sides, there is but one form of relation: if, for instance, in the equation [12]=0, which is the condition for a proper dodecagon, the function [12] could be decomposed into rational factors; then equating each of these factors to zero, we should have so many distinct forms of relation for a proper dodecagon. I believe that the assumption and reasoning are valid; but without entering further into this, I take it for granted that in the general case the functions [3], [4], &c. are in fact prime. But the coefficients β, γ, δ, or b, c, d, instead of being so many independent arbitrary quantities, may be given as rational functions of other quantities (if, for instance, the two conics are circles, radii R, r, and distance between the centres a, then β, γ, δ will be functions of R, r, a): and it is in a case of this kind quite conceivable that the functions [3], [4], &c., considered as functions of these new elements, should cease to be prime functions. In fact, in the case just referred to of the two circles (the original case of the Porism as considered by Füss), the functions [4], [6], &c., which correspond to a polygon of an *even* number of sides, appear to be each of them decomposable into two factors: the memoir contains some remarks tending to show *à priori* that in the case in question this decomposition takes place. I was led to examine the point by the elegant formulæ obtained in an essentially different manner by M. Mention, Bull. de l'Acad. de St. Pét. t. i. pp. 15, 30 and 507 (1860), in reference to the case of the two circles (it thereby appears that the decomposition takes place for the quadrangle and the hexagon); and these formulæ are reproduced in the memoir.

II. "On a New Auxiliary Equation in the Theory of Equations of the Fifth Order." By Arthur Cayley, Esq., F.R.S.

Received February 20, 1861.

(Abstract.)

Considering the equation of the fifth order, or quintic equation, $(*\between v, 1)^5=(v-x_1)(v-x_2)(v-x_3)(v-x_4)(v-x_5)=0$, and putting as usual $f\omega=x_1+\omega x_2+\omega^2 x_3+\omega^3 x_4+\omega^4 x_5$, where ω is an imaginary fifth root of unity, then, according to Lagrange's general theory for the solution of equations, $f\omega$ is the root of an equation of the order 24, called the Resolvent Equation, but the solution whereof depends

ultimately on an equation of the sixth order, viz. $(f\omega)^5$, $(f\omega^2)^5$, $(f\omega^3)^5$, $(f\omega^4)^5$ are the roots of an equation of the fourth order, each coefficient whereof is determined by an equation of the sixth order; and moreover the other coefficients can be all of them rationally expressed in terms of any one coefficient assumed to be known; the solution thus depends on a single equation of the sixth order. In particular the last coefficient, or $(f\omega\,.f\omega^2\,.f\omega^3\,.f\omega^4)^5$, is determined by an equation of the sixth order; and not only so, but its fifth root, or $f\omega\,.f\omega^2\,.f\omega^3\,.f\omega^4$ (which is a rational function of the roots, and is the function called by Mr. Cockle the Resolvent Product), is also determined by an equation of the sixth order: this equation may be called the Resolvent-Product Equation. But the recent researches of Mr. Cockle and Mr. Harley* show that the solution of an equation of the fifth order may be made to depend on an equation of the sixth order, originating indeed in, and closely connected with, the resolvent-product equation, but of a far more simple form: this is the auxiliary equation referred to in the title of the present memoir. The connexion of the two equations, and the considerations which led to the new one, are pointed out in the memoir; but I will here state synthetically the construction of the auxiliary equation. Representing for shortness the roots x_1, x_2, x_3, x_4, x_5, of the given quintic equation by 1, 2, 3, 4, 5, and putting moreover

$$12345=12+23+34+45+51, \text{ \&c.}$$

(where on the right-hand side 12, 23, &c. stand for x_1x_2, x_2x_3, &c.), then the auxiliary equation, say

$$(*\between\phi,\ 1)^6=0$$

has for its roots

$$\begin{aligned} \phi_1&=12345-24135, & \phi_4&=21435-13245,\\ \phi_2&=13425-32145, & \phi_5&=31245-14325,\\ \phi_3&=14235-43125, & \phi_6&=41325-12435, \end{aligned}$$

and, it follows therefrom, is of the form

$$(1,\ 0,\ C,\ 0,\ E,\ F,\ G\between\phi,\ 1)_6=0,$$

* Cockle, "Researches in the Higher Algebra," Manchester Memoirs, t. xv. pp. 131–142 (1858).

Harley, "On the Method of Symmetric Products, and its Application to the Finite Algebraic Solution of Equations," Manchester Memoirs, t. xv. pp. 172–219 (1859).

Harley, "On the Theory of Quintics," Quart. Math. Journal, t. iii. pp. 343–359 (1859).

where C, E, G are rational and integral functions of the coefficients of the given equation, being in fact seminvariants, and F is a mere numerical multiple of the square root of the discriminant.

The roots of the given quintic equation are each of them rational functions of the roots of the auxiliary equation, so that the theory of the solution of an equation of the fifth order appears to be now carried to its extreme limit. We have in fact

$$\begin{aligned}
\phi_1\phi_6+\phi_2\phi_4+\phi_3\phi_5&=(*\between x_1,\ 1)^4,\\
\phi_1\phi_2+\phi_3\phi_4+\phi_5\phi_6&=(*\between x_2,\ 1)^4,\\
\phi_1\phi_5+\phi_2\phi_3+\phi_4\phi_6&=(*\between x_3,\ 1)^4,\\
\phi_1\phi_3+\phi_2\phi_6+\phi_3\phi_5&=(*\between x_4,\ 1)^4,\\
\phi_1\phi_4+\phi_2\phi_5+\phi_3\phi_6&=(*\between x_5,\ 1)^4,
\end{aligned}$$

where $(*\between x_1,\ 1)^4$, &c. are the values, corresponding to the roots x_1, &c. of the given equation, of a given quartic function. And combining these equations respectively with the quintic equations satisfied by the roots x_1, &c. respectively, it follows that, conversely, the roots x_1, x_2, &c. are rational functions of the combinations $\phi_1\phi_6+\phi_2\phi_4+\phi_3\phi_5$, $\phi_1\phi_2+\phi_3\phi_4+\phi_5\phi_6$, &c. respectively, of the roots of the auxiliary equation.

It is proper to notice that, combining together in every possible manner the 6 roots of the auxiliary equation, there are in all 15 combinations of the form $\phi_1\phi_2+\phi_3\phi_4+\phi_5\phi_6$. But the combinations occurring in the above-mentioned equations are a completely determinate set of five combinations: the equation of the order 15, whereon depend the combinations $\phi_1\phi_2+\phi_3\phi_4+\phi_5\phi_6$, is not rationally decomposable into three quintic equations, but only into a quintic equation having for its roots the above-mentioned five combinations, and into an equation of the tenth order, having for its roots the other ten combinations, and being an irreducible equation. Suppose that the auxiliary equation and its roots are known; the direct method of ascertaining what combinations of roots correspond to the roots of the quintic equation would be to find the rational quintic factor of the equation of the fifth order, and observe what combinations of the roots of the auxiliary equation are also roots of this quintic factor. The direct calculation of the auxiliary equation by the method of symmetric functions would, I imagine, be very laborious. But the

coefficients are seminvariants, and the process explained in my memoir on the Equation of Differences was therefore applicable, and by means of it, the equation is readily obtained. The auxiliary equation gives rise to a corresponding covariant equation, which is given at the conclusion of the memoir.

III. "On Combustion in Rarefied Air." By Dr. EDWARD FRANKLAND, F.R.S. Received February 28, 1861.

In the autumn of 1859, whilst accompanying Dr. Tyndall to the summit of Mont Blanc, I undertook at his request some experiments on the effect of atmospheric pressure upon the amount of combustible matter consumed by a common candle. I found that, taking the average of five experiments, a stearin candle diminished in weight 9·4 grammes when burnt for an hour at Chamounix; whilst its ignition for the same length of time on the summit of Mont Blanc, perfectly protected from currents of air, reduced its weight to the extent of 9·2 grammes.

This close approximation to the former number under such a widely different atmospheric pressure,* goes far to prove that the rate of combustion is entirely independent of the density of the atmosphere.

It is impossible to repeat these determinations in a satisfactory manner with artificially rarefied atmospheres, owing to the heating of the apparatus which surrounds the candle, and the consequent guttering and unequal combustion of the latter; but an experiment in which a sperm candle was burnt first in air under a pressure of 28·7 inches of mercury, and then in air at 9 inches pressure, other conditions being as similar as possible in the two experiments, the consumption of sperm was found to be,—

At pressure of	28·7	inches	7·85	grms. of sperm	per hour,	
,,	9·0	,,	9·10	,,	,,	

thus confirming, for higher degrees of rarefaction, the result previously obtained.

In burning the candles upon the summit of Mont Blanc, I was much struck by the comparatively small amount of light which they emitted. The lower and blue portion of the flame, which under