

IV. "On the Double Tangents of a Curve of the Fourth Order."

By ARTHUR CAYLEY, Esq., F.R.S. Received May 30, 1861.

(Abstract.)

The present memoir is intended to be supplementary to that "On the Double Tangents of a Plane Curve," Phil. Trans. vol. cxlix. (1859) pp. 193-212. I take the opportunity of correcting an error which I have there fallen into, and which is rather a misleading one, viz. the emanants U_1, U_2, \dots were numerically determined in such manner as to become equal to U on putting (x_1, y_1, z_1) equal to (x, y, z) ; the numerical determination should have been (and in the latter part of the memoir is assumed to be) such as to render $H_1, H_2, \&c.$ equal to H , on making the substitution in question; that is, in the place of the formulæ

$$U_1 = \frac{1}{n} (x_1 \partial_x + y_1 \partial_y + z_1 \partial_z) U,$$

$$U_2 = \frac{1}{n(n-1)} (x_1 \partial_x + y_1 \partial_y + z_1 \partial_z)^2 U, \&c.,$$

there ought to have been

$$U_1 = \frac{1}{n-2} (x_1 \partial_x + y_1 \partial_y + z_1 \partial_z)^2 U,$$

$$U_2 = \frac{1}{(n-2)(n-3)} (x_1 \partial_x + y_1 \partial_y + z_1 \partial_z)^2 U, \&c.$$

The points of contact of the double tangents of the curve of the fourth order or quartic $U=0$, are given as the intersections of the curve with a curve of the fourteenth order $\Pi=0$; such last-mentioned curve is not absolutely determinate, since instead of $\Pi=0$, we may, it is clear, write $\Pi+MU=0$, where M is an arbitrary function of the tenth order. I have in the memoir spoken of Hesse's original form (say $\Pi_1=0$) of the curve of the fourteenth order obtained by him in 1850, and of his transformed form (say $\Pi_2=0$) obtained in 1856. The method in the memoir itself (Mr. Salmon's method) gives, in the case in question of a quartic curve, a third form, say $\Pi_3=0$. It appears by the paper "On the Determination of the Points of Contact of Double Tangents to an Algebraic Curve," Quart. Math. Journ. vol. iii. p. 317 (1859), that Mr. Salmon has verified by algebraic transformations the equivalence of the last-mentioned form with those of Hesse, but the process is not given. The object of the present memoir is to demonstrate the equivalence in question,

viz. that of the equation $\Pi_3=0$ with the one or other of the equations $\Pi_1=0$, $\Pi_2=0$, in virtue of the equation $U=0$. The transformation depends, 1st, on a theorem used by Hesse for the deduction of his second form $\Pi_2=0$ from the original form $\Pi_1=0$, which theorem is given in his paper "Transformation der Gleichung der Curven 14ten Grades welche eine gegebene Curve 4ten Grades in den Berührungspunkten ihrer Doppeltangenten schneiden," Crelle, t. lii. pp. 97-103 (1856), containing the transformation in question: I prove this theorem in a different and (as it appears to me) a more simple manner; 2nd, on a theorem relating to a cubic curve proved incidentally in my memoir "On the Conic of Five-pointic Contact at any point of a Plane Curve," Phil. Trans. vol. cxlix. (1859), see p. 385, the cubic curve being in the present case any first emanant of the given quartic curve: the demonstration occupies only a single paragraph, and it is here reproduced; and I reproduce also Hesse's demonstration of the equivalence of the two forms $\Pi_1=0$ and $\Pi_2=0$.

V. "Notes on the Atmospheric Lines of the Solar Spectrum, and on certain Spectra of Gases." By Dr. JOHN HALL GLADSTONE, F.R.S. Received May 30, 1861.

In the paper of Sir David Brewster and myself on the lines of the solar spectrum *, attention was drawn to the following among other phenomena:—

1st. "When the sun descends towards the horizon and shines through a rapidly increasing depth of air, certain lines which before were little if at all visible, became black and well defined, and dark bands appear even in what were formerly the most luminous parts of spectrum." These we termed "atmospheric lines." We did not wish to express by that term anything beyond the fact above mentioned; yet we threw out the idea that these lines may have their origin "in the air that encircles our globe."

2nd. In the case of those artificial flames whose spectra "consist of a series of luminous bands separated by dark spaces . . . these luminous bands sometimes coincide with the dark lines of the solar spectrum."

About the same time Kirchhoff† published his theory that this remarkable coincidence is due to the presence in the atmosphere of

* Philosophical Transactions, 1860, p. 149.

† Pogg. Ann. cix. pp. 148, 275; cx. p. 187.