

equation of the order $n-1$ is calculated by the method of symmetric functions ; and combining it with the linear equation, which is known, we have the required equation, except as regards the terms involving the last coefficient, which terms are found by the consideration that the coefficients of the required equation are seminvariants. The solution leads immediately to that of a more general question ; for if the product of the differences of all the roots except α , of the given equation $\phi v = (*\chi v, 1)^n = a(v-\alpha)(v-\beta) \dots = 0$ (which product is a function of the degree $n-2$ in regard to each of the roots $\beta \gamma \delta \dots$), is multiplied by $(x-\alpha y)^{n-2}$, the function so obtained will be the root of an equation of the order n , the coefficients of which are covariants of the quantic $(*\chi x, y)^n$, and these coefficients can be at once obtained by writing, in the place of the seminvariants of the former result, the covariants to which they respectively belong. In the case of the quintic equation, one of these covariants is, in regard to the coefficients, of the degree 6, which exceeds the limit of the tabulated covariants, the covariant in question has therefore to be now first calculated. The covariant equations for the cubic and the quartic might be deduced from the formulæ Nos. 119 and 142 of my Fifth memoir on Quantics, Phil. Trans. t. cxlviii. pp. 415-427 (1858) ; they are in fact the bases of the methods which are there given for the solution of the cubic and the quartic equations respectively ; and it was in this way that I was led to consider the problem which is here treated of.

II. "Description of a new Optical Instrument called the 'Stereotrope.'" By WILLIAM THOMAS SHAW, Esq. Communicated by WARREN DE LA RUE, Esq. Received Dec. 6, 1860.

This instrument is an application of the principle of the stereoscope to that class of instruments variously termed thaumatropes, phantascopes, phenakistoscopes, &c., which depend for their results on "persistence of vision." In these instruments, as is well known, an object represented on a revolving disc, in the successive positions it assumes in performing a given evolution, is seen to execute the movement so delineated ; in the stereotrope the effect of solidity is superadded, so that the object is perceived as if in motion and with

an appearance of relief as in nature. The following is the manner in which I adapt to this purpose the refracting form of the stereoscope.

Having procured eight stereoscopic pictures of an object—of a steam-engine for example—in the successive positions it assumes in completing a revolution, I affix them, in the order in which they were taken, to an octagonal drum, which revolves on a horizontal axis beneath an ordinary lenticular stereoscope and brings them one after another into view. Immediately beneath the lenses, and with its axis situated half an inch from the plane of sight, is fixed a solid cylinder, 4 inches in diameter, capable of being moved freely on its axis. This cylinder, which is called the eye-cylinder, is pierced throughout its entire length (if we except a diaphragm in the centre inserted for obvious reasons) by two apertures, of such a shape, and so situated relatively to each other, that a transverse section of the cylinder shows them as cones, with their apices pointing in opposite directions, and with their axes parallel to, and distant half an inch from, the diameter of the cylinder. Attached to the axis of the eye-cylinder is a pulley, exactly one-fourth the size of a similar pulley affixed to the axis of the picture-drum, with which it is connected by means of an endless band. The eye-cylinder thus making four revolutions to one of the picture-drum, it is evident that the axes of its apertures will respectively coincide with the plane of sight four times in one complete revolution of the instrument, and that, consequently, vision will be permitted eight times, or once for each picture.

The cylinder is so placed that at the time of vision the *large* ends of the apertures are next the eyes, the effect of which is that when the *small* ends pass the eyes, the axes of the apertures, by reason of their eccentricity, do not coincide with the plane of sight, and vision is therefore impossible. If, however, the position of the cylinder be reversed end for end, vision will be possible only when the small ends are next the eyes, and the angle of the aperture will be found to subtend exactly the pencil of rays coming from a picture, which is so placed as to be bisected at right angles by the plane of sight. Hence it follows that, the former arrangement of the cylinder being reverted to, the observer looking along the upper side of the aperture will see a narrow strip extending along the top of the picture; then, moving the cylinder on and looking along the lower side of the

aperture, he will see a similar strip at the bottom of the picture ; consequently, in the intermediate positions of the aperture, the other parts of the picture will have been projected on the retinae. The width of these strips is determined by that of the small ends of the apertures, which measure $\cdot 125$ inch ; and the diameter of the large ends is $1\cdot 5$ inch, the lenses being distant 9 inches from the pictures. The picture-drum being caused to revolve with the requisite rapidity, the observer will see the steam-engine constantly before him, its position remaining unchanged in respect of space, but its parts will appear to be in motion, and in solid relief, as in the veritable object. The stationary appearance of the pictures, notwithstanding the fact of their being in rapid motion, is brought about by causing their corresponding parts to be seen, respectively, *only* in the same part of space, and *that* for so short a time that while in view they make no sensible progression. As, however, there is an actual progression during the instant of vision, it is needful to take that fact into account—in order that it may be reduced as far as practicable—in regulating the diameter of the eye-cylinder, and of the apertures at their small ends ; and the following are the numerical data involved in the construction of an instrument with the relative proportions given above :—

The circumference of picture-drum = $22\cdot 5$ inches (A).

The circumference of eye-cylinder = 12 inches $\times 4$ revolutions = 48 inches (B).

The diameter of apertures at large ends = $1\cdot 5$ inch (C).

The diameter of apertures at small ends = $\cdot 125$ inch (D).

While the large end is passing the eye, the picture under view progresses $\frac{1\cdot 5 \text{ (C)}}{48 \text{ (B)}}$ of $22\cdot 5$ (A), or $\cdot 703$ inch.

This amount of progression ($\cdot 703$ in.), if perceived at one and the same instant, would be utterly destructive of all distinctness of definition ; but it is evident that the total movement brought under visual observation at any one moment is $\frac{\cdot 125 \text{ (D)}}{1\cdot 5 \text{ (C)}}$ of $\cdot 703$ inch, or $\cdot 058$ inch. This movement must necessarily occasion a corresponding slurring, so to speak, of the images on the retina ; and the fact of such slurring not affecting, to an appreciable extent, the distinctness of definition, seems to be referable to a faculty which the mind has

of correcting or disregarding certain discrepant appearances or irregularities in the organ of vision ; as a further illustration of which I may cite the fact, mentioned by Mr. Warren De la Rue in his "Report on Celestial Photography," that the retinal image of a star is, at least under some atmospheric conditions, made up of "a great number of undulating points," which, however, the mind rightly interprets as the effect of the presence before the eye of a single minute object. That this corrective power is, as might be supposed, very limited, may be proved experimentally by this instrument ; for if the small ends be enlarged in only a slight degree, so as to increase this slurring on the retinæ, a very marked diminution in clearness of definition is the immediate result.

That form of the stereotrope, in which Professor Wheatstone's reflecting stereoscope is made use of, and which is better adapted for the exhibition of movements that are not only local but progressive in space, it is needless to describe here, because the principles it involves are essentially the same as those which are stated above.

III. "On the Lunar-Diurnal Variation of the Magnetic Declination obtained from the Kew Photograms* in the years 1858, 1859, and 1860." By Major-General EDWARD SABINE, R.A., Treas. and V.P.R.S. Received December 19, 1860†.

Having communicated to the Royal Society in a recent paper an analysis of the *disturbances* of the declination in the years 1858 and 1859, shown by the photograms of the Kew Observatory, I propose in the present paper to submit the results of the *lunar-diurnal variation* of the declination in the years 1858, 1859, and 1860, obtained from the same source. The directions of the declination magnet at the instant of the commencement of every solar-hour having been tabulated from the photograms, and the final normals

* The term Photogram is adopted in place of Photograph in conformity with modern usage.

† [Note added on February 8th, 1861.] When this communication was read to the Royal Society on January 10th, 1861, it contained the lunar-diurnal variation for the years 1858 and 1859 only : whilst it was passing through the press, the calculation of the lunar-diurnal variation for 1860 was completed, and the results in that year have been added.