The total number of such systems or terms will be

$$\left\{\frac{\pi(m+n-2)}{\pi(m-1)\pi(n-1)}\right\}^2.$$

Every term in this determinant will itself be a sum of simple determinants of the $(m+n-1)$th order, corresponding (each to each) with the totality of the excylcic distributions of $(m+n-1)$ counters in respect of the particular systems of m capacities and n frequencies appertaining to that term; so that the number of simple determinants whose sum constitutes a term in the grand total determinant is always the product of two polynomial coefficients. In the particular case, where one of the systems contains only *two* variables, one of these polynomial coefficients becomes unity, and the other sinks down to a binomial coefficient. The only instance of a double determinant which is believed to have been considered up to the present moment is that given by Mr. Cayley in the 'Cambridge and Dublin Mathematical Journal,' vol. ix. 1854, for the case of $m=2$, $n=2$.

IV. "On a Theorem relating to Polar Umbræ." By J. J. Sylvester, M.A., F.R.S. Received April 27, 1863.

By polar umbræ I mean such as obey in the strictest manner the polar law of sign, so that not only any two appositions or products of such umbræ derivable from one another by an interchange of two of their elements are to be considered each as the negative of the other, but also any such apposition or product becomes zero if the same element is found in it more than once.

Thus Sir W. Hamilton's i, j, k are not polar umbræ, because although $ijk=-jik=kij$, &c., ii, jj, kk, instead of being *nulls*, are in the Calculus of Quaternions taken as *unities**.

Let us now define any set arranged either in line or column of such *umbral* quantities to be multiplied by a corresponding set of *actual* quantities when each term of the one set is multiplied by the corresponding one of the other, and the sum taken of the products so

* If we use Vandermonde's condensed notation for a determinant $\begin{bmatrix}1\ 2\ldots n\\1\ 2\ldots n\end{bmatrix}$ to represent a "determinant gauche," then, since on this supposition $rs=sr$ and $rr=0$, 1, 2, 3, ... n will be polar umbræ by definition.

obtained as in the ordinary case of the multiplication of the lines or columns of two determinants *inter se*. Thus, *ex. gr.* $(a,\ b,\ c \between x,\ y,\ z)$, as also $\left(\begin{matrix} a \\ b \end{matrix} \between \begin{matrix} x \\ y \\ z \end{matrix}\right)$ is to mean the same product, viz.

$$ax+by+cz.$$

Again, imagine a rectangular (square or oblong) matrix of polar umbræ, and that each line thereof is multiplied by the same line of *actual* quantities, the product of the products so obtained I call a Factorial of the Matrix. I also call the product similarly obtained when the columns of the matrix are substituted for the lines, a Factorial of the same, but distinguish between the two by giving to one the name of a Transverse, the second of a Longitudinal Factorial of the matrix. We are now in a position to enunciate the following remarkable theorem :—

The product of any longitudinal by any transverse factorial of the same polar umbral matrix is identically zero.

Ex. gr. Let $\left|\begin{matrix} a & b & c \\ d & e & f \end{matrix}\right|$ be a matrix of polar umbræ, but x, y, z and also ξ, η actual quantities. Then

$$(ax+by+cz)(dx+ey+fz)$$

is a transverse factorial,

$$(a\xi+d\eta)(b\xi+e\eta)(c\xi+f\eta)$$

a longitudinal factorial of the above matrix, and by the theorem their product should be zero. This is easily verified.

The two *factorials* expanded are respectively

$$adx^2+bey^2+cfz^2+(ae+bd)xy+(bf+ce)yz+(af+dc)zx,$$
$$abc\xi^3+(abf+aec+dbc)\xi^2\eta+(dec+dbf+aef)\xi\eta^2+def\eta^3;$$

in their product the coefficient of

$$\begin{aligned} x^2\xi^3 &= abcad=0, \\ xy\xi^3 &= abcae+abcbd=0, \\ x^2\xi^2\eta &= abfad+aecad+dbcad=0, \\ xy\xi^2\eta &= abfae+abfbd+aecae+aecbd+dbcae+dbcdb \\ &= aecbd+dbcae=aecbd-aecbd=0, \end{aligned}$$

and so for all the other terms.

This is the fundamental theorem by aid of which I obtain the

resultant of a lineo-linear system of equations in its most perfect form. It is easy to obtain two different solutions, each of them unsymmetrical in respect of the data of the question; the conversion and fusion of each of these into one and the same determinant, symmetrical in all its relations to the data, is effected instantaneously by a process derived from the above theorem. In that particular application of it, the umbræ involved each represent columns of actual quantities in number equal to the number of places in the width and length of the umbral matrix to which they belong, so that each coefficient in the product of a lateral by a longitudinal factorial represents an ordinary determinant made up of these columns, from which it is evident that the polar law of sign and nullity necessary for the truth of the theorem are satisfied in the case supposed.

V. "Notes, principally on Thermo-electric Currents of the Ritterian Species." By C. K. Akin, Esq. Communicated by Professor Stokes, Sec. R.S. Received March 26, 1863.

(Abstract.)

The electromotive force of a thermo-electric couple is a function of the nature of the metals of which it is composed, and of the temperatures of the junctions. It is expressed in this paper by

$$[x, y]_t^{\mathrm{T}},$$

where x and y are names of metals, and T and t are temperatures. In this notation Becquerel's two laws become

$$[a, b]_{t'}^{t''}=[a, b]_t^{t''}-[a, b]_t^{t'}; \quad \ldots \ldots \quad \text{(I.)}$$

and

$$(a, c)_t^{\mathrm{T}}=[a, b]_t^{\mathrm{T}}+[b, c]_t^{\mathrm{T}}. \quad \ldots \ldots \quad \text{(II.)}$$

From (I.) we learn that the electromotive force of a couple may be expressed as the difference of two quantities which are functions of the temperature and of the nature of the circuit, or

$$[x, y]_t^{\mathrm{T}}=[x, y]_{\mathrm{T}}-[x, y]_t. \quad \ldots \ldots \quad \text{(III.)}$$