No. 2. Report kept by Leeds Philosophical Society.
Leeds, May 7, 1862, 5 P.M.

Barometer.	At. Therm.	Dry Bulb.	Wet Bulb.	Wind.	Force.	Cloud.	Shade.		Max. Sun.
							Max.	Min.	
29·380 in.	70°	64°	60°	N.E.	1	10	70	51	100

I am, &c.,
THOMAS SUTCLIFFE.

III. "On the true Theory of Pressure as applied to Elastic Fluids." By R. MOON, M.A., late Fellow of Queen's College, Cambridge. Communicated by Professor SYLVESTER. Received June 26, 1862.

(Abstract.)

It is the author's object—

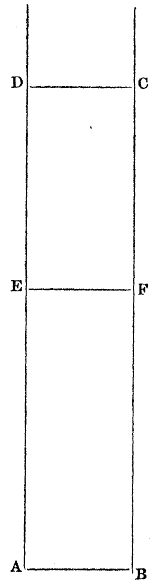
I. To show that, in elastic fluids in motion, or tending to move, it is not generally true, or at least not accurately true, that the pressure depends solely on the density, as is assumed in the ordinary theory of the motion of elastic fluids.

II. To show that, within certain limits and under certain circumstances, pressure may be transmitted instantaneously from one point of an elastic fluid to other points situated at finite distances from the first, before any change has been effected in the density of the intermediate fluid—in a manner analogous to that in which, in the theory of dynamics as applied to rigid bodies, force is assumed to be propagated instantaneously from one point to another.

III. To show that in elastic fluids in motion, or tending to move, the pressure at any point in a given direction will consist of two parts:—one depending solely on the density, which will be equal in all directions; the other depending on the state of motion throughout the fluid generally, and which will vary with the direction in which the pressure is estimated. The former of these two constituents the author proposes to designate the statical pressure; the latter, the instantaneous pressure. The true pressure at any point in a given direction will be found by taking the sum or difference of the statical and instantaneous pressures, according to circumstances.

IV. To indicate the manner in which the instantaneous pressure may be represented mathematically.

V. To show the bearing of the proposed correction on the received theory of sound.



A B C D is a vertical cylinder closed at the base A B, and having an air-tight piston C D capable of moving freely in the upper part of it.

Below the piston the tube is filled with air, which at the time t is wholly free from impressed velocity, but in which the density varies in the following manner: viz., from A B up to an imaginary horizontal plane E F, the density is uniform; while from E F the density gradually increases up to C D, in such a manner that the effective force at every point of the air between E F and C D is exactly the same, and equal to f*. Above the piston a vacuum exists. The piston is supposed to have weight, but, for the sake of simplicity, the air under the piston is supposed to be unaffected by gravity. The weight of the piston is supposed to be such that the effective force on each particle of the piston is the same as that on each particle of the mass of fluid E C, viz. f.

If the pressure exerted by the air which originally occupied the space A F on that which originally occupied the space E C were to continue during the time t_1 the same that it was at the time t, every particle of the former mass of air (which we will designate as the air in A F) would during the time t_1 be under the action of the same effective force f, and would therefore in that time describe the same length of path, viz. $\frac{ft_1^2}{2}$; and on this supposition no change would take place in the density of the air in E C during the time t_1. But, according to the received theory, the pressure of the air in A F on that in E C will continue unchanged until the density of the part of the air in A B which abuts on the common boundary of the two

* This will be the case if $\frac{1}{\rho}\frac{dp}{dx}=f$, or putting $p=a^2\rho$, $a^2 \log_2\rho = fx+c$; where ρ denotes the density at the distance x measured vertically, and c is a const.

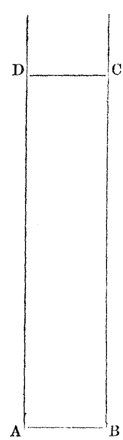
masses of fluid has changed. Hence *change in the density of the air in A F must precede change in the density of E C.*

On the other hand, so long as the pressure of the air in E C on the air in A F remains unchanged, the air in A F will remain at rest, and will therefore undergo no change of density. But as, according to the received theory, the pressure of the air in E C on the air in A F depends on the density of the part of the air in E C which abuts on the common boundary of the two masses of air, it follows that *change in the density of the air in E C must precede change in the density of the air in A F.*

But we have before proved the exact contrary, viz. that change in the density of the air in A F must precede change in the density of the air in E C. It is evident therefore that, according to the received theory, no change can, under the circumstances above supposed, take place in the density of either mass of air.

If, however, the density in A F remain unchanged, we have already seen that every particle in E C will in the time t_1 describe a space equal to $\frac{ft_1^2}{2}$; and if the density in E C remain unchanged, we have equally seen that every particle of A F will have remained at rest during t_1; which is a contradiction. It appears therefore that in the case we have been considering the received theory leads us to an absurd result.

It can with still more facility be shown that the received theory leads to an absurd result in the following case.



A B C D is such a tube as before described; but in the present case we shall suppose it filled below the piston with air of uniform density in equilibrium, the pressure of the air being such as to exactly sustain the weight W_1 of the piston. As before, a vacuum is supposed to exist above the piston, and the air is assumed to be unaffected by gravity.

If a second weight W_2 be placed upon the piston, we know that the equilibrium will be destroyed. But if it be true, as the received theory asserts, that the pressure of an elastic fluid depends solely on its density, the pressure of the air on the lower surface of the piston will be exactly the same after W_2 has

been introduced as it was before W_2 was introduced; and, since action and reaction are equal and opposite, whatever be the pressure of the air in the piston, the same will be the pressure of the piston on the air; so that the pressure downwards of the piston on the air beneath will be the same after W_2 was introduced as it was before; and the system therefore will continue in equilibrium after W_2 has been introduced; which is absurd.

By an argument too elaborate to be indicated within the limits of this abstract, the cause of the failure of the existing theory in the instance first above considered is shown; and it is proved that in the second case the effect of the introduction of the weight W_2 is instantaneously to propagate through the air to a definite distance below the piston a finite increase of pressure; such increase of pressure having its maximum immediately underneath the piston, and thence gradually diminishing till, if the tube be long enough, it finally vanishes. The depth to which the instantaneous increase of pressure will extend will be defined by means of two considerations:—1st, that the effective force on every particle of the piston and weight must be exactly the same as that on the air immediately below it; and 2nd, that the aggregate moving force developed in the piston W, the weight W_2, *and the portion of the air in the tube through which the instantaneous pressure extends,* must be equal to the moving force developed by gravity in W_2 when free to move *in vacuo.*

It is also shown that if instead of the weight on the piston being suddenly increased it were to be suddenly diminished, exactly analogous results, *mutatis mutandis,* would occur,—the effect of the sudden removal of part of the weight being instantaneously to *diminish* the pressure to a finite distance below the piston—such diminution having its maximum immediately beneath the piston, and thence gradually diminishing till, at a certain distance below the piston, the whole pressure will be exactly the same as it was before any part of the weight was removed.

If the piston were wholly removed, the pressure of the air originally in contact with it at the instant of removal would be zero.

It is then shown that the addition to or diminution from the weight on the piston in the case last considered will produce no immediate change in the horizontal pressure in the air below the piston.

It is next shown that in cases where there is no impressed velocity,

as in the case first considered in this paper, the instantaneous pressure p_i may be expressed in terms of its partial differential coefficients, and of the density at the point where the pressure is being considered.

It is also shown that, in the general case, where the whole or a portion of the fluid is endued with velocity, the instantaneous pressure may be ascertained by adding to the expression of the last paragraph a term involving the density and the partial differential coefficients of the velocity at the point where the pressure is being considered.

It is finally shown that, in the case of the transmission of a pulse through a cylindrical tube where the motions are small, the equation of motion will be of this form,

$$\frac{d^2y}{dt^2}=\frac{a^2d^2y}{dx^2}-\frac{b^2d^2y}{dxdt};$$

where x denotes the distance from the origin measured parallel to the axis of a given stratum in the state of rest, y the same distance at the time t, and a^2 and b^2 are constants, the value of a^2 being the same as in the ordinary theory.

As this equation leads to the conclusion that there are two velocities, it results that, except perhaps in very rare instances, in which a duplication has been observed in sounds heard at very great distances, the proposed correction of the theory of the motion of elastic fluids will not practically affect the theory of sound.

By the method adopted in the case of elastic fluids, the author conceives himself to have established that, in what are commonly termed inelastic fluids, the pressure during motion will not be equal in all directions.

IV. "On the Nerves of the Liver, Biliary Ducts, and Gall-bladder." By Robert Lee, M.D., F.R.S. Received August 18, 1862.

(Abstract.)

After adverting to the deficiency of existing knowledge respecting the distribution and arrangement of the nerves of the liver, the author states that he has recently made dissections which "prove that all the arteries which ramify throughout the substance of the liver, even the most minute, are accompanied with nerves, on which