January 9, 1868.

Lieut.-General SABINE, President, in the Chair.

The following communications were read :—

I. "On the Conditions for the existence of Three Equal Roots, or of Two Pairs of Equal Roots of a Binary Quartic or Quintic." By A. Cayley, F.R.S. Received November 26, 1867.

(Abstract.)

In considering the conditions for the existence of given systems of equalities between the roots of an equation, we obtain some very interesting examples of the composition of relations. A relation is either onefold, expressed by a single equation $U=0$, or it is, say, k-fold, expressed by a system of k or more equations. Of course, as regards onefold relations, the theory of the composition is well known: the relation $UV=0$ is a relation compounded of the relations $U=0$, $V=0$; that is, it is a relation satisfied if, and not satisfied unless one or the other of the two component relations is satisfied. The like notion of composition applies to relations in general; viz., the compound relation is a relation satisfied if, and not not satisfied unless one or the other of the two component relations is satisfied. The author purposely refrains at present from any further discussion of the theory of composition. The conditions for the existence of given systems of equalities between the roots of an equation furnish instances of such composition; in fact, if we express that the function $(*\between x, y)^n$, and its first-derived function in regard to x, or, what is the same thing, the first-derived functions in regard to x, y respectively, have a common quadric factor, we obtain between the coefficients a certain twofold relation, which implies either that the equation $(*\between x, y)^n=0$ has three equal roots, or else that it has two pairs of equal roots; that is, the relation in question is satisfied if, and it is not satisfied unless there is satisfied either the relation for the existence of three equal roots, or else the relation for the existence of two pairs of equal roots; or the relation for the quadric factor is compounded of the last-mentioned two relations. The relation for the quadric factor, for any value whatever of n, is at once seen to be expressible by means of an oblong matrix, giving rise to a series of determinants which are each to be put $=0$; the relation for three equal roots and that for two pairs of equal roots in the particular cases $n=4$ and $n=5$, are given in the author's "Memoir on the Conditions for the existence of given Systems of Equalities between the roots of an Equation," Phil. Trans. t. cxlvii. (1857), pp. 727–731; and he proposes in the present Memoir to exhibit, for the cases in question $n=4$ and $n=5$, the connexion between the compound relation for the quadric factor and the component relations for the three equal roots and for the two pairs of equal roots respectively.