use of, to serve as the origin and insertion of the supplemental muscle. The Table just given shows that in the employment of these supplemental muscles they are always so arranged as to work to the maximum advantage.

Thus the action of skew muscles, which, on a hasty examination, would be pronounced to be *imperfections*, furnishes another proof of my postulate.

POSTULATE.—*The muscles, bones, and joints of all animals are so related geometrically to each other as to produce in every case the maximum amount of work.*

IV. "On the Rings produced by Crystals when submitted to Circularly Polarized Light." By WILLIAM SPOTTISWOODE, M.A., Treas. R.S. Received April 24, 1872.

The general optical arrangements here used are known. Particular cases of the phenomena resulting from it have been described by Fresnel and by Airy; and more have doubtless been observed by others. The main part of the apparatus consists, so far as polarization is concerned, of the ordinary polarizer P, analyzer A, and crystal plate to be examined, C. To this are added two quarter-undulation plates of mica, Q, Q_1, one of which, Q, is placed below and the other, Q_1, above the crystal C. Let i, a, b, j be the angles between the principal sections of P, Q, C, Q_1, A, taken two and two together in the order written, all the angles being considered to be of the same sign when measured in the same direction—say, positive with that of a clock-hand. Then, if θ be the retardation produced in any ray, whose wave-length is λ, by the crystal C, the intensity of the ordinary image at any point is given (Verdet, 'Leçons d'Optique Physique,' tome ii. p. 201 *) by the formula

$$\begin{aligned}I^2 = {} & \cos^2(j-i)\cos^2(a+b)+\sin^2(j+i)\sin^2(a+b)\\ & +(\cos 2i\sin 2a\sin 2b\cos 2j-\sin 2i\sin 2j)\sin^2\frac{\theta}{2}\\ & +(\cos 2i\sin 2a\sin 2j+\sin 2i\sin 2b\cos 2j)\sin\frac{\theta}{2}\cos\frac{\theta}{2}.\end{aligned}$$

Of this general case four particular instances have been studied, viz. :—

$$a = b = 45°, \text{ whence } I^2=\sin^2\left(j+i+\frac{\theta}{2}\right); \quad . \quad . \quad . \quad \text{(I.)}$$

$$a = -b = 45°, \text{ whence } I^2=\cos^2\left(j-i-\frac{\theta}{2}\right); \quad . \quad . \quad . \quad \text{(II.)}$$

$$i = j = 45°, \text{ whence } I^2=\cos^2\frac{\theta}{2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

$$i = -j = 45°, \text{ whence } I^2=\sin^2\frac{\theta}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IV.)}$$

* From which work the greater part of the discussion of the cases I., II., III., IV. has been taken.

In all these cases the light falling on the analyzer is plane polarized; and the following Table will give the angle through which the plane of polarization has been turned in its passage through the system Q, C, Q_1.

Case.	Angle between plane of polarization and principal plane of A.	Angle between principal planes of P and A.	Sum = angle through which plane of polarization is turned.
I.	$\frac{\pi}{2}-i-j-\frac{\theta}{2}$	$\frac{\pi}{2}+j+i$	$\pi-\frac{\theta}{2}$
II.	$j-i-\frac{\theta}{2}$	$j+i$	$2j-\frac{\theta}{2}$
III.	$\frac{\theta}{2}$	$\frac{\pi}{2}+a+b$	$\frac{\pi}{2}+a+b+\frac{\theta}{2}$
IV.	$\frac{\pi}{2}\quad-\frac{\theta}{2}$	$a+b$	$\frac{\pi}{2}+a+b-\frac{\theta}{2}$

Cases I. and II. represent Fresnel's experiments, cases III. and IV. Airy's. In all four the rotation depends upon θ, that is, upon the wave-length; and consequently if the analyzer be turned round, we shall have a succession of colours which recur after every 180°. Thus far the effects will resemble those produced by quartz, excepting that the tints will not be exactly the same; because in these cases the rotation of the plane of polarization is approximately proportional to λ^{-1}, while in that of quartz it is proportional to λ^{-2}. It may be noticed that if I., II., IV. represent a right-handed, III. will represent a left-handed quartz.

The result in case I., being independent of i and j, shows that if the system of plates Q, C, Q_1 be turned round bodily in its own plane there will be no change in the phenomena produced.

The result in case II. shows that if the system of plates be turned bodily in its own plane, we have a sequence of colours the reverse of those produced when the analyzer is turned round, and returning at every 90° instead of every 180°.

The results in cases III. and IV., being independent of the separate values of a and b, but depending only upon the sum $a+b$, show that if the system P, Q, Q_1, A remain fixed, and the crystal C be turned in its own plane, the phenomena will undergo no change.

If the experiments be made with convergent light, two general results are manifest: first, that rings are formed as with plane-polarized light, but better defined, because in the absence of any constant term in the value of I^2 the minima values are absolutely zero; and secondly, that no dark brushes will exist.

If we now consider the rings produced with convergent light and the system Q, C, Q_1, in which the axes of Q, Q_1 are crossed, and the axis of

Q is inclined at 45° to the principal section of P, we have the conditions

$$a+b=90^\circ,\ i=45^\circ;$$

whence

$$\sin(a+b)=1,\quad \cos(a+b)=0,$$
$$\sin 2a=\sin 2b,\quad \cos 2a=-\cos 2b,$$
$$\sin 2(i+j)=\cos 2j,\quad \sin 2i=1,\quad \cos 2i=0.$$

Introducing these into the general expression for the intensity, we obtain

$$I^2=\tfrac{1}{2}\{1+\sin 2j\cos\theta+\sin 2b\cos 2j\sin\theta\}.$$

This shows that if the analyzer be placed parallel to the polarizer ($j=45^\circ$) and then turned round in a positive direction until $j=90^\circ$, $\sin 2j$ will retain its sign $+$, while $\cos 2j$ will take a negative sign. Also if we exchange the crystal supposed positive for a negative one, $\cos\theta$ will retain its sign while $\sin\theta$ will change it. It follows, therefore, that if in examining uniaxal crystals with this arrangement the analyzer be turned in one direction, the distortion of the rings from a circular to an oval form will take place in a vertical line for positive crystals and in an horizontal for negative; while if the analyzer be turned in the opposite sense, the distortion of the rings will likewise be reversed. This process, therefore, will give a method of distinguishing positive from negative crystals.

That the same law holds good for biaxal crystals is clear from the formulæ which apply equally to both classes; but it may be experimentally verified by applying the method to the series of mica plates crossed in different groups so as to show the transition from the biaxal to the uniaxal condition. In the case of biaxal crystals, however, the readiest method of making the experiment is to observe the rings near the centre of the field where the lemniscate loops meet, and to notice whether, on turning the analyzer in a given direction, the rings appear to stream in towards one another from the axes, or to be drawn out towards the axes. If for a given arrangement of the plates the rings of a negative uniaxal crystal are drawn out horizontally when the analyzer is turned to the right, for the same arrangement and motion the rings of a negative biaxal crystal will appear to flow together.

The Society adjourned over Ascension Day to Thursday, May 16.