

were pointed out at the commencement of the paper as forming the essentials of an eye, and considers that it affords a contradiction to the position of Agassiz, "That every great type is formed on a distinct plan,—so peculiar, indeed, that homologies cannot be extended from one type to the other, but are strictly limited to each of them." On the contrary, the eye of an *Asterias* is formed on the same plan as that of a *Planaria* and a *Daphnia*; and the eye of the leech possesses the same parts as that of the *Helix*, while the similarity of the eye of the cephalopod to that of the vertebrata is obvious to all.

January 24, 1861.

Major-General SABINE, R.A., Treasurer and Vice-President,
in the Chair.

The following communications were read :—

- I. "On the Calculus of Symbols, with Applications to the Theory of Differential Equations." By W. H. A. RUSSELL, A.B. Communicated by ARTHUR CAYLEY, Esq. Received December 20, 1860.

(Abstract.)

The calculus of generating functions, discovered by Laplace, was, as is well known, highly instrumental in calling the attention of mathematicians to the analogy which exists between differentials and powers. This analogy was perceived at length to involve an essential identity, and several analysts devoted themselves to the improvement of the new methods of calculation which were thus called into existence. For a long time the modes of combination assumed to exist between different classes of symbols were those of ordinary algebra; and this sufficed for investigations respecting functions of differential coefficients and constants, and consequently for the integration of linear differential equations, with constant coefficients. The laws of combination of ordinary algebraical symbols may be divided into the commutative and distributive laws; and the number of symbols in the higher branches of mathematics, which are commutative with respect to one another, is very small. It became

then necessary to invent an algebra of non-commutative symbols. This important step was effected by Professor Boole, for certain classes of symbols, in his well-known and beautiful memoir published in the Transactions of this Society for the year 1844; and the object of the paper which I have now the honour to lay before the Society is to perfect and develop the methods there employed.

For this purpose I have constructed systems of multiplication and division for functions of non-commutative symbols, subject to the same laws of combination as those assumed in Professor Boole's memoir; and I thus arrive at equations of great utility in the integration of linear differential equations with variable coefficients.

I then proceed to develop certain general theorems, which will, I hope, be found interesting. I have applied the methods of multiplication, as just explained, to deduce theorems for non-commutative symbols analogous to the binomial and multinomial theorems of ordinary algebra.

Lastly, I have shown how to employ the equations deduced in the earlier part of this paper in the integration of linear differential equations. I have, for this purpose, made use of methods closely resembling the method of divisors which has so long been used in resolving ordinary algebraical equations. The whole paper will, I hope, be found to be a step upwards in the important subject of which it treats.

I shall just observe, that the symbolical combinations used in this paper may also be applied to the calculus of finite differences, as may be seen in Professor Boole's memoir.

II. "On the Properties of Liquid Carbonic Acid." By GEORGE GORE, Esq. Communicated by Professor TYNDALL. Received January 17, 1861.

(Abstract.)

In this communication the author has shown how a small quantity of liquid carbonic acid may be readily and safely prepared in glass tubes closed by stoppers of gutta percha, and be brought in a pure state into contact with any solid substance upon which it may be desired to ascertain its chemical or solvent action, or be submitted