	27	$b_3-b=38$	1444
$b_4=126$	14	$b_4-b=25$	625
$b_5=125$	40	$b_5-b=9$	81
$b_6=125$	20	$b_6-b=29$	841
$b_7=126$	0	$b_7-b=11$	121
$b=125$	49	$\Sigma=$	3953

See Method 1.

Log. of 3953	$=3{\cdot}5969268$
Log. of ·4549 &c.	$=\bar{1}{\cdot}6579503$
	$3{\cdot}2548771$
Log. of $6=m-1$	$=0{\cdot}7781513$
$e^2=299''{\cdot}73$	$=2{\cdot}4767258$

$$P=\frac{1}{\sqrt{\left\{\left(\frac{6}{6\times 299{\cdot}73+2498{\cdot}5}\right)+\left(\frac{6}{6\times 299{\cdot}73+2498{\cdot}5}\right)+\left(\frac{10}{10\times 299{\cdot}73+2498{\cdot}5}\right)+\left(\frac{20}{20\times 299{\cdot}73+2498{\cdot}5}\right)+\left(\frac{30}{30\times 299{\cdot}73+2498{\cdot}5}\right)+\left(\frac{20}{20\times 299{\cdot}73+2498{\cdot}5}\right)+\left(\frac{20}{20\times 299{\cdot}73+2498{\cdot}5}\right)\right\}}}$$

$6\times 299{\cdot}73=1798{\cdot}38$			
	$+2498{\cdot}5$	$\lambda\ 6=$	$0{\cdot}77815$
	$4296{\cdot}88$	& λ	$=3{\cdot}63315$
1st W	$={\cdot}001396$		$=\bar{3}{\cdot}14500$
2nd W	$={\cdot}001396$		
$10\times 299{\cdot}73=2997{\cdot}3$			
	$+2498{\cdot}5$	$\lambda\ 10=$	$1{\cdot}$
	$5495{\cdot}8$	& λ	$=3{\cdot}74003$
3rd W	$={\cdot}001820$		$=\bar{3}{\cdot}25997$

$20\times 299{\cdot}73=5994{\cdot}6$			
	$+2498{\cdot}5$	$\lambda\ 20=$	$1{\cdot}30103$
	$8493{\cdot}1$	& λ	$=3{\cdot}92907$
4th W	$={\cdot}002355$		$=\bar{3}{\cdot}37196$
$30\times 299{\cdot}73=8991{\cdot}9$			
	$+2498{\cdot}5$	$\lambda\ 30=$	$1{\cdot}47712$
	$11490{\cdot}4$	& λ	$=4{\cdot}06034$
5th W	$={\cdot}002611$		$=\bar{3}{\cdot}41678$
6th W	$={\cdot}002355$		
7th W	$={\cdot}002355$		

Method 3 (*continued*).

1st $=\cdot 001396$
2nd $=\cdot 001396$
3rd $=\cdot 001820$
4th $=\cdot 002355$
5th $=\cdot 002611$
6th $=\cdot 002355$
7th $=\cdot 002355$

W $=\cdot 014288$

λ of 1 $= 0\cdot$
λ of $\cdot 014288 = \bar{2}\cdot 15497$

$1\cdot 84503 = \lambda$ of P^2
$0\cdot 92252 = \lambda$ of P

$P^2 = 69''\cdot 99$
$P = 8''\cdot 366$
$W = \cdot 01429$

		$125^\circ +$
$a_1 = 28$	$W_1 = 14$	$28 \times 14 = 392$
$a_2 = 29$	$W_2 = 14$	$29 \times 14 = 406$
$a_3 = 87$	$W_3 = 18$	$87 \times 18 = 1566$
$a_4 = 74$	$W_4 = 24$	$74 \times 24 = 1776$
$a_5 = 40$	$W_5 = 26$	$40 \times 26 = 1040$
$a_6 = 20$	$W_6 = 24$	$20 \times 24 = 480$
$a_7 = 60$	$W_7 = 24$	$60 \times 24 = 1440$
	144	144) 7100(49·3

576
———
1340
1296
———
440
432

$\therefore$ Mean result $= 125^\circ\ 49'$
Zero $= 450\ \ 15$

Mean Angle $= 324^\circ\ 26'$

Summary of Results.

Name of Star.	Mean Position.	No. of Measures.	Sum of ϵ^2.	e^2.	f^2.	P^2.	P.	W.	Method
Σ 796	61° 8′	60	271,370	......′	......′	34′·88	5′·91	29	1
	61 8	,,	....	......	......	53·97	7·35	19	2
	61 4	,,	....	310·1	2026·7	117·7	10·85	9	3
Σ 1596, 2 Com. Ber.	238 38	156	3,529,521	......	......	66·41	8·15	15	1
Early observations	238 36	,,	....	......	......	545·61	23·36	2	2
1843–8.	238 36	,,	....	8731·0	2178·0	559·85	23·66	2	3
Σ 1596 *continued.*	238 30	215	937,659	......	......	9·27	3·05	108	1
Observations of	238 30	,,	....	......	......	12·57	3·55	80	2
1859.	238 30	,,	....	88·7	2032·4	21·70	4·66	46	3
Σ 3049, σ Cassio-	324 26	112	630,536	......	......	23·07	4·80	43	1
peæ.	324 27	,,	....	......	......	32·50	5·70	31	2
	324 26	,,	....	299·7	2498·5	69·99	8·37	14	3