order to determine the capacity of the sliding condenser at the lower extremity of its range, a spherical condenser, so constructed that its capacity could be accurately determined in absolute measure, was employed. An apparent discrepancy in the results obtained, arising from an inequality in the condensers forming the platymeter, is then considered, and the method of deducing the true result investigated. A series of experiments is then described which gave 1·975 as the specific inductive capacity of paraffine, that of air being taken as unity, but failed to show whether this alters with variations of temperature. An improved form of condenser, composed of concentric brass cylinders with paraffine for the dielectric, and the results obtained from it, are then described. The measurements made at different temperatures show no variation of specific inductive capacity. In order to allow to the paraffine freedom of expansion with temperature, another form of condenser was employed, and the same results obtained. A series of experiments was then made on the expansion of paraffine with temperature, in order to estimate the effect of this upon the capacity of paraffine condensers. As a mean of the results, it was found that the linear expansion of paraffine at 9° C. is ·000237 per degree. Some further measurements of the cylindrical condenser were made with the same result as before. Thus all the measurements of this condenser made at temperatures ranging from —12°·15 to 24·35 C. show no variation of specific inductive capacity of paraffine with temperature. This was found to be 1·977, that of air being taken as unity.

In a note added to the paper a description is given of an improved form of sliding condenser.

III. "On the Uniform Flow of a Liquid." By Henry Moseley, M.A., D.C.L., Canon of Bristol, F.R.S., and Corresponding Member of the Institute of France. Received December 1, 1870.

(Abstract.)

The resistance of every molecule of a liquid at rest which a solid (by moving through it) disturbs, contributes its share to the resistance which the solid experiences; so that the inertia of each molecule so disturbed and its shear must be taken into account in the aggregate, which represents the resistance the liquid offers to the motion of the solid. The motions communicated to the molecules of a liquid by a solid passing through it, and the resistances opposed to them, however, are so various, and so difficult to be represented mathematically, that in the present state of our knowledge of hydrodynamics the problem of the resistance of a liquid at rest to a solid in motion is perhaps to be considered insoluble. As it regards the opposite problem of the resistance of a solid at rest to a liquid in motion (as in the case of a liquid conveyed through a pipe), there are in like manner to be taken into account the disturbances created by that re-

sistance in what would otherwise have been the motion of each individual molecule of the liquid so disturbed.

This problem, however, is by no means so difficult as the other. There is, indeed, a case in which it admits of solution. It is that of a liquid flowing from a reservoir, in which its surface is kept always at the same level, through a circular pipe which is perfectly straight, and of the same diameter throughout, and of a uniform smoothness or roughness of internal surface, and always full of the liquid. The liquid would obviously in such a pipe arrange itself in infinitely thin cylindrical films coaxial with the pipe, all the molecules in the same film moving with the same velocity, but the molecules of different films with velocities varying from the axis of the pipe to its internal surface. The direction of the motions of the molecules of such a liquid being known, and all in the same film moving with the same velocity, which velocity is a function of the radius of the film, and the law of the resistance of each film to the slipping over it of the contiguous film being assumed to be known, as also the head of water, it is possible to express mathematically

(1st) the work done per unit of time by the force which gives motion to the liquid, and

(2nd) the work per unit of time of the several resistances to which the liquid in moving through the pipe is subjected, and

(3rd) the work accumulated per unit of time in the liquid which escapes— and thus to constitute an equation in which the dependent variables are the radius of any given film, and the velocity of that film. This equation being differentiated and the variables separated, and the resulting differential equation being integrated, there is obtained the formula

$$v_0 \epsilon^{-\frac{250 r}{l}},$$

where v is the velocity of the film whose radius is r, and v_0 that of the central filament, and l the length of the pipe—the unit of length being one metre, and of time one second.

The method by which the author has arrived at this formula is substantially the same as that which he before used in a paper read before the Society on the "Mechanical Impossibility of the Descent of Glaciers by their weight only," and which he believes to be a method new to mechanical science. It was indeed to verify it in its application to liquids that he undertook the investigations which he now submits to the Society, which, however, he has pursued beyond their original object.

The recent experiments of MM. Darcy and Bazin* have supplied him with the means of this verification. These experiments, made with admirable skill and precision, on pipes upwards of 100 metres in length, and varying in diameter from $0^{m}\cdot0122$ to $0^{m}\cdot5$, under heads of water varying

* Recherches Expérimentales relatives au mouvement de l'Eau dans les Tuyaux, par H. Darcy: Paris, 1857. Recherches Hydrauliques, par MM. Darcy et Bazin: Paris, 1865.

in height from $0^{m}\cdot027$ to $30^{m}\cdot714$, include (together with numerous experiments on the quantity of water which flows per second from such pipes under different conditions) experiments on the velocities of the films of water at different distances from the axes of the pipes, made by means of an improved form and adaptation of the well-known tube of Pitot. These last-mentioned experiments afford the means of verifying the above-mentioned formulæ. With a view to this verification, the author has compared the formula with sixty of the experiments of M. Darcy, and stated the results in the first two Tables of his paper.

The discharge per 1″ from a pipe of a given radius may be calculated from the above formula in terms of the velocity of the central filament. This calculation the author has made, and compared it with the results of eleven of M. Darcy's experiments.

Where in the formula which thus represents the discharge from a pipe of given radius, in terms of the velocity of the central filament, the radius is made infinite, an expression is obtained for the volume of liquid of a cylindrical form, but of infinite dimensions (laterally), which would be put in motion by a *single filament* of liquid which traversed its axis; and, conversely, it gives the volume of such a liquid in motion which would be held back by a filament of liquid kept at rest along its axis. Thus it explains the well-known retarding effect of filaments of grass and roots in retarding the velocities of streams.

It is the relation of the velocity of any film to that of the central filament which the author establishes in the above formula. To the complete solution of the problem it is necessary that he should further determine the actual velocity v_0 of the central filament. This is the object of the second part of his paper. This velocity being known, the actual discharge per 1″ is known. The following is the formula finally arrived at:—

$$Q=C.\left[\epsilon^{-\frac{250\,R}{l}}-\frac{250\,R}{l}-1\right]R_{11}\ h^{\frac{1}{2}}\,l^{\frac{5}{3}},$$

where

Q=discharge per 1″ in cubic metres.

R=radius of pipe in metres.

l =length of ditto.

h =head of water.

C =a constant dependent on the state of the internal surface of the pipe.

The values of this constant C, as deduced from the experiments of M. Darcy are given,

1st, for new cast-iron pipes;

2nd, for the same covered with deposit;

3rd, for the above *cleaned*;

4th, for iron pipes coated internally with bitumen;

5th, for new leaden pipes;

6th, for glass pipes.

The author compares this formula with sixty-two of M. Darcy's experiments, and records the results of this comparison in the last three Tables of his paper.

The paper concludes with an investigation of the rise in the temperature of a liquid flowing through a pipe caused by the resistances which its coaxial films oppose to their motions on one another (or, as it is termed, their *frictions* on one another) and on the internal surface of the pipe. The pipe is in this investigation supposed to be of a perfectly non-conducting substance.

February 9, 1871.

General Sir EDWARD SABINE, K.C.B., President, in the Chair.

The following communications were read:—

I. "On the Effect of Exercise upon the Bodily Temperature." By T. CLIFFORD ALLBUTT, M.A., M.D. Cantab., F.L.S., Member of the Alpine Club, &c. Communicated by Mr. BUSK. Received November 12, 1870.

(Abstract.)

The object of the author in carrying out the experiments recorded in the present paper was to inquire whether the regulating-power of the organism held good under great variations of muscular exertion. For this purpose he made frequent daily examinations of his own temperatures during a short walking tour in Switzerland, and found that the effect of continuous muscular exertion upon himself was to sharpen the curve of daily variation—the culmination being one or two tenths higher than usual, and the evening fall coming on more rapidly and somewhat earlier. Charts of the daily temperatures were handed in with the paper. The author made reference also to some observations of M. Lortet, which differed from his own. These observations, which did not come into Dr. Clifford Allbutt's hands until his own experiments were partially completed, were adduced by M. Lortet to prove that the human body was very defective in regulating-power under the demands of the combustion needed to supply the force expended in muscular exertion. Dr. Clifford Allbutt's results were very decidedly opposed to those of M. Lortet; for only on two occasions did he note the depressions of temperature which M. Lortet regards as constant. It would seem, however, that the body is more or less liable to such depressions when engaged in muscular exertion; but the cause of them is very obscure. Of the two low temperatures noted by the author, one occurred during a very easy ascent of lower slopes, and the second was observed during a descent. The author thinks that they may be due to some accidental deficiency in combustion, and inquires whether the capacity of the chest in different individuals may account for the varying in-