of correcting or disregarding certain discrepant appearances or irregularities in the organ of vision; as a further illustration of which I may cite the fact, mentioned by Mr. Warren De la Rue in his "Report on Celestial Photography," that the retinal image of a star is, at least under some atmospheric conditions, made up of "a great number of undulating points," which, however, the mind rightly interprets as the effect of the presence before the eye of a single minute object. That this corrective power is, as might be supposed, very limited, may be proved experimentally by this instrument; for if the small ends be enlarged in only a slight degree, so as to increase this slurring on the retinæ, a very marked diminution in clearness of definition is the immediate result.

That form of the stereotrope, in which Professor Wheatstone's reflecting stereoscope is made use of, and which is better adapted for the exhibition of movements that are not only local but progressive in space, it is needless to describe here, because the principles it involves are essentially the same as those which are stated above.

## III. "On the Lunar-Diurnal Variation of the Magnetic Declination obtained from the Kew Photograms* in the years 1858, 1859, and 1860." By Major-General Edward Sabine, R.A., Treas. and V.P.R.S. Received December 19, 1860†.

Having communicated to the Royal Society in a recent paper an analysis of the *disturbances* of the declination in the years 1858 and 1859, shown by the photograms of the Kew Observatory, I propose in the present paper to submit the results of the *lunar-diurnal variation* of the declination in the years 1858, 1859, and 1860, obtained from the same source. The directions of the declination magnet at the instant of the commencement of every solar-hour having been tabulated from the photograms, and the final normals

* The term Photogram is adopted in place of Photograph in conformity with modern usage.

† [Note added on February 8th, 1861.] When this communication was read to the Royal Society on January 10th, 1861, it contained the lunar-diurnal variation for the years 1858 and 1859 only: whilst it was passing through the press, the calculation of the lunar-diurnal variation for 1860 was completed, and the results in that year have been added.

for each month and hour computed, after the omission from the record of all the hourly directions which deviated 3′·3 from their final normals,—the *differences* were taken between each of the remaining hourly directions and the final normal of the same month and hour, and were entered afresh in *lunar* monthly tables, having the lunar days in successive horizontal lines, and the twenty-four lunar hours in vertical columns, each difference being placed under the lunar hour to which it most nearly approximated. The entries in these tables should consequently represent directly the lunar influence at the different lunar hours, subject only to minor disturbances; the effects of the solar-diurnal variation as well as of the larger disturbances having been eliminated. The differences were marked with a + sign when the north end of the magnet was east of its mean direction, and with the − sign when west of the same. The differences were then summed up, and hourly, monthly, and annual means taken by the non-commissioned officers of the Royal Artillery employed at Woolwich, under the superintendence of Mr. Magrath.

Having in the former paper exhibited the results of the disturbances at Kew in comparison with those at Hobarton, I propose to do the same with the lunar-diurnal variation treated of in this communication; believing that such comparisons are very conducive to a just appreciation of the systematic character and natural reality of the results, and instructive both by the agreements and disagreements which they exhibit. The lunar-diurnal variation at Hobarton has been obtained for the purpose of this comparison, by a similar process to that which has been described above, from observations at every solar hour during five years (Sundays excepted), from Oct. 1, 1843 to Sept. 30, 1848; omitting as disturbed such observations as deviated 2′·13 from their respective final normals. The total number of hourly observations was 36,832; the disturbed observations 2606; and the number employed in the lunar-diurnal variation 34,226. As it has been customary to represent such periodical variations by formulæ of well-known character, the results at Kew and Hobarton are here represented by formulæ in which $a$, corresponding to $x$ (the lunar time for which the lunar-diurnal variation is desired), is counted in hours and parts of an hour, multiplied by 15°, from the epoch of the moon's upper culmination. The + sign corresponds (as before)

to a deflection of the north end of the magnet to the east of its mean place, and the — sign to the west.

Kew $\Delta x = +0''\cdot64 - 2''\cdot54 \sin(a+6^\circ\cdot2) - 9''\cdot74 \sin(2a+59^\circ\cdot8)$.

Hobarton $\Delta x = -0''\cdot1 + 1''\cdot14 \sin(a+344^\circ\cdot7) + 6''\cdot8 \sin(2a+43^\circ\cdot2)$.

In computing the lunar-diurnal variation by means of these formulæ, the coefficient of the term which includes the sine of twice the hour-angle is of principal importance: the subsequent terms are comparatively of little significance, and are therefore omitted on the present occasion. When all the terms are employed, the original observed values are reproduced.

Table I. exhibits, at Kew, in column 2 the lunar-diurnal variation as actually observed on the mean of the three years, and in column 3, the same computed by the formula. Column 4 is the lunar-diurnal variation at Hobarton on the mean of the five years as observed, and column 5 the same computed by the formula.

Table I.—Lunar-diurnal Variation at Kew and Hobarton.

| Lunar Hours. | Kew. | | Hobarton. | | Lunar Hours. |
|---|---|---|---|---|---|
| | Observed. | Computed. | Observed. | Computed. | |
| Col. 1. | Col. 2. | Col. 3. | Col. 4. | Col. 5. | Col. 6. |
| | $''$ | $''$ | $''$ | $''$ | |
| 0 | − 6·0 | − 8·0 | + 4·8 | + 4·3 | 0 |
| 1 | −11·4 | −10·0 | + 6·1 | + 6·4 | 1 |
| 2 | − 8·6 | − 9·3 | + 5·2 | + 6·8 | 2 |
| 3 | − 5·0 | − 6·2 | + 5·9 | + 5·4 | 3 |
| 4 | − 3·2 | − 1·7 | + 4·2 | + 2·7 | 4 |
| 5 | + 1·4 | + 3·0 | 0·0 | − 0·6 | 5 |
| 6 | + 5·4 | + 6·5 | − 4·9 | − 3·7 | 6 |
| 7 | + 7·6 | + 8·0 | − 6·1 | − 5·5 | 7 |
| 8 | + 8·6 | + 7·0 | − 4·9 | − 5·6 | 8 |
| 9 | + 4·3 | + 3·9 | − 3·3 | − 4·1 | 9 |
| 10 | + 2·8 | − 0·4 | − 3·2 | − 4·3 | 10 |
| 11 | − 3·0 | − 4·6 | + 3·6 | + 2·0 | 11 |
| 12 | −10·6 | − 7·5 | + 4·9 | + 4·9 | 12 |
| 13 | −10·4 | − 8·2 | + 6·6 | + 6·4 | 13 |
| 14 | − 7·0 | − 6·3 | + 5·9 | + 6·2 | 14 |
| 15 | − 2·2 | − 2·3 | + 4·1 | + 4·3 | 15 |
| 16 | + 4·8 | + 3·0 | + 1·4 | + 1·1 | 16 |
| 17 | +10·4 | + 8·0 | − 3·4 | − 2·7 | 17 |
| 18 | +13·2 | +11·6 | − 6·4 | − 5·9 | 18 |
| 19 | +12·6 | +12·7 | − 6·5 | − 7·7 | 19 |
| 20 | + 7·2 | +11·1 | − 6·6 | − 7·8 | 20 |
| 21 | + 6·2 | + 7·1 | − 8·4 | − 6·0 | 21 |
| 22 | − 0·4 | + 1·7 | − 1·9 | − 2·9 | 22 |
| 23 | − 1·4 | − 4·6 | + 0·8 | + 0·9 | 23 |

The aspect of the lunar-diurnal variation at Kew and Hobarton presents features of great simplicity as well as accord. The form at both stations is a division of the 24 lunar hours into four equal or nearly equal portions, in which the magnet is attracted alternately to the east and to the west of its mean position, which is passed through four times in the progress of the magnet towards two extreme easterly and two extreme westerly deflections: the easterly extremes are about 12 hours apart, and the westerly the same. As far as our present experience goes, this appears to be the general form of the lunar-diurnal variation of the declination at all the stations at which it has been examined; it is also that of the corresponding variations of the Dip and Total force. At Hobarton, where the results are obtained from five years of observation, there is scarcely any difference deserving of notice between the amplitudes of the extremes on either side of the upper culmination and those on either side of the lower culmination. At Kew, where the results are obtained from only three years, the extreme deflections are not quite so symmetrical in amount, but they may become more so as additional years are brought into the account. The amplitude of the oscillation on a mean of the two alternations is $9''\cdot74$ at Kew and $6''\cdot8$ at Hobarton, a difference in correspondence with the difference in the opposite direction of the antagonistic retaining force of the earth's magnetism at the two stations, which is 3·7 at Kew and 4·5 at Hobarton. On inspecting the Table, we see that the lunar times when the moon's influence produces no deflection (or the times when the variation is zero), are four, and are nearly the same at Kew and at Hobarton, two of them being a little more than an hour before the moon's passage of the meridian, both at her upper and lower culminations, and the other two intermediate. So far the two stations are alike; but in regard to the direction towards which the magnet is deflected (if in conformity with general usage we speak in both hemispheres of the north end of the magnet, as is done in the Table), we see that the variation becomes west at Kew when it becomes east at Hobarton, and *vice versâ*; the phases, while agreeing in hours at the two stations, having throughout opposite signs.

By extending the comparison of the lunar hours at which the lunar variation passes through its zero-points to other stations than Kew and Hobarton, we are made aware of differences which appear to

deserve particular attention in theoretical respects. At Pekin, for example—which may be advantageously compared with Kew, being both in the same hemisphere, but Pekin some degrees nearer the equator—the variation is zero in the passage of the north end of the magnet from east to west at $20\frac{1}{2}$ lunar hours, or $2\frac{1}{2}$ hours earlier than the corresponding epoch at Kew. Again, at the Cape of Good Hope, situated in the same hemisphere with Hobarton, but some degrees nearer the equator, the variation is zero in the passage of the north end of the magnet from west to east also at $20\frac{1}{2}$ lunar hours, or $2\frac{1}{2}$ hours earlier than the corresponding epoch at Hobarton. Thus there is an accord of precisely the same kind between Pekin and the Cape of Good Hope that there is between Kew and Hobarton, whilst there is a difference between the two pairs of stations of $2\frac{1}{2}$ hours in the position of the moon relatively to the meridian at which she ceases to exercise a deflecting influence on the magnet. Again, at St. Helena, which is in the same (geographical) hemisphere as Hobarton and the Cape of Good Hope, but still nearer to the equator than either, the lunar influence is zero in the passage from west to east at $19\frac{1}{2}$ lunar hours, being one hour earlier than at the Cape, and $3\frac{1}{2}$ hours earlier than at Hobarton.

Where the whole range of the variation of which we have been treating is so small (not more than a few seconds of arc in each lunar day), it may be desirable to show by the accordance of the independent evidence obtained in single years, the degree of confidence which may be placed in the mean results of several years. This may be seen in the Table on the next page, which contains the separate results in each of the five successive years of observation at Hobarton, as well as their mean.

In this Table the principal features of the variation are seen to be substantially alike in each year. The individual results at the several hours in single years are of course somewhat less regular than in the mean of the five years: such small discrepancies are no doubt in great part due to the lesser disturbances which, being below the separating value of 2′·13, have been left in the body of the observations. They slightly disfigure the symmetry of the results in single years, but almost entirely disappear when the mean of several years is taken. In order to appreciate justly and fully the confidence to which the whole investigation is entitled, it must be borne in mind that every single

entry in the Table (exclusive of course of the column which exhibits the mean of the five preceding columns) is derived from a wholly independent body of observations which belong to itself alone, and are not employed in the deduction of any of the other entries.

TABLE II. — Lunar-diurnal Variation at Hobarton in the several years from October 1843 to September 1848; omitting disturbed observations differing 2′·13 from their final normals.

| Lunar Hours. | Years ending September 30th. | | | | | Means. | Lunar Hours. |
|---|---|---|---|---|---|---|---|
| | 1844. | 1845. | 1846. | 1847. | 1848. | | |
| 0 | + 0″·6 | + 7″·8 | + 3″·6 | + 2″·4 | + 9″·6 | + 4″·8 | 0 |
| 1 | + 6·6 | + 9·0 | + 1·2 | + 0·6 | +13·2 | + 6·1 | 1 |
| 2 | + 4·8 | + 5·4 | + 5·4 | + 6·0 | + 4·2 | + 5·2 | 2 |
| 3 | + 9·6 | + 7·8 | + 7·8 | + 3·6 | + 0·6 | + 5·9 | 3 |
| 4 | + 4·8 | + 6·6 | + 6·0 | + 3·0 | + 0·6 | + 4·2 | 4 |
| 5 | − 3·0 | + 2·4 | + 3·0 | − 1·8 | − 0·6 | 0·0 | 5 |
| 6 | − 7·8 | − 6·0 | − 1·8 | − 7·8 | − 1·2 | − 4·9 | 6 |
| 7 | − 6·0 | − 9·6 | − 0·6 | −10·8 | − 3·6 | − 6·1 | 7 |
| 8 | − 4·2 | − 8·4 | − 1·2 | − 7·8 | − 3·0 | − 4·9 | 8 |
| 9 | 0·0 | − 9·0 | − 0·6 | − 4·8 | − 3·0 | − 3·3 | 9 |
| 10 | − 2·4 | − 4·8 | − 1·8 | − 0·6 | − 6·6 | − 3·2 | 10 |
| 11 | + 3·0 | + 0·6 | + 3·0 | + 8·4 | + 3·0 | + 3·6 | 11 |
| 12 | + 7·2 | + 2·4 | + 4·8 | + 7·8 | + 2·4 | + 4·9 | 12 |
| 13 | +12·0 | + 6·6 | + 6·6 | + 4·8 | + 3·0 | + 6·6 | 13 |
| 14 | + 3·0 | + 8·4 | + 7·2 | + 7·8 | + 3·0 | + 5·9 | 14 |
| 15 | + 7·8 | + 4·8 | + 4·2 | + 3·0 | + 0·6 | + 4·1 | 15 |
| 16 | + 1·8 | + 3·6 | + 0·6 | + 0·6 | + 0·6 | + 1·4 | 16 |
| 17 | 0·0 | − 1·2 | − 3·6 | − 2·4 | − 9·6 | − 3·4 | 17 |
| 18 | − 6·6 | − 6·6 | − 5·4 | − 6·6 | − 6·6 | − 6·4 | 18 |
| 19 | − 4·8 | − 5·4 | − 8·4 | − 7·2 | − 6·6 | − 6·5 | 19 |
| 20 | − 6·6 | −10·2 | −13·2 | − 4·2 | − 5·4 | − 6·6 | 20 |
| 21 | − 9·6 | −12·6 | −10·8 | − 4·2 | − 4·8 | − 8·4 | 21 |
| 22 | − 4·2 | − 0·6 | −10·8 | + 2·4 | + 3·6 | − 1·9 | 22 |
| 23 | − 2·4 | 0·0 | − 2·4 | + 0·6 | + 8·4 | + 0·8 | 23 |

It may operate as an encouragement to those who have not yet subjected their observations to any process of examination or analysis, to perceive, by this example, how substantially satisfactory are the results which may be obtained from even a single year of hourly observations, after the larger disturbances and the solar-diurnal variation have been eliminated.

I have spoken in a recent paper of an unexceptionable test by which we may satisfy ourselves as to the confidence which may be reposed in a series of observations, whether obtained by the eye or tabulated from instrumental traces. Such a test is furnished when the entries at solar hours are rewritten according to the lunar hours to which they most nearly approximate, and when consequently their

original order and relations are changed and are replaced by others which were wholly unforeseen, so that the observations must necessarily be free from the possibility of having been influenced by any mental bias. When we find the effects of a natural law, represented by such minute values as that of the lunar-diurnal variation, exhibited by the observations of a single year with the degree of symmetry shown in Table II, we may safely conclude that the observations themselves are worthy of the labour bestowed in eliciting their results. In this view the Hobarton observations prove themselves to have been not only a faithful, but also an extremely careful series, highly creditable to Captain Kay, R.N., and to the Naval Officers who with him and their Civil Assistant Mr. Jeffery, maintained for so many years the laborious and monotonous duty of hourly observation.

Table III. exhibits the separate results in each of the three years at Kew, as well as their mean.

TABLE III.—Lunar-diurnal Variation at Kew in the years 1858, 1859, and 1860; omitting disturbed observations differing 3′·3 from their final normals.

| Lunar Hours. | Year ending December 31. | | | Means. | Lunar Hours. |
|---|---|---|---|---|---|
| | 1858. | 1859. | 1860. | | |
| | ″ | ″ | ″ | ″ | |
| 0 | − 6·0 | + 0·6 | −12·6 | − 6·0 | 0 |
| 1 | −14·4 | − 7·2 | −12·6 | −11·4 | 1 |
| 2 | −10·8 | − 9·6 | − 5·4 | − 8·6 | 2 |
| 3 | − 7·8 | − 4·2 | − 3·0 | − 5·0 | 3 |
| 4 | − 3·0 | − 4·2 | − 2·4 | − 3·2 | 4 |
| 5 | + 5·4 | − 6·6 | + 5·4 | + 1·4 | 5 |
| 6 | + 2·0 | + 1·2 | + 3·0 | + 5·4 | 6 |
| 7 | + 9·0 | + 4·2 | + 9·6 | + 7·6 | 7 |
| 8 | +19·6 | + 8·4 | + 7·8 | + 8·6 | 8 |
| 9 | + 7·2 | + 6·6 | − 0·9 | + 4·3 | 9 |
| 10 | + 3·0 | + 7·2 | − 1·8 | + 2·8 | 10 |
| 11 | − 3·6 | − 1·2 | − 4·2 | − 3·0 | 11 |
| 12 | − 4·8 | − 9·0 | −18·0 | −10·6 | 12 |
| 13 | − 3·0 | −13·2 | −15·0 | −10·4 | 13 |
| 14 | − 3·0 | − 8·4 | − 9·6 | − 7·0 | 14 |
| 15 | − 7·2 | − 3·6 | + 4·2 | − 2·2 | 15 |
| 16 | + 3·0 | + 3·6 | + 7·8 | + 4·8 | 16 |
| 17 | + 7·8 | + 9·6 | +13·8 | +10·4 | 17 |
| 18 | + 7·8 | +14·4 | +17·4 | +13·2 | 18 |
| 19 | + 4·8 | +18·0 | +15·0 | +12·6 | 19 |
| 20 | + 3·0 | +12·6 | + 6·0 | + 7·2 | 20 |
| 21 | − 2·4 | +18·6 | + 2·4 | + 6·2 | 21 |
| 22 | − 7·8 | + 9·6 | − 3·0 | − 0·4 | 22 |
| 23 | − 6·0 | + 5·4 | − 3·6 | − 1·4 | 23 |

In conclusion, it may be useful to call the attention of the Society, and of those Fellows in particular who interest themselves in tracing up the phenomena of nature to their physical causes, to the assemblage of facts which are now available for such inquiries, in a branch of magnetical science which may not inappropriately be called *celestial magnetism.* In the introductory discussion prefixed to the 2nd volume of the St. Helena Magnetical Observations, p. cxliv to cxlviii, the lunar-diurnal variation is given for each of the three magnetic elements, the Declination, the Dip, and the Intensity of the force, at the four stations of Toronto, St. Helena, the Cape of Good Hope and Hobarton, and for the Declination at two additional stations Kew and Pekin. The variations are given both in formulæ and in tables; the latter exhibiting the amount of the lunar influence at each of the 24 lunar hours, in the several magnetic elements at each station. These data are directly applicable to inquiries into the nature of the moon's magnetism; and into the mode by which the moon's magnetism acts either on the magnetism of the earth itself, or on the magnetic needle stationed at different points of the earth's surface, so as to produce a small but systematic and perfectly appreciable variation in each of the magnetic elements, having a double period in every lunar day.

The lunar-diurnal variation of the Declination at Kew and Hobarton, as given in this communication, is slightly different from the figures in the 2nd St. Helena volume referred to, because the results at Kew are a mean of 3 years instead of 2, as in the St. Helena volume; and at Hobarton a lower standard has been taken for the disturbances, causing a larger number of the disturbed observations to be omitted in the calculation of the lunar-diurnal variation.