John Hall Gladstone, Esq., Ph.D.; Joseph Dalton Hooker, M.D.; Henry Bence Jones, M.A., M.D.; Prof. James Clerk Maxwell, M.A.; Prof. William Pole, C.E.; Archibald Smith, Esq., M.A.; Prof. Henry J. Stephen Smith, M.A.; The Earl Stanhope, P.S.A., D.C.L.; Prof. James Joseph Sylvester, M.A.; Thomas Watson, M.D., D.C.L.; Prof. Charles Wheatstone, D.C.L.; Rev. Prof. Robert Willis, M.A.

The question of Captain Ibbetson's readmission into the Society was put to the ballot, and, the ballot having been taken, Captain Ibbetson was declared to be readmitted.

The following communications were read:—

I. "Account of Magnetic Observations made between the years 1858 and 1861 inclusive, in British Columbia, Washington Territory, and Vancouver Island." By Captain R. W. Haig, R.A. Communicated by the President. Received November 4, 1863.

(Abstract.)

This paper contains the results of magnetic observations made between the years 1858 and 1861 inclusive, in British Columbia, Washington Territory, and Vancouver Island. The results are tabulated; and from them the direction and position of the lines of equal dip, total force, and declination or variation are determined.

Three maps at the end show the position of these lines, the stations of observation, and the observed values of the three magnetic elements at each station.

II. "On Plane Water-Lines." By W. J. Macquorn Rankine, C.E., LL.D., F.R.SS.L. & E., Assoc. Inst. N.A., &c. Received July 28, 1863.

(Abstract.)

1. By the term "Plane Water-Line" is meant one of those curves which a particle of a liquid describes in flowing past a solid body when such flow takes place in plane layers. Such curves are suitable for the water-lines of a ship; for during the motion of a well-formed ship, the vertical displacements of the particles of water are small, compared with the dimensions of the ship; so that the assumption that the flow takes place in plane layers, though not absolutely true, is sufficiently near the truth for practical purposes*.

2. The author refers to the researches of Professor Stokes (Camb. Trans. 1842), "On the Steady Motion of an Incompressible Fluid," and of Pro-

* As water-line curves have at present no single word to designate them in mathematical language, it is proposed to call them *Neoïds*, from *νηὸς*, the Ionic genitive of *ναῦς*.

fessor William Thomson (made in 1858, but not yet published), as containing the demonstration of the general principles of the flow of a liquid past a solid body.

3. Every figure of a solid, past which a liquid is capable of flowing smoothly, generates an endless series of water-lines, which become sharper in their forms as they are more distant from the primitive water-line of the solid. The only exact water-lines whose forms have hitherto been completely investigated, are those generated by the cylinder in two dimensions, and by the sphere in three dimensions. In addition to what is already known of those lines, the author points out that, when a cylinder moves through still water, the orbit of each particle of water is one loop of an elastic curve.

4. The profiles of waves have been used with success in practice as water-lines for ships, first by Mr. Scott Russell (for the explanation of whose system the author refers to the Transactions of the Institution of Naval Architects for 1860–62), and afterwards by others. As to the frictional resistance of vessels having such lines, the author refers to his own papers —one read to the British Association in 1861, and printed in various engineering journals, and another read to the Royal Society in 1862, and printed in the Philosophical Transactions. Viewed as plane water-lines, however, the profiles of waves are not exact, but approximate; for the "solitary wave of translation," investigated experimentally by Mr. Scott Russell (Reports of the British Association, 1844), and mathematically by Mr. Earnshaw (Camb. Trans. 1845), is strictly applicable to a channel of limited dimensions only, and the trochoidal form belongs properly to an endless series of waves, whereas a ship is a solitary body.

5. The author proceeds to investigate and explain the properties of a class of water-lines comprising an endless variety of forms and proportions. In each series of such lines, the primitive water-line is a particular sort of oval, characterized by this property, that the ordinate at any point of the oval is proportional to the angle between two lines drawn from that point to two foci. Ovals of this class differ from ellipses in being considerably fuller at the ends and flatter at the sides.

6. The length of the oval may bear any proportion to its breadth, from equality (when the oval becomes a circle) to infinity.

7. Each oval generates an endless series of water-lines, which become sharper in figure as they are further from the oval*. In each of those derived lines, the excess of the ordinate at a given point above a certain minimum value is proportional to the angle between a pair of lines drawn from that point to the two foci.

8. There is thus an endless series of ovals, each generating an endless series of water-lines; and amongst those figures, a continuous or "fair" curve can always be found combining any proportion of length to breadth,

* As a convenient and significant name for these water-lines, the term "Oögenous Neoïds" is proposed (from 'Ωογενής, generated from an egg, or oval).

from equality to infinity, with any degree of fullness or fineness of entrance, from absolute bluffness to a knife-edge.

9. The lines thus obtained present striking likenesses to those at which naval architects have arrived through practical experience; and every successful model in existing vessels can be closely imitated by means of them.

10. Any series of water-lines, including the primitive oval, are easily and quickly constructed with the ruler and compasses.

11. The author shows how to construct two algebraic curves traversing certain important points in the water-lines, which are exactly similar for all water-lines of this class. One is a rectangular hyperbola, having its vertex at the end of the oval. It traverses all the points at which the motion of the particles, in still water, is at right angles to the water-lines. The other is a curve of the fourth order, having two branches, one of which traverses a series of points, at each of which the velocity of gliding of the particles of water along the water-line is less than at any other point on the same water-line; while the other branch traverses a series of points, at each of which the velocity of gliding is greater than at any other point on the same water-line.

12. A certain point in the second branch of that curve divides each series of water-lines into two classes,—those which lie within that point having three points of minimum and two of maximum velocity of gliding, while every water-line which passes through or beyond the same point has only two points of minimum and one of maximum velocity of gliding. Hence the latter class of lines cause less commotion in the water than the former.

13. On the water-line which traverses the point of division itself, the velocity of gliding changes more gradually than on any other water-line having the same proportion of length to breadth. Water-lines possessing this character can be constructed with any proportion of length to breadth, from $\sqrt{3}$ (which gives an oval) to infinity. The finer of those lines are found to be nearly approximated to by wave-lines, but are less hollow at the bow than wave-lines are.

14. The author shows how horizontal water-lines at the bow, drawn according to this system, may be combined with vertical plane lines of motion for the water at the stern, if desired by the naval architect.

15. In this, as in every system of water-lines, a certain relation (according to a principle first pointed out by Mr. Scott Russell) must be preserved between the form and dimensions of the bow and the maximum speed of the ship, in order that the appreciable resistance may be wholly frictional and proportional to the square of the velocity (as the experimental researches of Mr. J. R. Napier and the author have shown it to be in well-formed ships), and may not be augmented by terms increasing as the fourth and higher powers of the velocity, through the action of vertical disturbances of the water.