Let $x=a \sin\theta$, $y=a \sin\phi$, then the general equation of the rth order may be written

$$\alpha \sin(m\theta+n\phi)+\alpha' \sin(m'\theta-n'\phi)+\alpha'' \sin(m''\theta+n''\phi)+\ldots = a \sin\theta.$$

Let a number of machines like the foregoing be placed side by side with their ordinate wheels rolling in one another, and their abscissa wheels duly connected. Let one abscissa wheel describe an angle $m\theta$, and the corresponding ordinate wheel the angle $n\phi$, then a nut placed on the corresponding addition wheel, at a distance α from its centre, will cause a horizontal bar to descend vertically through a space $\alpha \sin(m\theta+n\phi)$. In the same way a nut properly placed on the subtraction wheel will cause a horizontal bar to descend vertically through a space $\alpha' \sin(m\theta-n\phi)$. By means of the adjacent machines we may in like manner cause bars to descend through the vertical spaces, $\alpha'' \sin(m'\theta+n'\phi)$, $\alpha''' \sin(m'\theta-n'\phi)$, &c. Now let motion be communicated to the ordinate wheels, and let all the vertical motions due to the addition and subtraction wheels be combined together and made to act vertically upon a nut in one of the abscissa wheels; then the angles θ, ϕ, will satisfy the equation

$$\alpha \sin(m\theta+n\phi)+\alpha' \sin(m\theta-n\phi)+\alpha'' \sin(m'\theta+n'\phi)\ldots = a \sin\theta,$$

which is the general equation of the rth order.

Therefore two bars moved respectively horizontally and vertically by nuts in the wheels describing the angles θ and ϕ will trace by their intersection the required curve.

Communications received since the end of the session.

I. "Spectroscopic Observations of the Sun."—No. V.

By J. Norman Lockyer, F.R.S. Received July 8, 1869.

Since the date of my last communication under the above title the weather has, if possible, been worse for telescopic work than during the winter and spring; my opportunities of observation, therefore, have been very limited: still the sun has occasionally been in such a disturbed state, and our atmosphere has at times been so pure, that several new facts of importance have come out.

I will state them here as briefly as possible, reserving a discussion of them and my detailed observations for a future occasion.

I. The extreme rates of movement in the chromosphere observed up to the present time are:—

Vertical movement	40 miles a second
Horizontal or cyclonic movement .	120 ,,

II. I have carefully observed the chromosphere when spots have been near the limb. The spots have sometimes been accompanied by prominences, at other times they have not been so accompanied. Such observations show that we may have spots visible without prominences in the same region,

and prominences without spots; but I do not say that a spot is not accompanied by a prominence *at some stage of its life*, or that it does not result from some action which, in the majority of cases, is accompanied by a prominence.

III. At times, when a prominence is seen bright on the sun itself, the bright F line varies considerably, both in thickness and brilliancy, within the thickness of the dark line. The appearances presented are exactly as if we were looking at the prominences through a grating.

IV. Bright prominences, when seen above spots on the disk, if built up of other substances besides hydrogen, are indicated by the bright lines of those substances in addition to the lines of hydrogen. The bright lines are then seen very thin, situated centrally (or nearly so) on the broad absorption-bands caused by the underlying less-luminous vapours of the same substances.

V. I have at last detected an absorption-line corresponding to the orange line in the chromosphere. Father Secchi states* that there is a line corresponding to it much brighter than the rest of the spectrum. My observation would seem to indicate that he has observed a bright line less refrangible than the one in question, which bright line is at times excessively brilliant. It requires absolutely perfect atmospheric conditions to see it in the ordinary solar spectrum. It is best seen in a spot-spectrum when the spot is partially covered by a bright prominence.

VI. In the neighbourhood of spots the F bright line is sometimes observed considerably widened out in several places, as if the spectroscope were analyzing injections of hydrogen at great pressure in very limited regions into the chromosphere.

VII. The brilliancy of the bright lines visible in the ordinary solar spectrum is extremely variable. One of them, at 1871·5, and another, at 1529·5 of Kirchhoff's scale, I have detected in the chromosphere at the same time that they were brilliant in the ordinary solar spectrum.

VIII. Alterations of wave-length have been detected in the sodium-, magnesium-, and iron-lines in a spot-spectrum. In the case of the last substance, the lines in which the alteration was detected were *not* those observed when iron (if we accept them to be due to iron alone) is injected into the chromosphere.

IX. When the chromosphere is observed with a tangential slit, the F bright line close to the sun's limb shows traces of absorption, which gradually diminish as the higher strata of the chromosphere are brought on to the slit, until the absorption-line finally thins out and entirely disappears. The lines of other substances thus observed do not show this absorption.

X. During the most recent observations I have been able to detect traces of magnesium and iron in nearly all solar latitudes in the chromosphere. If this be not merely the result of the good definition lately, it would indicate an increased general photospheric disturbance as the maximum sun-

* Comptes Rendus, 1869, 1^{r} sem. p. 358.

spot period is approached. Moreover I suspect that the chromosphere has lost somewhat of its height.

I append a list of the bright lines, the positions of which in the chromosphere I have determined absolutely, with the dates of discovery, remarking that in the case of C and F my observations were anticipated by M. Janssen :—

Hydrogen.

C.	October 20, 1868.
F.	October 20, 1868.
near D.	October 20, 1868*.
near G.	December 22, 1868.
h.	March 14, 1869.

Sodium.

D.	February 28, 1869.

Barium.

1989·5†.	March 14, 1869.
2031·2.	July 5, 1869.

Magnesium and included line.

b^1, b^2, b^3, b^4	February 21, 1869.

Other Lines.

Iron.....	1474.	June 6, 1869.
?	1515·5.	June 6, 1869.
Bright line	1529·5.	July 5, 1869.
?	1567·5.	March 6, 1869
?	1613·8.	June 6.
Iron.....	1867·0.	June 26.
Bright line	1871·5.	„
Iron.....	2001·5.	„
?	2003·4.	„
? band or line near black line, very delicate....	2054·0	July 5.

I have seen other lines besides these at different times; but I do not include them, as their positions have not been determined absolutely.

I refrain from dwelling on this list at present, except to point out that, taking iron as an instance, and assuming that the iron-lines mapped by Ångström and Kirchhoff are due to iron only, I have only been able, up to the present time, to detect 3 lines out of the total number (460) in the spectrum of the lower regions of the chromosphere,—a fact full of promise as

* [*Hydrogen* ?—G. G. S.]

† This reference is to Kirchhoff's scale.

regards the possible results of future laboratory work. The same remark applies to magnesium and barium.

Dr. Frankland and myself have determined that the widening out of the sodium-line in the spectrum of a spot which I pointed out in 1866, and then stated to be possibly an evidence of greater absorption, indicates a greater absorption due to greater pressure.

The continuous widening out of the sodium-line in a spot must therefore be regarded as furnishing an additional argument (if one were now needed) in favour of the theory of the physical constitution of the sun first put forward by Dr. Frankland and myself—namely, that the chromosphere and the photosphere form the true atmosphere of the sun, and that under ordinary circumstances the absorption is continuous from the top of the chromosphere to the bottom of the photosphere, at whatever depth from the bottom of the spot that bottom may be assumed to be.

This theory was based upon all our observations made from 1866 up to the time at which it was communicated to the Royal Society and the Paris Academy of Sciences, and has been strengthened by all our subsequent work; but several announcements made by Father Secchi to the Paris Academy of Sciences and other learned bodies are so opposed to it, and differ so much from my own observations, that it is necessary that I should refer to them, and give my reasons for still thinking that the theory above referred to is not in disaccord with facts. At the same time I must state that Father Secchi does not combat this theory; indeed it is not to be gathered from any of his communications that he has seen any of the papers communicated by myself to the Royal Society.

Father Secchi states that the chromosphere is often separated from the photosphere, and that between the chromosphere and the photosphere there exists a stratum giving a continuous spectrum, which he considers to be the base of the solar atmosphere, and in which he thinks that the inversion of the spectrum takes place.

With regard to the first assertion, I may first state that all the observations I have made have led me to a contrary conclusion. Secondly, in an instrument of comparatively small dispersive power, such as that employed by Father Secchi, in which the widening out of the F line at the base of the chromosphere is not clearly indicated, it is almost impossible to determine, by means of the spectroscope, whether the chromosphere rests on the sun or not, as the chromosphere is an envelope and we are not dealing merely with a section. But an instrument of great dispersive power can at once settle the question; for since the F line widens out with pressure, and as the pressure increases as the sun is approached, the continuous curvature of the F line must indicate really the spectrum of a section; and if the chromosphere were suspended merely at a certain height above the photosphere, we should not get a widening due to pressure: but we always do get such a widening.

With regard to the second assertion, I would remark that if such a continuous-spectrum-giving envelope existed, I entirely fail to see how it could be regarded as a region of selective absorption. Secondly, my observations have indicated no such stratum, although injections of sodium, magnesium, &c. into the chromosphere not exceeding the limit of the sun's limb by 2″ have been regularly observed for several months past. To-day I have even detected a low level of barium in the chromosphere not 1″ high. This indicates, I think, that my instrument is not lacking in delicacy; and as I have never seen anything approaching to a continuous spectrum when my instrument has been in perfect adjustment, I am inclined to attribute the observation to some instrumental error. Such a phenomenon might arise from a local injection of solid or liquid particles into the chromosphere, if such injection were possible. But I have never seen such an injection. If such an occurrence could be observed, it would at once settle that part of Dr. Frankland's and my own theory, which regards the chromosphere as the last layer of the solar atmosphere; and if it were possible to accept Father Secchi's observation, the point would be settled in our favour.

The sodium experiments to which I have referred, however, and the widening out of the lines in the spot-spectra, clearly indicate, I think, that the base of the atmosphere is below the spot, and not above it. I therefore cannot accept Father Secchi's statement as being final against another part of the theory to which I have referred—a conclusion which Father Secchi himself seems to accept in other communications.

Father Secchi remarks also that the F line is produced by the absorption of other bodies besides hydrogen, because it never disappears. This conclusion is also negatived by my observations; for it has very often been observed to disappear altogether and to be replaced by a bright line. At times, as I pointed out to the Royal Society some months ago, when a violent storm is going on accompanied by rapid elevations and depressions of the prominences, there is a black line on the less-refrangible side of the bright one; but this is a phenomenon due to a change of wave-length caused by the rapid motion of the hydrogen.

With regard to the observation of spot-spectra, I find that every increase of dispersive power renders the phenomenon much more clear, and at the same time more simple. The selective absorption I discovered in 1866 comes out in its most intense form, but without any of the more complicated accompaniments described by Father Secchi. I find, however, that by using three prisms this complexity vanishes to a great extent. We get portions of the spectrum here and there abnormally bright, which have given rise doubtless to some of the statements of the distinguished Roman observer; but the bright lines, properly so-called, are as variable as they are in any other part of the disk, but not much more so. I quite agree that the "interpretation" of sun-spot phenomena to which Father Secchi has referred*,

* Comptes Rendus, 1869, 1r sem. p. 764.

which ascribes the appearances to anything but selective plus general absorption, is erroneous. But as I was not aware that it had ever been propounded, I can only refer to my own prior papers in support of my assertion, and to Mr. Huggins's indorsement of my observations, which were communicated to the Royal Society some three years ago.

II. "Researches on Gaseous Spectra in relation to the Physical Constitution of the Sun, Stars, and Nebulæ."—Third Note. By E. Frankland, F.R.S., and J. Norman Lockyer, F.R.S. Received July 14, 1869.

1. It has been pointed out by one of us that the vapours of magnesium, iron, &c. are sometimes injected into the sun's chromosphere and are then rendered sensible by their bright spectral lines *.

2. It has also been shown (1) that these vapours, for the most part, attain only a very low elevation in the chromosphere, and (2) that on rare occasions the magnesium vapour is observed like a cloud separated from the photosphere.

3. It was further established on the 14th of March, 1869, and a drawing was sent to the Royal Society indicating, that when the magnesium vapour is thus injected the spectral lines do not all attain the same height.

Thus of the b lines, b^1 and b^2 are of nearly equal height, but b^4 is much shorter.

4. It has since been discovered that of the 450 iron lines observed by Ångström, only a very few are indicated in the spectrum of the chromosphere when iron vapour is injected into it.

5. Our experiments on hydrogen and nitrogen enabled us at once to connect these phenomena, always assuming, as required by our hypothesis†, that the great bulk of the absorption to which the Fraunhofer lines are due takes place in the photosphere itself.

It was only necessary, in fact, to assume that, as in the case of hydrogen and nitrogen, the spectrum became simpler where the density and temperature were less, to account at once for the reduction in the number of lines visible in those regions where, on our theory, the pressure and temperature of the absorbing vapours of the sun are at their minimum.

6. It became important, therefore, to test the truth of this assumption by some laboratory experiments, the preliminary results of which we beg to communicate in this Note, reserving details, and an account of the further experiments we have already commenced, for another paper under the above title.

We took the spark in air between two magnesium poles, so separated that the magnesium spectrum did not extend from pole to pole, but was visible only for a little distance, indicated by the atmosphere of magnesium vapour, round each pole.

* Proc. Roy. Soc. vol. xvii. p. 351.

† *Ibid.* p. 290.