while my experiments for the same form of head, but with much higher velocities, give

$$2b = {\cdot}000060\frac{R^2}{W} = {\cdot}000000104\,\frac{d^2}{W}.$$

There is reason to expect that my value of b will require a small reduction for the low velocities used in M. Hélie's experiments; but it is extremely improbable that it can be reduced to M. Hélie's value. It will thus appear that M. Hélie and I agree in adopting a law of the resistance of the air, but that we have followed quite independent methods in experimenting, and have arrived at different numerical results.

February 27, 1868.

Lieut.-General SABINE, President, in the Chair.

The following communications were read :—

I. "On the Resistance of the Air to Rifled Projectiles." By J. A. Longridge, C.E. Communicated by C. Manby, Esq. Received February 13, 1868.

(Abstract.)

The introduction of elongated rifled projectiles having rendered it necessary to reconsider the laws of resistance which had been deduced by Robins, Hutton, and more recent authors, such an investigation is the object of this paper.

It is first shown that Hutton's law,

$$R = av + bv^2,$$

if applied to the results obtained by the Special Armstrong and Whitworth Committee, 1866, leads to the following equation,

$$x = 1620\log_{10}\left\{\frac{V - 1015{\cdot}4}{v - 1015{\cdot}4}\right\},$$

where V is the initial velocity,

v the residual velocity at the distance x from the gun.

In like manner it is shown that the law adopted by Piobert,

$$R = Av^2 + Bv^3,$$

leads to the equation

$$x = 2197\log_{10}\left\{\frac{V - 994}{v - 994}\cdot\frac{v}{V}\right\},$$

and the law

$$R = Av^3 + Bv^4$$

to the equation

$$x = 2668\log_{10}\left\{\frac{V^2 - 958850}{v^2 - 958850}\cdot\frac{v^2}{V^2}\right\}.$$

These equations all fail by x becoming infinite when $v=1015$, 994, and 979 respectively.

It is, however, observed that, in the assumption of the law of the resistance, the higher the power of velocity the longer does the corresponding equation give rational results; and by assuming $R=av^p$ with the same data, the following equation was obtained,

$$x=\frac{\log^{-1} 23\cdot618}{V^{8\cdot747}}\left\{\left(\frac{V}{v}\right)^{8\cdot747}-1\right\},$$

which gives consistent results for all values of v.

The value of p here is 8·747, which would give the resistance varying nearly as the ninth power of the velocity.

This result led the author of the paper to doubt the accuracy of the experiments, and to seek for further and more correct data, which were obtained from a minute (No. 23,351) of the Ordnance Select Committee, dated 21st September 1867, containing the results of experiments showing the loss of velocity of two projectiles, one of 8·818 lbs., and the other of 251 lbs., in passing through certain given distances with given initial velocities, varying from about 1500 feet to 600 feet per second.

From these results a diagram was constructed, and for each projectile an equation was found which agreed tolerably well with the experimental results.

The form of the equation assumed was

$$(x+a)\,v^n=C\,;$$

and the resulting equation was for the small shot

$$(x+665)\,v^{2\cdot4}=\log^{-1} 10\cdot1473853,$$

and for the large shot

$$(x+2032)\,v^{3}=\log^{-1} 12\cdot6696158,$$

the maximum error being about $1\frac{1}{2}$ per cent. of the velocity.

Introducing into these equations the diameter and weight of the respective projectiles, and taking the index $n=2\cdot5$, the values of C were found to be,

small shot, $\quad C=\log^{-1} 10\cdot7295585\,\frac{W}{d^2}$,

large shot, $\quad C=\log^{-1} 10\cdot7454405\,\frac{W}{d^2}$,

the mean being $C=\log^{-1} 10\cdot7375745\,\frac{W}{d^2}$,

and the resulting general equation

$$\left(x+\frac{\log^{-1} 10\cdot7375745\,.\,W}{d^2\,V^{2\cdot5}}\right)\frac{d^2\,.\,v^{2\cdot5}}{W}=\log^{-1} 10\cdot7375745.$$

The maximum error in velocity, as calculated by this formula, was for the small shot $1\frac{1}{2}$ per cent., and for the large shot $2\frac{1}{2}$ per cent.

From the above equation the resistance per square inch of sectional area is found,

$$R = \frac{v^{4\cdot5}}{\log^{-1} 13\cdot0154756},$$

from which the following Table is constructed, the third column showing the resistance, as calculated by Hutton's formula:—

Table of Resistances to a Rifled Projectile.

Velocity, feet per second.	Resistance, in lbs., per square inch.	Hutton, p. 218.	Velocity, feet per second.	Resistance, in lbs., per square inch.	Hutton, p. 218.
1500	18·89	18·94	700	0·613	3.12
1400	13·87	16·23	600	0·306	2·20
1300	9·94	13·67	500	0·135	1·49
1200	6·92	11·29	400	0·0494	0·93
1100	3·722	9·14	300	0·01354	0·52
1000	3·052	7·24	200	0·00218	0·23
900	1·900	5·61	100	0·0000965	0·556
800	1·118	4·24			

It is next shown that the hypothesis of the great increase of resistance at velocities exceeding 1100 feet per second being due to the vacuum behind the projectile is untenable, because the actual resistance at 1300 feet per second is only 9·94 lbs. per square inch, whilst, according to that hypothesis, the back resistance alone would be 15 lbs. per square inch.

It is suggested that the true reason of the great increase of resistance may be found in the fact that a wave-impulse cannot be propagated at a greater velocity than 1100 feet per second, and that consequently a great condensation of air must take place in front of the projectile at all velocities exceeding this, and the resisting force of such condensed air will increase at a greater rate than indicated by Mariotte's law, owing to the evolution of heat due to the condensation.

A comparison is then instituted between the resistances as ascertained by the above law and those given by Hutton's formula.

It is stated that in experiments made on May 17th, 1867, the small shot weighing 8·8 lbs., moving with a mean velocity of 986 feet per second, lost $58\frac{1}{2}$ feet of velocity in a distance of 900 feet.

The time of flight being ·96 of a second, the resisting force must have been nearly twice the weight of the shot, or more accurately 17·2 lbs.

Now, according to the formula given in this paper, the resistance is found to be 17·75 lbs., whilst Hutton's formula gives a resistance of $46\frac{1}{2}$ lbs.

Having thus obtained a law which gives, with considerable accuracy, the residual velocity at any point of the flight, the corresponding equation to the trajectory is deduced for low degrees of elevation when the length of

the arc differs very slightly from the horizontal distance, or $ds=dx$ nearly; and the following is the resulting equation :—

$$y=x\tan\phi+\mathrm{A}\left\{\frac{n}{2(n+1)}a^{\frac{2(n+1)}{n}}+a^{\frac{n+2}{n}}x-\frac{n}{2(n+1)}(x+a)^{\frac{2(n+1)}{n}}\right\},$$

where $\mathrm{A}=\frac{g}{\mathrm{C}^{\frac{2}{n}}}\cdot\frac{n}{n+2}$, and c and a are the constants, and n the index in the general equation

$$(x+a)\,v^n=\mathrm{C}.$$

Examples of the application of this are given, showing the calculated elevation for the 12-pounder muzzle-loading Armstrong gun for ranges of 2855 yards and 4719 yards, the gun being 17 feet above the planes.

The calculated elevations were 6° 56′ and 14° 6′, the actual elevations being 7° and 15° respectively.

It is not intended to claim more than approximate accuracy for the formulæ in this paper. The general formula has been shown to be derived by taking mean values of n and c, whereas the actual results would indicate that the value of n increases with the diameter of the projectile; and it is shown in a note that the values of n which agree best with experiment are,

for the small shot $n=2{\cdot}4$,

for the large shot $n=4$,

corresponding to the following resistances,

small shot $\mathrm{R}=v^{4\cdot4}$,

large shot $\mathrm{R}=v^6$.

Whether in reality the index does increase with the diameter of the shot must be left to be determined by more extended experiments; meantime it may be assumed that the general formula in this paper represents with tolerable accuracy the law of resistance and the loss of velocity of projectiles varying from 8·8 lbs. to 251 lbs. in weight, from 3 inches to 9 inches in diameter, and from 1500 to 600 feet per second in velocity.

II. "On the Theory of Probability, applied to Random Straight Lines." By M. W. Crofton, B.A., of the Royal Military Academy, Woolwich, late Professor of Natural Philosophy in the Queen's University, Ireland. Communicated by Prof. Sylvester. Received February 5, 1868.

(Abstract.)

This paper relates to the Theory of Local Probability—that is, the application of Probability to geometrical magnitude. This inquiry seems to have been originated by the great naturalist Buffon, in a celebrated problem proposed and solved by him. Though the subject has been more than once touched upon by Laplace, yet the remarkable depth and beauty of this new Calculus seem to have been little suspected till within the last