for me by Mr. W. Barrett Davis, by the aid of a grant from the Donation Fund at the disposal of the Royal Society. The paragraphs of the present memoir are numbered consecutively with those of the former memoirs on Quantics.

II. "On the Cause and Theoretic Value of the Resistance of Flexure in Beams." By W. H. Barlow, F.R.S. Received April 13, 1870.

(Abstract.)

The author refers to his previous papers, read in 1855 and 1857, wherein he described experiments showing the existence of an element of strength in beams, which varied with the degree of flexure, and acts in addition to the resistance of tension and compression of the longitudinal fibres. It was pointed out that the ratio of the actual strength of solid rectangular beams to the strength as computed by the theory of Liebnitz is,

In cast iron, as about $2\frac{1}{4}$ to 1.
In wrought iron as $1\frac{3}{5}$ and $1\frac{3}{4}$ to 1.
And in steel, as $1\frac{3}{5}$ and $1\frac{3}{4}$ to 1.

The theory of Liebnitz assumes a beam to be composed of longitudinal fibres only, contiguous, but unconnected, and exercising no mutual lateral action. But it is remarked that a beam so constituted would possess no power to resist transverse stress, and would only have the properties of a rope.

Cast iron and steel contain no actual fibre, and wrought iron (although some qualities are fibrous) is able to resist strain nearly equally in any direction.

The idea of fibre is convenient as facilitating investigation; but the word fibre, as applied to a homogenous elastic solid, must not be understood as meaning filaments of the material. In effect it represents lines of direction, in which the action of forces can be ascertained and measured; for in torsion-shearing and "*angular deformation*" the fibres are treated by former writers as being at the angle of 45°, because it has been shown that the diagonal resistances have their greatest manifestation at that angle.

Elastic solids being admitted to possess powers of resistance in the direction of the diagonals, attention is called to omission of the effect of resistance in the theory of beams.

The author then states, as the result of his investigation, that compression and extension of the diagonal fibres constitute an element of strength equal to that of the longitudinal fibres, and that *flexure* is the consequence of the relative extensions and compressions in the direct and diagonal fibres, arising out of the amount, position, and direction of applied forces.

Pursuing the subject, it is shown that certain normal relations subsist

between the strains of direct fibres and their relative diagonals, evenly distributed strain being that in which the strain in the direct fibres is accompanied by half the amount of strain in the relative diagonal fibres.

Any disturbance of this relation indicates the presence of another force.

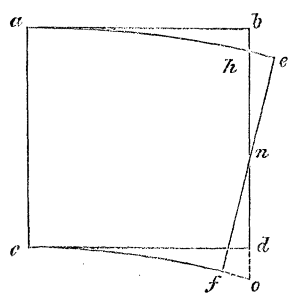
Thus tensile forces applied at *right angles* to compressive forces of equal amount, produce no strain in the diagonals. But if forces applied at right angles to each other are both tensile, or both compressive, the strain in the diagonal is as great as that in the direct fibres.

It is also pointed out that in a given fibre *a b c*, the point *b* may be moved with regard to *a* and *c*, thus producing plus and minus strains in the same fibre.

Treating a solid as being made up of a series of laminæ, and showing that every change of figure can be represented by the variation in length of the diagonals, taken in connexion with those of the direct fibres, the author proceeds to trace the effects of the application of tensile and compressive forces acting longitudinally on either side of the neutral plane, and shows that curvature is the result of the relation between the strains in direct fibres and those in the diagonals.

The operation of a single tensile force applied along one side of the plate and a transverse stress are likewise traced out, and the conditions of "*elastic equilibrium*" referred to.

The amount of resistance offered by the diagonal fibres is shown as follows:—



a b c d represents a portion of a beam strained by transverse forces into the circular curve *a e*.

Two resistances arise.

1. That due to the extension and compression of the longitudinal fibres produced by the rotation of *b d* about the neutral axis, which is the resistance considered in the theory of Leibnitz.
2. That due to the extension and compression of the diagonal fibres, caused by the deformation of the square *a b c d* into the figure *a h o c*, which is the resistance of flexure.

It is then shown that in a solid rectangular beam, the second resistance is equal to the first, and that both resistances act independently, and

consequently that the true theoretic resistance of a solid rectangular beam is exactly twice that arrived at by the theory of Leibnitz,

The strength so computed is in general accordance with the results of experiments in cast iron, wrought iron, steel, and other materials, the maximum strength being found in cast iron, which is one-eighth above, and the minimum in glass, which is one-fourth below the calculated strength.

The author considers this treatment of the subject as arising necessarily out of Dr. Hook's law "ut tensio sic vis," and that it is in effect completing the application of those principles which were only partially applied by Leibnitz.

The paper concludes with some practical illustrations (accompanied by photographs) of the effect of diagonal action.

The appendix contains the results of experiments on the tensile, compressive, and transverse resistances of steel.

III. "On Deep-sea Thermometers." By Staff-Commander John E. Davis, R.N. Communicated by Capt. Richards, R.N., Hydrographer of the Admiralty. Received April 18, 1870.

(Abstract.)

The results of thermometric observations at great depths in the ocean not being of a satisfactory nature, the attention of the Hydrographer of the Navy was directed to the defects in the construction of the Six's self-registering thermometers then in use, and also to the want of knowledge of the effects of compression on the bulb ; and as it was known that a delicate thermometer was affected *in vacuo*, it was natural to suppose that an opposite effect would be had by placing them under pressure, and particularly such as they would be subjected to at great depths.

Several thermometers, of a superior construction, were made by different makers, and permission was granted to make experiments by pressure in an hydraulic press ; but much delay was caused by not being able to obtain a press suitable to the requirements, until Mr. Casella, the optician, had a testing-apparatus constructed at his own expense, and the experiments were commenced.

Previous to the experiments being made, Dr. W. A Miller, V.P.R.S., proposed, or rather revived, a mode of protecting the bulb from compression by encasing the full bulb in glass, the space between the case and the bulb being nearly filled with alcohol*.

A wrought-iron bottle had been made to contain a thermometer, for the purpose of comparison with those subjected to compression ; but it failed, and finally burst under great compression ; it proved, however, of but little consequence, as those designed by Dr. Miller showed so little difference under pressure that they were at once accepted as standards.

* *Vide* Proceedings of the Royal Society, vol. xvii. No. 113, June 17, 1869.