February 4, 1864.

Major-General SABINE, President, in the Chair.

The following communication was read:—

"Experiments to determine the effects of impact, vibratory action, and a long-continued change of Load on Wrought-iron Girders." By WILLIAM FAIRBAIRN, LL.D., F.R.S. Received January 20, 1864.

(Abstract.)

The author observes that the experiments which were undertaken, nearly twenty years ago, to determine the strength and form of the Tubular Bridges which now span the Conway and Menai Straits, led to the adoption of certain forms of girder, such as the tubular, the plate, and the lattice girder, and other forms founded on the principle developed in the construction of these bridges. It was at first designed that the ultimate strength of these structures should be six times the heaviest load that could ever be laid upon them, after deducting half the weight of the tubes. This was considered a fair margin of strength; but subsequent considerations, such as generally attend a new principle of construction with an untried material, showed the expediency of increasing it; and instead of the ultimate strength being six times, it was in some instances increased to eight times the weight of the greatest load.

The proved stability of these bridges gave increased confidence to the engineer and the public, and for several years the resistance of six times the heaviest load was considered an amply sufficient provision of strength.

But a general demand soon arose for wrought-iron bridges, and many were made without due regard to first principles, or to the law of proportion necessary to be observed in the sectional areas of the top and bottom flanges, so clearly and satisfactorily shown in the early experiments. The result of this was the construction of weak bridges, many of them so ill-proportioned in the distribution of the material as to be almost at the point of rupture with little more than double the permanent load. The evil was enhanced by the erroneous system of contractors tendering by weight, which led to the introduction of bad iron, and in many cases equally bad workmanship.

The deficiencies and break-downs which in this way followed the first successful application of wrought iron to the building of bridges led to doubts and fears as to their security. Ultimately it was decided by the Board of Trade that in wrought-iron bridges the strain with the heaviest load should not exceed 5 tons per square inch; but on what principle this standard was established does not appear.

The requirement of 5 tons per square inch did not appear sufficiently definite to secure in all cases the best form of construction. It is well

known that the powers of resistance to strain in wrought iron are widely different, according as we apply a force of tension or compression; it is even possible so to disproportion the top and bottom areas of a wrought-iron girder calculated to support six times the rolling load, as to cause it to yield with little more than half the ultimate strain or 10 tons on the square inch. For example, in wrought-iron girders with solid tops it requires the sectional area in the top to be nearly double that of the bottom to equalize the two forces of tension and compression; and unless these proportions are strictly adhered to in the construction, the 5-ton strain per square inch is a fallacy which may lead to dangerous errors. Again, it was ascertained from direct experiment that double the quantity of material in the top of a wrought-iron girder was not the most effective form for resisting compression. On the contrary, it was found that little more than half the sectional area of the top, when converted into rectangular cells, was equivalent in its powers of resistance to double the area when formed of a solid top plate. This discovery was of great value in the construction of tubes and girders of wide span, as the weight of the structure itself (which increases as the cubes, and the strength only as the squares) forms an important part of the load to which it is subjected. On this question it is evident that the requirements of a strain not exceeding 5 tons per square inch cannot be applied in both cases, and the rule is therefore ambiguous as regards its application to different forms of structure. In that rule, moreover, there is nothing said about the dead weight of the bridge; and we are not informed whether the breaking-weight is to be so many times the applied weight plus the multiple of the load, or, in other words, whether it includes or is exclusive of the weight of the bridge itself.

These data are wanting in the railway instructions; and until some fixed rinciple of construction is determined upon, accompanied by a standard measure of strength, it is in vain to look for any satisfactory results in the erection of road and railway bridges composed entirely of wrought iron.

The author was led to inquire into this subject with more than ordinary care, not only on account of the imperfect state of our knowledge, but from the want of definite instructions. In the following experimental researches he has endeavoured to ascertain the extent to which a bridge or girder of wrought iron may be strained without injury to its ultimate powers of resistance, or the exact amount of load to which a bridge may be subjected without endangering its safety—in other words, to determine the fractional strain of its estimated powers of resistance.

To arrive at correct results and to imitate as nearly as possible the strain to which bridges are subjected by the passage of heavy trains, the apparatus specially prepared for the experiments was designed to lower the load quickly upon the beam in the first instance, and next to produce a considerable amount of vibration, as the large lever with its load and shackle was left suspended upon it, and the apparatus was sufficiently elastic for that purpose.

The girder subjected to vibration in these experiments was a wrought-iron plate beam of 20 feet clear span, and of the following dimensions :—

Area of top......................	4·30 square inches.
Area of bottom	2·40 „
Area of vertical web	1·90 „
Total sectional area	8·60 „
Depth..........................	16 inches.
Weight	7 cwt. 3 qrs.
Breaking-weight (calculated)........	12 tons.

The beam having been loaded with 6643 lbs., equivalent to one-fourth of the ultimate breaking-weight, the experiments commenced as follows :—

Experiment I.

Experiment on a wrought-iron beam with a changing load equivalent to one-fourth of the breaking-weight.

Date.	Number of changes of Load.	Deflection produced by Load.	Remarks.
1860.			
March 21	0	0·17	Strap loose on the 24th March.
April 7	202,890	0·17	Strap broken on the 20th April.
May 1...........	449,280	0·16	
May 14	596,790	0·16	

The beam having undergone about half a million changes of load by working continuously for two months night and day, at the rate of about eight changes per minute, without producing any visible alteration, the load was increased from one-fourth to two-sevenths of the statical breaking-weight, and the experiments were proceeded with till the number of changes of load reached a million.

Experiment II.

Experiment on the same beam with a load equivalent to two-sevenths of the breaking weight, or nearly $3\frac{1}{2}$ tons.

Date.	Number of changes of Load.	Deflection, in inches.	Remarks.
1860.			
May 14	0	0·22	In this experiment the number of changes of load is counted from 0,
May 22	85,820	0·22	although the beam had already undergone 596,790 changes, as shown
June 9	236,460	0·21	in the preceding Table.
June 26	403,210	0·23	The beam had now suffered one million changes of load.

After the beam had thus sustained one million changes of load without apparent alteration, the load was increased to 10,486 lbs., or $\frac{2}{5}$ths of the breaking-weight, and the machinery again put in motion. With this additional weight the deflections were increased, with a permanent set of 05 inch, from ·23 to ·35 inch, and after sustaining 5175 changes the beam broke by tension at a short distance from the middle. It is satisfactory here to observe that during the whole of the 1,005,175 changes none of the rivets were loosened or broken.

The beam broken in the preceding experiment was repaired by replacing the broken angle-irons on each side, and putting a patch over the broken plate equal in area to the plate itself. A weight of 3 tons was placed on the beam thus repaired, equivalent to one-fourth of the breaking-weight, and the experiments were continued as before.

Experiment III.

Date.	Number of changes of Load.	Deflection, in inches.	Permanent set, in inches.	Remarks.
1860.				
August 9	158	....	....	The load during these changes was equivalent to 10,500 lbs., or 4·6875 tons at the centre. With this weight the beam took a large but unmeasured set.
August 11	12,950			During these changes the load in the beam was 8025 lbs., or 3·58 tons.
August 13	25,900	0·22	?	
August 13	25,900	0·18	0	Load reduced to 2·96 tons, or $\frac{1}{4}$th the breaking-weight.
December 1 ...	768,100	0·18	0·01	
1861.				
March 2.........	1,602,000	0·18	0·01	
May 4	2,110,000	0·17	0·01	
September 4 ...	2,727,754	0·17	0·01	
October 16......	3,150,000	0·17	0·01	

At this point, the beam having sustained upwards of 3,000,000 changes of load without any increase of the permanent set, it was assumed that it might have continued to bear alternate changes to any extent with the same tenacity of resistance as exhibited in the foregoing Table. It was then determined to increase the load from one-fourth to one-third of the breaking-weight; and accordingly 4 tons were laid on, which increased the deflection to ·20.

Experiment IV.

Date.	Number of changes of Load.	Deflection, in inches.	Permanent set, in inches.	Remarks.
1861.				
October 18......	0	0·20		
November 18...	126,000	0·20	0	
December 18...	237,000	0·20		
1862.				
January 9	313,000	....	....	Broke by tension across the bottom web.

Collecting the foregoing series of experiments, we obtain the following summary of results.

Summary of Results.

No. of Expt.	Date.	Weight on middle of the beam, in tons.	Number of changes of Load.	Strain per sq. inch on bottom.	Strain per sq. inch on top.	Deflection, in inches.	Remarks.
1	From March 21 to May 14, 1860...	2	596,790	4·62	2·58	·17	
2	From May 14 to June 26, 1860...	3·50	403,210	5·46	3·05	·23	
3	From July 25 to July 28, 1860...	4·68	5,175	7·31	4·08	·35	Broke by tension a short distance from the centre of the beam.
			Beam repaired.				
4	Aug. 9, 1860	4·68	158	7·31	4·08	...	The apparatus was accidentally set in motion.
5	Aug. 11 & 12	3·58	25,742	3·59	3·12	·22	
6	From Aug. 13, 1860 to Oct. 16, 1861	2·96	3,124,100	4·62	2·58	·18	
7	From Oct. 18, 1861 to Jan. 9, 1862 ...	4·00	313,000	6·25	3·48	·20	Broke by tension as before, close to the plate riveted over the previous fracture.

From these experiments it is evident that wrought-iron girders of ordinary construction are not safe when submitted to violent disturbances equivalent to one-third the weight that would break them. They, however, exhibit wonderful tenacity when subjected to the same treatment with one-fourth the load; and assuming therefore that an iron girder bridge will bear with this load 12,000,000 changes without injury, it is clear that it would require 328 years at the rate of 100 changes per day before its security was affected. It would, however, be dangerous to risk a load of one-third

the breaking-weight upon bridges of this description, as, according to the last experiment, the beam broke with 313,000 changes; or a period of eight years, at the same rate as before, would be sufficient to break it. It is more than prohable that the beam had been injured by the previous 3,000,000 changes to which it had been subjected; and assuming this to be true, it would follow that the beam was undergoing a gradual deterioration which must some time, however remote, have terminated in fracture.

February 11, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read :—

I. "On the Calculus of Symbols.—Fourth Memoir. With Applications to the Theory of Non-linear Differential Equations." By W. H. L. Russell, A.B. Communicated by Professor Cayley. Received July 31, 1863.

(Abstract.)

In the preceding memoirs on the Calculus of Symbols, systems have been constructed for the multiplication and division of non-commutative symbols subject to certain laws of combination; and these systems suffice for linear differential equations. But when we enter upon the consideration of non-linear equations, we see at once that these methods do not apply. It becomes necessary to invent some fresh mode of calculation, and a new notation, in order to bring non-linear functions into a condition which admits of treatment by symbolical algebra. This is the object of the following memoir. Professor Boole has given, in his 'Treatise on Differential Equations,' a method due to M. Sarrus, by which we ascertain whether a given non-linear function is a complete differential. This method, as will be seen by anyone who will refer to Professor Boole's treatise, is equivalent to finding the conditions that a non-linear function may be externally divisible by the symbol of differentiation. In the following paper I have given a notation by which I obtain the actual expressions for those conditions, and for the symbolical remainders arising in the course of the division, and have extended my investigations to ascertaining the results of the symbolical division of non-linear functions by linear functions of the symbol of differentiation.

II. "On Molecular Mechanics." By the Rev. Joseph Bayma, of Stonyhurst College, Lancashire. Communicated by Dr. Sharpey, Sec. R.S. Received January 5, 1864.

The following pages contain a short account of some speculations on molecular mechanics. They will show how far my plan of molecular