III. "On the Problem of the In- and Circumscribed Triangle." By A. Cayley, F.R.S. Received December 30, 1870.

(Abstract.)

The problem of the in and circumscribed triangle is a particular case of that of the in- and circumscribed polygon: the last-mentioned problem may be thus stated—to find a polygon such that the angles are situate in and the sides touch a given curve or curves. And we may in the first instance inquire as to the number of such polygons. In the case where the curves containing the angles and touched by the sides respectively are all of them distinct curves, the number of polygons is obtained very easily and has a simple expression: it is equal to twice the product of the *orders* of the curves containing the several angles respectively into the product of the *classes* of the curves touched by the several sides respectively; or, say, it is equal to twice the product of the orders of the angle-curves into the product of the classes of the side-curves. But when several of the curves become one and the same curve, and in particular when the angles are all of them situate in and the sides all touch one and the same curve, it is a much more difficult problem to find the number of polygons. The solution of this problem when the polygon is a triangle, and for all the different relations of identity between the different curves, is the object of the present memoir, which is accordingly entitled "On the Problem of the In- and Circumscribed Triangle;" the methods and principles, however, are applicable to the case of a polygon of any number of sides, the method chiefly made use of being that furnished by the theory of correspondence.

IV. "On the Unequal Distribution of Weight and Support in Ships, and its Effects in Still Water, in Waves, and in Exceptional Positions on Shore." By E. J. Reed, C.B., Vice-President of the Institution of Naval Architects. Communicated by Prof. G. G. Stokes, Sec. R.S. Received December 31, 1870.

(Abstract.)

The object of this paper is to bring within the grasp of calculation what the author considers a much neglected division of shipbuilding science and art, by investigating the actual longitudinal bending- and shearing-strains to which the structure is exposed in ships of various forms under the varying conditions to which all ships are more or less liable. The weakness exhibited by many ships has long pointed to the necessity of further investigation in this direction; and two modern events (the use of iron and steel in shipbuilding, and the introduction of armoured ships) have added much to the urgency of the inquiry.

After glancing briefly at the state of the question as presented in the writings of Bouguer, Bernoulli, Euler, Don Juan D'Ulloa, Romme, Dupin,

and Dr. Young, the author proceeds to show that the introduction of steam as a propelling agent, and of largely increased lengths and proportions for ships, has brought about a comparative distribution of weight and buoyancy very different from that which those writers contemplated. He has taken the cases of three or four typical modern ships, and has had the relative distributions of the weight and buoyancy very carefully and fully calculated and graphically recorded. Owing to the great labour involved, only the most meagre and unsatisfactory attempts to measure and exhibit the actual strains of ships had previously been made; and the author's results are wholly unlike any that have before been worked out and published. The first case is that of the royal yacht 'Victoria and Albert,' which represents the conditions of long fine-lined paddle-steamers, with great weights of engines, boilers, and coals concentrated in the middle, combined with very light extremities. The second case s that of the 'Minotaur,' which represents long fine-lined ships with great weights distributed along their length. The iron-clad 'Bellerophon' is the third case, representing shorter ships with fuller lines and very concentrated midship weights; and the last case is that of the 'Invincible' class, in which the weights of armour &c. are still more concentrated. All these ships are divided into very numerous short lengths; and the weight of hull, weight of equipment, and buoyancy or displacement of each short length are separately calculated, curves of weight and buoyancy being constructed from these items used as ordinates. A third curve, of which the ordinates are the differences between the curves of total weight and of displacement, known as the curve of loads, is constructed. By summing the ordinates, or calculating the areas of this curve, from point to point, a curve of shearing or racking forces is formed; and by employing the products of the areas of the curve of loads (taken step by step) into the distances of their centres of gravity from one end as ordinates of a new curve, a curve of bending-moments is constructed.

These operations are performed for all the ships previously named, first when they are floating in still water, next when they are respectively floating on the crests of waves of their own lengths, and thirdly when they are floating in the hollow of two adjacent waves of those lengths. The maximum breaking-strains of all the ships when supported on shore, first at the extremities and next at the middle, are also calculated, and compared with the still-water and sea strains.

In considering still-water strains, the author shows that remarkable contrasts of strain occur between ships light and laden, and that the theories of former writers on the subject require to be greatly modified. In some cases the breaking-strain is increased as the ship is lightened. In discussing the shearing-strains, he points out that the sections of maximum shearing-strain in a ship coincide with the balanced or "water-borne sections" (at which the weight and buoyancy are equal), and that in most ships the number of these sections is equal. The position of absolute

maximum shearing-force occupies very different positions in different types of ship. Sections of zero shearing-force coincide with "sections of water-borne division," on either side of which the weight balances the buoyancy; and their number is usually odd. He afterwards shows that maximum and minimum bending-moments are experienced by sections of water-borne division, and that between two sections of maximum "hogging"-moment there must fall either a section of minimum hogging-moment or a section of maximum "sagging"-moment, and that it is an error to suppose (as all former writers on the subject have done) that the absolute maximum bending-moment falls amidships. In the 'Victoria and Albert' the last-named moment is in the forebody; in the 'Bellerophon' and 'Audacious' it is in the afterbody. The effect of the horizontal fluid pressure in the longitudinal bending-moments is also discovered, and shown to be important.

The dynamical aspect of the question—showing the strains brought upon ships at sea—is admitted to be both the more difficult and the more important. In discussing the strains, the author calculates them approximately under the following assumptions:—(1) That for the moment the effect of the ship's vertical motion may be neglected. (2) That for the moment the ship may be regarded as occupying a position of hydrostatical equilibrium. (3) That the methods of calculating bending- and shearing-strains previously used for still water may be employed here also, in order to approximate to the momentary strains. The following particulars of the 'Minotaur' and 'Bellerophon' floating on the crests and in the hollows of waves of their own length respectively and of proportionate heights, illustrate the results to which the calculations before named have led for those ships.

On Wave-crest.

	'Minotaur.'	'Bellerophon.'
Excess of weight forward	1,275 tons.	445 tons.
,, ,, aft	1,365 ,,	555 ,,
,, buoyancy amidships.	2,640 ,,	1,000 ,,
Maximum shearing-strain	1,365 ,,	555 ,,
,, bending-moment ..	140,300 foot-tons.	43,600 foot-tons.
In Wave-hollow.		
Excess of buoyancy forward ..	685 tons.	640 tons.
,, ,, aft	695 ,,	600 ,,
,, weight amidships ..	1,380 ,,	1,240 ,,
Maximum shearing-strain	695 ,,	640 ,,
,, bending-moment ..	74,800 foot-tons.	48,800 foot-tons.

The strains of ships supported on shore, first at the extremities and then at the middle, are next investigated. The following Table gives the ap-

proximate quantitative values of the shearing-forces and bending-moments obtained for the three ships, 'Minotaur,' 'Bellerophon,' and 'Victoria and Albert':—

	'Minotaur.'		'Bellerophon.'		'Victoria and Albert.'	
	Shearing-force.	Bending-moment.	Shearing-force.	Bending-moment.	Shearing-force.	Bending-moment.
	Displacement.	Displace. × length.	Displacement.	Displace. × length.	Displacement.	Displace. × length.
In still water	$\frac{1}{22}$	$\frac{1}{88}$	$\frac{1}{33}$	$\frac{1}{176}$	$\frac{1}{16}$	$\frac{1}{139}$
On a wave-crest	$\frac{1}{7}$	$\frac{1}{28}$	$\frac{1}{13}$	$\frac{2}{97}$	$\frac{1}{11}$	$\frac{1}{43}$
In a wave-hollow	$\frac{1}{14}$	$\frac{1}{53}$	$\frac{1}{11}$	$\frac{1}{43}$	$\frac{1}{6}$	$\frac{1}{23}$
Supported at the extremities.	$\frac{1}{2}$	$\frac{1}{7}$	$\frac{1}{2}$	$\frac{1}{7}$	$\frac{1}{2}$	$\frac{1}{6}$
Supported at the middle	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{2}{3}$	$\frac{1}{12}$

February 16, 1871.

General Sir EDWARD SABINE, K.C.B., President, in the Chair.

The following communications were read:—

I. "On some of the more important Physiological Changes induced in the Human Economy by change of Climate, as from Temperate to Tropical, and the reverse" (concluded)*. By ALEXANDER RATTRAY, M.D. (Edinb.), Surgeon R.N., H.M.S. 'Bristol.' Communicated by Mr. BUSK. Received January 6, 1871.

IV. *The influence of Tropical Climates on the Kidneys and Skin.*

None of the organs of the body are more visibly affected by great changes of climate than these, and their secretions, the urine and perspiration. As with the lungs† and other internal viscera, the congestion of the kidneys lessens, while that of the skin increases, when the blood is attracted to the surface by heat. The reverse happens when it is driven inward by cold. This involves their special and vicarious, waste-product and water-excreting functions alike. In the tropics the skin doubtless excretes much of the water thrown off by the kidneys and lungs in colder regions, as well as the nitrogen and carbon of the former, and carbonic acid of the latter. The elimination of surplus water, one of the most important uses of all of the four great depurating organs, is largely effected by these two. Their intimate relation in this office in cold latitudes is already known. We shall here attempt to show what it is in the tropics.

* Continued from Proceedings of the Royal Society, June 16, 1870, vol. xviii. p. 529.

† Ibid. p. 523.