March 14, 1861.

Major-General SABINE, R.A., Vice-President and Treasurer, in the Chair.

The following communications were read :—

I. "On an Application of the Theory of Scalar and Clinant Radical Loci." By ALEXANDER J. ELLIS, Esq., B.A., F.C.P.S. Communicated by ARTHUR CAYLEY, Esq. Received February 20, 1861.

(Abstract.)

This investigation is in correction and extension of Plücker's theory of transversals (System der Geometrie, § 3, art. 64), and is founded on the theories explained in the 'Proceedings,' vol. x. pp. 415–426.

It is shown that if $f(x, y)$ be an algebraical formation (function) of $n+2m$ dimensions, such that when x is scalar (possible) $f=0$ has n scalar and $2m$ clinant (imaginary) roots, and λ is the coefficient of y^{n+2m}, the value of $f(x_1, y_1)$ may be represented geometrically by

$$\lambda \,.\, \frac{M_1Q}{OB} \,.\, \frac{M_2Q}{OB} \,..\, \frac{M_nQ}{OB} \times \frac{M'_1Q}{OB} \,.\, \frac{M'_2Q}{OB} \cdots \frac{M_1{}^{(m)}Q}{OB} \,.\, \frac{M_2{}^{(m)}Q}{OB},$$

where O is the origin, $OP=x_1 \,.\, OI$, $PQ=y_1 \,.\, OB$, and PQ is any straight line, cutting the curve whose equations are

$$OM=x \,.\, OI+y \,.\, OB, \qquad f(x, y)=0,$$

(where x, y are scalar, OI is in the direction OP, OB is of the same length as OI and in the direction PQ) in the n points $M_1, M_2 \,..\, M_n$, and where $M'_1, M'_2 \ldots M_1{}^{(m)}, M_2{}^{(m)}$ are determined as follows.

Put $y=r+\sqrt{-1} \,.\, s$, where r and s are scalar, reduce $f(x, y)$ to the form $F_1(x, r, s)+\sqrt{-1} \,.\, s^n \,.\, F_2(x, r, s)$, put $F_1=0$, $F_2=0$, from which equations find by elimination $F_3(x, r)=0$, $F_4(x, s)=0$, and construct the loci of R and S, where

$$OR=x \,.\, OI+r \,.\, OB, \qquad F_3(x, r)=0,$$
$$OS=x \,.\, OI+s \,.\, OB, \qquad F_4(x, s)=0.$$

Then determine the points R', R'' ... $R^{(m)}$ in which PQ cuts the first, and the corresponding pairs of points S_1', S_2', S_1'', S_2'' .... $S_1^{(m)}$, $S_2^{(m)}$ in which it cuts the second, and draw $R'M_1'=\sqrt{-1}\,.\,PS_1'$, $R'M_2'=\sqrt{-1}\,.\,PS_2'$, ... $R^{(m)}M_1^{(m)}=\sqrt{-1}\,.\,PS_1^{(m)}$, $R^{(m)}M_2^{(m)}=\sqrt{-1}\,.\,PS_2^{(m)}$.

The loci of M, R, S are the principal scalar and clinant radical loci of the formation $f(x, y)$.

From this it is concluded, that if $O_1 O_2 \ldots O_n O_1$ be a completely enclosed polygon, the sides of which are taken in order, to replace PQ, first in the direction $O_1 O_2$, $O_2 O_3$, &c., and then in the direction $O_2 O_1$, $O_3 O_2$, &c., and the points M_1, $M_2 \ldots M_n$, M_1', $M_2' \ldots M_1^{(m)}$, $M_2^{(m)}$ be determined for each direction and each side, and $[O_{1,2} M \div O_{2,1} M]$ represent the product of the ratios $O_1 M \div O_2 M$, &c., the $O_1 M$ referring to the origin O_1 and the M being determined with reference to the direction $O_1 O_2$, and the $O_2 M$ referring to the origin O_2 and the M being determined with reference to the direction $O_2 O_1$, and so on, then we shall have in all cases,

$$[O_{1,2} M \div O_{2,1} M]\,.\,[O_{2,3} M \div O_{3,2} M] \ldots . [O_{n,1} M \div O_{1,n} M]=1,$$

and not $=\pm 1$, as supposed by Plücker, whose error is traced to its origin, and displayed in all the examples he has given, where the roots of $f=0$ are scalar. This result is then applied to a simple case where some of the roots of $f=0$ are clinant, and a result obtained in accordance with other considerations.

II. "A Seventh Memoir on Quantics." By Arthur Cayley, Esq., F.R.S. Received February 28, 1861.

(Abstract.)

The present memoir relates chiefly to the theory of ternary cubics. Since the date of my Third Memoir on Quantics, M. Aronhold has published the continuation of his researches on ternary cubics, in the memoir "Theorie der homogenen Functionen dritten Grades von drei Veränderlichen," Crelle, t. lv. pp. 97–191 (1858). He there considers two derived contravariants, linear functions of the fundamental ones, and which occupy therein the position which the funda-