

XXI. "Supplement to Two Papers published in the Philosophical Transactions (1820 and 1825) on the Science connected with Human Mortality." By BENJAMIN GOMPERTZ, Esq., F.R.S. Received June 19, 1861.

(Abstract.)

The first of the two Papers referred to is entitled "A Sketch of an Analysis and Notation applicable to the estimation of the value of Life Contingencies." It is especially devoted to the explanation of a new notation and a new mode of analysis applicable to the subject. The second Paper, entitled "On the Nature of the Function expressive of the Law of Human Mortality, and on a new mode of determining Life Contingencies," treats, in its first part, of the equation  $L_x = d \cdot g^x$ , in which  $L_x$  denotes the number of persons who would be living at the age  $x$ , out of the number of persons who may have been living at some given common previous age, in which  $d, g, q$  may be considered constant quantities through a long series of years, and the equations to give the real rate of mortality during that period within a very satisfactory approach to coincidence, say, for instance, from the age ten to the age sixty; a theorem from whence the number living at an assigned age may be derived, by what the author calls the vital rule of three, from the number living, for instance, at the age twenty, forty, and sixty, given in a table of mortality. But the author observes that though the above formula with suitably determined constant values of  $d, g, q$  appears within a great approach of the observed result, to agree with the tabular data through so long a period, still that actually  $d, g, q$  are not constant quantities, as every different selection from the table of the living to which the formula is applied, for the purpose of determining the three constants contained in it, even if that table throughout were an accurate representation of the law of mortality, which no table based on obtainable data can be expected to be, would give different values of  $d, g, q$ , as they are not really absolutely constant, but slightly variable throughout, because the formula  $L_x = d \cdot g^x$  does not represent perfectly the law of mortality, if  $d, g, q$  are perfectly constant; and the author complains of its misinterpretation in this respect by a subsequent writer, who lays claim to discovery on this subject.

The other part of the Paper of 1825 is devoted to a mode with accompanying tables, to enable a calculator with any given table of mortality to calculate the value of an annuity on any number of joint lives to within any degree of accuracy at pleasure ; provided the periods, if long, are divided into parts.

The present supplement, written so many years after the original papers were communicated to the Society, appeared to the author to be called for in order that he might further illustrate the subject, and add his later speculations and improvements. The supplement the author commenced writing about two years ago from collections of his manuscripts, but was prevented by continued ill-health from proceeding with it ; but having received an invitation last year from the International Congress to assist in their meritorious labours, and being unable from indisposition to attend the Meeting, as he stated to the Congress, he offered to send them some hints respecting his recent labours, the results of which he intended to present to the Royal Society, if his health should permit him to finish the papers. The hints which he gave, which were honoured by a place in the Reports of the Congress, were, he believes, deemed interesting, but he thinks they were sufficiently separated from the strictly mathematical part of the subject, and also from the most important portions of the result, to allow him to consider the present paper a new work, or sufficiently new, to be thought worthy of presentation to the Royal Society, and especially because since the notice above alluded to was furnished, the author has been able to introduce important improvements. The original formulæ give one uniform law of mortality from birth to the utmost limit of the table ; but in the case of the table presented to the Congress, and of the present further improved formulæ, if a comparison be made, for instance, with the Carlisle table from birth to the age of 100, and even from birth to one month, to two months, to three months, to six months, and to one year, where the tables appear to be so irregular, and formed on no law, the result appears to show that a law really exists and is available.

The author shows the vast use and applicability to solutions of all intricate inquiries of life contingencies of his present deductions. He gives a theory, which he believes to be quite new, which he calls special, single, and specially influenced contingencies ; a subject which he states he had not lost sight of when officially engaged in

the science of assurance, as he considered that all cases of connected lives, where there might be a connexion of any sort, of influences acting with respect to the probability of co-existence, all questions solved without reference to such influence, were faulty, and might be importantly faulty. He states a striking instance with respect to this observation as follows:—If it be required to assure a sum on the joint lives of A and B, that is, to pay the sum on the first of the death of A and B, whichever may be that first, and if in another case it were required to assure the sum on the death of B in particular, provided he should die in the lifetime of A, it is evident that if A and B are of the same age, and not subject to any influential connexion, that if they are both subject to the same uninfluenced mortality, the price of the first assurance would be exactly double the other. But if the assurance were for a very short time, for instance, in the time a ship was passing from Dover to Calais, and they were both going in the same ship together, the risk of the ship being wrecked would affect them both equally, and the chance of their separate escape would depend on circumstances, for instance, one being a better swimmer than the other; but there would be a chance that neither of them could escape, and as far as that chance is concerned, the assurance on one of the two, whichever it might be, who died first, the assurance on one in particular, or the assurance on the longest of their lives during that voyage would be all of equal value; and the method the author has adopted for all uninfluenced contingencies would be available in all cases of influenced, by the introduction of certain formulæ which the author regards as of a very interesting nature.

In addition to the above particulars, the author presents a theory of sickness, which appears to him to be a near approach to actual statements of sickness occurring among Friendly Societies.

The Society then adjourned over the Long Vacation, to Thursday, November 21.