part of the plate in front of the rivet. The amount of that bending stress is approximately estimated, and a rule found for the distance of the rivet-hole from the edge of the plate. When the plate gives way by tearing from rivet-hole to rivet-hole, it is commonly assumed that the stress on the part of the plate between the rivets is a uniformly distributed stress. This is shown to be not strictly correct, and the want of uniformity of stress will cause the plate to give way with a lower average intensity of stress than that which corresponds to the ultimate resistance of the plate to tension. How much the plate may be weakened by the want of uniformity in the distribution of the stress it is impossible to calculate. Probably the loss of strength due to this cause is in ductile plates very small; but it is pointed out that this weakening may account, at least in part, for the apparent loss of strength of the plates at joints as compared with the same plates broken in an unperforated condition. This loss of strength has been hitherto ascribed entirely to injury done to the plate in the punching-process. When the rivet gives way by shearing, the stress on the section is also not uniform. In consequence of the great deformation of the rivet before fracture, it is subjected to bending as well as shearing action. The friction between the plates induced by the contraction of the rivets in cooling has been supposed sometimes to add to the apparent resistance of the rivet to shearing. It is shown that a considerable displacement of the plates takes place before ultimate fracture, and that the deformation of the rivets is so great that it can hardly be supposed that they exert any tension, holding the plates together at the moment of fracture. The friction should therefore be entirely neglected in estimating the ultimate resistance of riveted joints; and this, indeed, has been done by most English writers.

The question of the resistance to deformation is then discussed, and a limit fixed for the safe intensity of the pressure on the bearing surface of the rivet.

The practical considerations affecting the diameter of the rivet are then stated; and joining these to the considerations given above, a series of rules for proportioning riveted joints are drawn up, and some Tables of the proportions of rivets and joints are given.

II. "On the Employment of Meteorological Statistics in determining the best Course for a Ship whose Sailing Qualities are known." By Francis Galton, F.R.S. Received March 13, 1873.

If we desire to estimate which of two alternative passages between the same ports would be performed most quickly on the average of many voyages, no knowledge can be more immediately useful than that of the distance which the ship could accomplish at various points of the routes

in a unit of time. The intention of the present memoir is to show how this desideratum may be most readily obtained, and the precise method by which, when it has been obtained, it should be turned to account. It should be added that in the earlier part of it I am obliged to recapitulate views which I have already published in the Transactions of the British Association, 1866 (Transactions of Sections), p. 17.

Suppose the meteorological statistics of some ocean district to show that, on the average, out of every 100 ships that visited it the weather recorded in the following Table was experienced :—

Table I. Statistics of weather.

30	ships	find	the wind	N.,	with	an average	force	of	3
25	”	”	”	E.,	”	”	”		2
15	”	”	”	S.,	”	”	”		1
10	”	”	”	W.,	”	”	”		2
20	”	”	”	calm,	”	”	”		0
100									

At first I will suppose no current to exist. I have grouped the winds under the 4 cardinal points for the sake of simplicity in explanation; but it must be recollected that in practice they would be grouped under at least 8 points, and probably 16.

Let the sailing qualities of the ship be those specified in Table II., in which the figures have been extracted from an elaborate but, I fear, only approximate schedule of the performances of the standard ship of meteorologists, commonly described as the "Beaufort Ship." I have

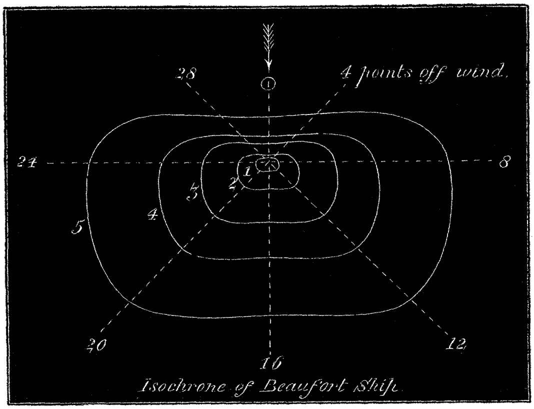
Table II. Sailing qualities of Ship.

Force of wind.	Number of miles made good in one day's sail. Number of points at which the course of the ship lies off the wind, reckoning to the right.			
	0 points, or wind right ahead.	8 points, or wind abeam (same value as 24 points).	16 points, or wind astern.	24 points, or wind abeam (same value as 8 points).
1	5	12	12	12
2	10	38	34	38
3	24	89	79	89
4	38	139	125	139
5	62	230	202	230

protracted the data contained in the original schedule, and formed from them the curves represented in the annexed diagram (fig. 1).

From the materials contained in Tables I. and II. we have to calcu-

Fig. 1.

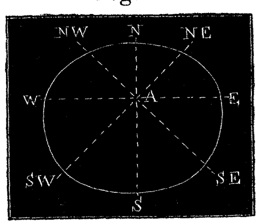


late the average distance towards each of the 4 cardinal points that the ship is capable of accomplishing in a day.

The ship that on each of the 100 occasions makes the best of her way to the N. experiences the following weather. The wind is right ahead of her (that is, is N.) on 30 occasions, with an average force of 3; therefore, under those circumstances, as we learn from Table II., she will sail an aggregate distance of $30\times24=720$ miles. On 25 occasions the wind is abeam (E.) of her, with a force of 2, and she sails under these conditions $25\times38=950$ miles; on 15 occasions it is astern (S.), with force 1, by which she makes $15\times12=180$ miles; and, lastly, it is abeam (W.) on 10 occasions, with force 2, contributing 380 miles. Therefore the total distance she would sail in the 100 voyages, each of one day's duration, and in every case to the N., is $720+950+180+380=2230$ miles; and consequently her average daily performance to that point of the compass is 22·30 miles.

Similar computations give 34·40 for an E. course, 37·75 for a S., and 38·00 for one to the W. The form of calculation is appended in Table III.

Fig. 2.



If we take a point A, fig. 2, and mark from it the distances we have just obtained, and draw a contour, with a free hand, enclosing the dots, we shall have a figure such as is represented, which determines the performance of the ship from A to every point of the compass. It should be borne in mind that, although there must be much guess work in drawing the curve under the guidance of only 4 dots, there will be considerably less chance of error when we have 8, and none of appreciable

amount when we have 16. In calculating to only 4 points, we have, as is shown in Table III., no more than 4 lines and 4 columns, or $4^2=16$ entries. If we dealt with 8 points, the number of entries would amount to $8^2=64$, and if with 16, to $16^2=256$ entries; but the amount of labour involved in such tedious computations need excite no apprehension, because I shall show how calculation may be entirely dispensed with, and the results obtained by the aid of a machine with remarkable facility and quickness.

Table III.

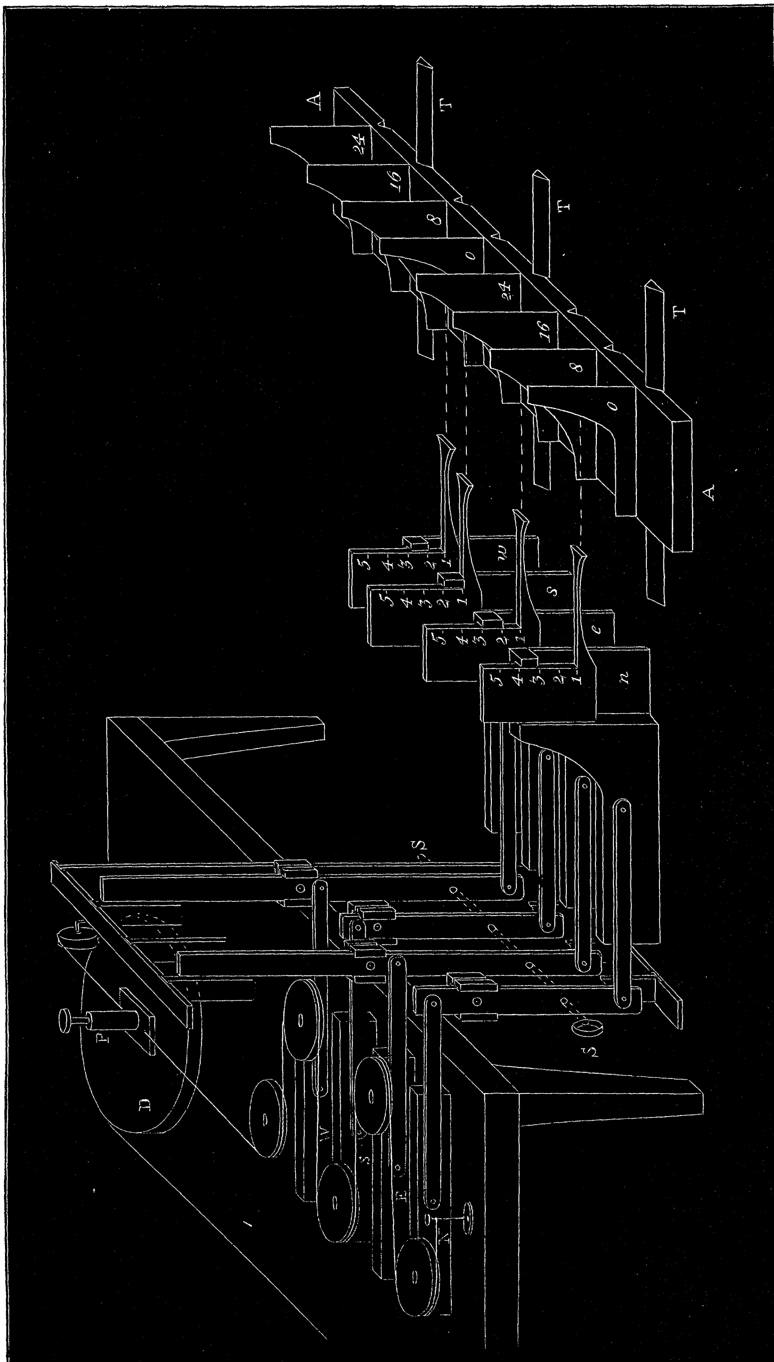
Course of ship.	Relative frequency and force of wind.				Total miles on each course.
	N. 30 per cent. Force 3.	E. 25 per cent. Force 2.	S. 15 per cent. Force 1.	W. 10 per cent. Force 2.	
N.	Wind ahead, 0 point. 720 miles.	Abeam, 8 950	Astern, 16 180	Abeam, 24 380	2230
E.	Abeam, 24 2670	Ahead, 0 250	Abeam, 8 180	Astern, 16 340	3440
S.	Astern, 16 2370	Abeam, 24 950	Ahead, 0 75	Abeam, 8 380	3775
W.	Abeam, 8 2670	Astern, 16 850	Abeam, 24 180	Ahead, 0 100	3800

We have thus far supposed no current to exist; if there should be a current, the above contour would be incorrect as regards the position of A; but it would be perfectly correct in regard to the position in which a float would be found at the end of the day which had been dropped in the water at the beginning, because the float and the ship would have drifted simultaneously in parallel lines. If, then, A be the point of departure and A′ be the position of the float at the end of the day, and if we draw a contour, as described above, round A′, it will be true for the joint effects of winds and currents for A. Conversely, in order to construct a contour for their joint effects from A, we have first to draw A A′ to represent the drift of the current in one day from A, and then to use A′ as a point of construction for a contour calculated upon the data of the winds alone; the contour so drawn will be the required figure, with A for the point of departure.

It seems strange that so useful and definite a conception as that of the contour we have been considering should not long since have acquired a name. The idea in its general application is not unfamiliar to us, because we are all accustomed to reflect how far we can travel in an hour's walk, ride, or drive from our house, in various directions, and we know that the distances in different directions vary according to the goodness or straightness of the road. Nevertheless our vocabulary does not admit of an expression of the idea upon which this memoir is based, without a tedious paraphrase including references to (1) equal and specified times, (2) direct geographical distance, (3) universality of directions, and (4) a bounding line. I therefore propose to employ the word *isochrone* (equality of time) in a special sense in this memoir, meaning thereby all that is expressed by the contour just described, which I should call the "isochrone from A," the unit of measurement being understood to be a single day's sail, unless otherwise specified. The isochrone *towards* A will be the same figure reversed and inverted.

The lines of swiftest passage from one port to another can only be determined after computation of the isochrones for a sufficient number of ocean districts within the region of inquiry to enable those at any particular spot, or as much of them as is needed, to be found by interpolation. It seems to me perfectly impossible to draw any portion of an isochrone, except in a rudely approximative way, without previous computation. Calculation to 8 or 16 points takes so long to perform, and small differences in the mean force of the winds have so large an influence, that no human brain is competent to deduce correct results after a mere inspection of the data. As an example, I may be allowed to mention that I asked a naval officer of unusually large experience in the construction of weather charts, and who was familiar with the sailing qualities of a "Beaufort standard ship," to estimate portions of isochrones in certain cases; and I found the mean error of his estimates to exceed 15 per cent. The guesses of ordinary navigators would necessarily be much more wide of the truth. Now we must recollect that a very small saving on the average length of voyages would amount to an enormous aggregate of commercial gain, and that, where precision is practicable, we should never rest satisfied with the rule of thumb. Our meteorological statistics afford the best information attainable at the present moment, and they exceed by some hundredfold the experiences of any one navigator; their probable errors may nevertheless be large, but that is no reason for needlessly associating them with additional subjects of doubt. The probable error of a navigator's estimate of an isochrone, and consequently of the data which he must use, whether consciously or not, whenever he attempts to calculate his best track, is due at the present time to no less than three distinct sets of uncertainties :—*a*, the average weather; *b*, the performance of his ship on different courses with winds of different force (which I understand to be hardly ever ascertained with much precision); *c*, the computa-

Fig. 3.



tion of the isochrone. If A, B, and C be the probable errors of these three respectively, then the probable error of the navigator's estimate will be $\sqrt{A^2+B^2+C^2}$. My desire is to reduce this large item to a simple A, which may itself be minimized until it ceases to be of any practical inconvenience. It is no new thing that statistics should require discussion and elaborate calculation before they can be turned to account and be made the familiar basis of vast commercial undertakings. Classified lists of ages at death are no more fitted to be the immediate guides of those who grant annuities or engage to pay reversions, than are the crude meteorological statistics of the ocean to be the immediate guides of the navigator.

It is probable that most vessels may practically admit of division into some moderate number of classes, and that it would suffice to calculate isochrones for each of these classes; but in any case the number of calculations must be very large, because they would differ not only for the class of ship and the particular destination, but also for the season of the year when the voyage was made. It is therefore important that even individual ships should be enabled to have isochrones drawn for their especial use at a trifling cost. I will now show how this may be effected by mechanical means.

The drawing I give (fig. 3) is only a diagram to explain the principle of the machine; the framework is left out, and the proportions are somewhat varied for the convenience of illustration. Also, for simplicity of explanation, I have supposed the machine constructed to apply to no more than 4 points of the compass, and it is represented as adjusted to work out the example already given in Table III.

A long tray, open at the top and almost wholly open along the front, has 8 grooves, into which pieces of zinc, thin wood, or even stout cardboard may be dropped, much like the glass plates in a photographer's box. In fig. 3 we only see the base of this tray, A A, and the pieces of zinc standing upon it in the position in which they would be held by the grooves. The zinc plates have curved edges in front, which refer to the sailing qualities of the particular ship under consideration; these are cut out from the data in Table II., each plate corresponding to the column whose heading it bears. The ordinate of the curve is proportionate to the force of the wind, and the abscissa to the distance sailed in one day on the specified course (0, 8, 16, or 24) with that force of wind. There are grooves cut in A A, one under each zinc plate, and there are tramways, T, upon which A A may be set, in gear with those grooves. In the figure it is so set that 0 is opposite to *n*, 8 to *e*, 16 to *s*, and 24 to *w*; but if it were lifted up and laid one groove more to the left, 0 (on the right hand) would be opposite to *w*, 8 to *n*, 16 to *e*, and 24 to *s*. Similarly, by setting it two or three grooves to the left, the other possible variations would be gone through. The slides *n*, *e*, *s*, and *w* refer to the course of the ship

shown in the first column of Table III., and the four different settings correspond to the four lines in the body of that Table.

The slides *n*, *e*, *s*, and *w* move to and fro parallel to the tramways, and are each furnished with bars that can be pushed vertically up and down, and a rod projects horizontally from each of the bars towards the zinc plate opposite to it. Graduations referring to the force of the wind are placed on the bars, which are thereby adjusted to come in contact at the proper levels with the zinc plates, and to be pushed back through a distance equal to the length of the abscissæ at those levels, when A A is run forwards on its tramway.

Thus far we have obtained the result that *n*, *e*, *s*, and *w* shall be severally pushed back to the distances which would be sailed over in one day if the wind blew on 100 occasions with specified forces from *each* of those quarters, and if the ship were sailing to that quarter whose initial is opposite to the zinc plate marked 0. As A A is adjusted in the figure, that quarter would be N.; if it were moved one step to the left it would be W., if 2 steps it would be S., and if 3 steps it would be E.

We have now to diminish the movements impressed upon *n*, *e*, *s*, and *w* in the ratio of the percentage of occurrence of the several winds. This is effected by linking them to another series of slides, N, E, S, and W, as shown in the figure, and by attaching adjustable centres to the arms, there shown in a vertical position. The standards to which those centres would be clamped are almost wholly removed in the figure, but the top and bottom of them can be seen. The reduction will be correct within such limits as we need, when the links are somewhat longer than in the figure and the arm does not swing through more than 40°. I therefore do not care to propose in this case the somewhat more complicated, but perfectly accurate arrangement which I contrived for the parallel slides of the pantagraphs now in use at the Meteorological Office, of which a description is given in the Report of the Meteorological Committee for 1870, p. 30.

I place the adjustable centres between the links in the cases of N and S, and above them in those of E and W; consequently, when *n*, *e*, *s*, and *w* are all pushed back, N and S will advance, and E and W will retreat: the reason for doing this will be seen presently. In order to graduate the arms for the adjustable centres, we must take the $\frac{1}{100}$ part of the distance between the pivots of the links as the unit of measurement, and measure in both cases from the pivot of the upper link as the zero-point. Then if p = the percentage to which the movements are to be reduced, the graduations for p between the links are determined by the formula $\frac{100p}{100+p}$, and when above the links by $\frac{100p}{100-p}$.

The results we have now attained are that, when A A is pushed forward, N, E, S, W shall move alternately forwards and backwards, each through a distance corresponding to that which a ship would sail towards the

quarter whose initial is opposite to the zinc plate marked 0, under the several influences of the N., E., S., and W. winds as they are found to occur. What remains is to sum up these movements.

I put a pulley, running easily on its axis, upon each of the upper slides, as in the figure, and pass a band, one of whose ends is secured to a fixed peg, round these pulleys, alternately over and under them. The free end of the band is kept stretched by a light weight, and a framework, carrying a vertical pricker, is urged to and fro by the movements of the band. A disk (D), upon which the drawing-paper is secured, has its centre exactly below the pricker when the machine is at zero; and whenever A is pushed home, the pricker travels in a radial distance to an amount equal to twice the sum of the movements of the several slides, and therefore through a distance proportionate to a day's sail of the ship towards the quarter whose initial letter is opposite to the zinc plate marked 0. If desired, the numerical value of the movement of the string could of course be read off.

The manipulation of the instrument would be as follows:—

1. Remove A A. 2. Push or pull the slides *n*, *e*, *s*, *w* into their mean position. 3. Thrust a skewer, S S, through the holes in the arms and framework to hold every thing fast. 4. Adjust the centres, and clamp them if necessary. 5. Adjust the bars for force of wind. 6. Remove the skewer. 7. Push the slides *n*, *e*, *s*, *w* as far back as they will go. 8. Replace A A, and pull the slides forward to it. The machine is now in working order. *a*. Push A A home. *b*. Press the pricker, not forgetting to mark the N point. *c*. Turn the disk through a quadrant. *d*. Pull back A A, and set it one step in advance on the tramways. *e*. Pull the slides up to it.

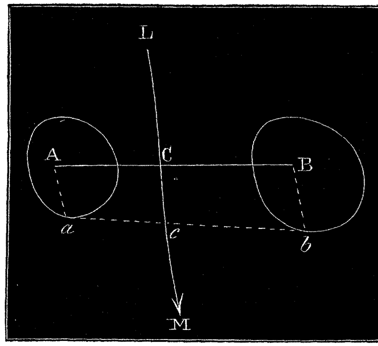
The series 1 to 8 has to be gone through once for all for each isochrone; that from *a* to *e* for each of the 4 points of the compass. The whole of these actions are simple and rapid, and the adjustments are of the easiest kind. An isochrone based upon 4 points ought to be leisurely plotted out in $2\frac{1}{2}$ minutes, and one based upon 8 points in 4 minutes.

This step-by-step arrangement is far easier of construction than one, which may suggest itself to many persons, in which the movement should be continuous, using a curved surface instead of a set of curved edges, and by which the entire curve should be drawn instead of a few points pricked out; also it is far more convenient and compact not to arrange the machine in a circular form, which is that which would most naturally first be thought of.

When the isochrones have been drawn to scale on a chart, isochronic lines at various points along any proposed route could readily be found by graphical interpolation. Thus, in fig. 4, let the route be from L to M, and let the isochrones round A and B be known. Draw A *a*, B *b* parallel to L M, and join A B, *a b*, cutting L M in C and *c*; then C *c* is one day's sail from C. We can do more than this; for we may find the distance of

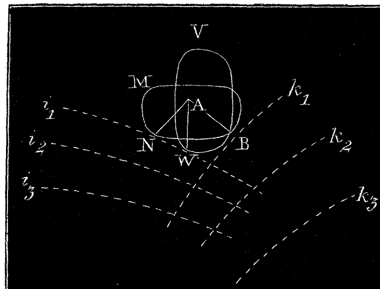
a day's sail from *any* point of L M by the following device. Erect C c' perpendicular to L M, and equal to C c; similarly, erect other perpendiculars (L l', M m') from points L, M, &c., of which the day's sail along the route has been laid down in the same way that C c was, and draw with a free hand a line through l', c', m'; then the perpendicular distance X x' from any point X on the line L M to its intersection at x' with the line l', c', m' will be the length of a day's sail from X, and it can be transferred with a pair of compasses to find x on the line L, X, M.

Fig. 4.



Another application of the principle of isochrones is to draw them at distances of 1, 2, 3, &c. days for ships bound *towards*, not from, a given port. The first day's isochrone requires no explanation. For that of the second day, a few neighbouring points must be selected on the contour already drawn, and from these such small portions as are needed of other isochrones must be constructed; the line sketched with a free hand to bound these figures will be the isochrone of two days' journey towards the port, and similarly for the third and subsequent days. The lines of shortest passage from curve to curve join the points whence the subsidiary isochrones were drawn with the points where the latter are in contact with their bounding line. The appearance of a chart so constructed would be that of a series of roughly concentric curves round the port and of radial lines of shortest passage. I give no illustration of such a chart, because it seemed a waste of labour to calculate one upon the scanty data available at the present moment, when far more ample and trustworthy materials have been collected and are in the course of gradual publication by the Meteorological Office; and calculations would have been necessary, because the machine just described exists only in the form of a rude model.

Fig. 5.



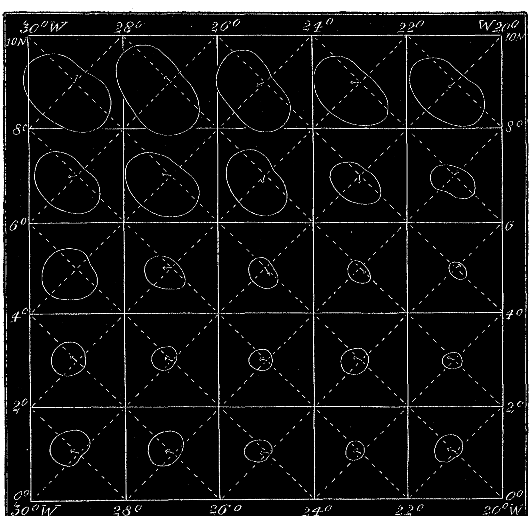
We have, lastly, to explain how the navigator would use one of these charts on specific occasions. Suppose (fig. 5) he finds himself at A with such and such weather, present and probable, how should he steer? He must use an isochrone appro-

priate to the occasion, most likely out of a stock which he would keep ready to hand; and applying it to his map, he would note the direction in which it will cut the largest number of isochrones towards the port. If we take cases where the isochrones for the occasion are (1) N M B and (2) W V B, the first being with a N. and the second with a W. wind, then, if the isochrones towards the port be as in the series k_1, k_2, &c., the course to be steered in both instances is A B; but if they are as in the series i_1, i_2, &c., the course in case (1) will be A N, and in case (2) A W, quite independently of the direction in which the port may happen to lie.

There is much to say about the proper method of discussing the crude statistics derived from ocean districts artificially bounded by lines of latitude and longitude in order to obtain the most probable meteorological values, but I will only allude to them here. First, homogeneous districts and periods of time have to be made out; secondly, the crude observations in each subdivision of those districts have to be discussed in connexion with those made at adjacent subdivisions in the same district; and, thirdly, they have to be discussed in reference to those made in preceding and succeeding periods of time. There is no doubt but that labour spent in these discussions would after a time become more remunerative than the same amount of labour in accumulating fresh observations.

I submit (fig. 6) a series of 8-hour isochrones computed to 8 points from the

Fig. 6.



crude observations taken in each "2-degree square" of the ocean between

10° north latitude and the equator, and between 20° and 30° west longitude, in the month of January. The uniformity of their sequence is very striking; and it would no doubt have been still more so if the data had previously been discussed in the above-mentioned manner. The short line with an arrow-head shows the direction and amount of current; but the centre of each square is the point of departure, for which the contour shows the joint effects of winds and current.

To recapitulate. I have shown in this memoir:—1, what isochrones are, and their great importance; 2, how to calculate them; 3, how to construct a machine to supersede their calculation and to make it possible to have them drawn for special cases at a trifling cost; 4, how to make an isochronal chart; and, 5, how to use it on individual occasions.

I should be glad if one result of this memoir were to bring into greater prominence than at present the high value of the ocean statistics collected and now being published by the Meteorological Office, and the fact that no degree of precision of meteorological knowledge need be thrown away in the practice of navigation. Such knowledge will be good for all time, and will always afford the requisite data whence isochrones conformable to the varying performances of new varieties of ships and to new lines of commerce may be calculated.

May 1, 1873.

WILLIAM SPOTTISWOODE, M.A., Treasurer and Vice-President, in the Chair.

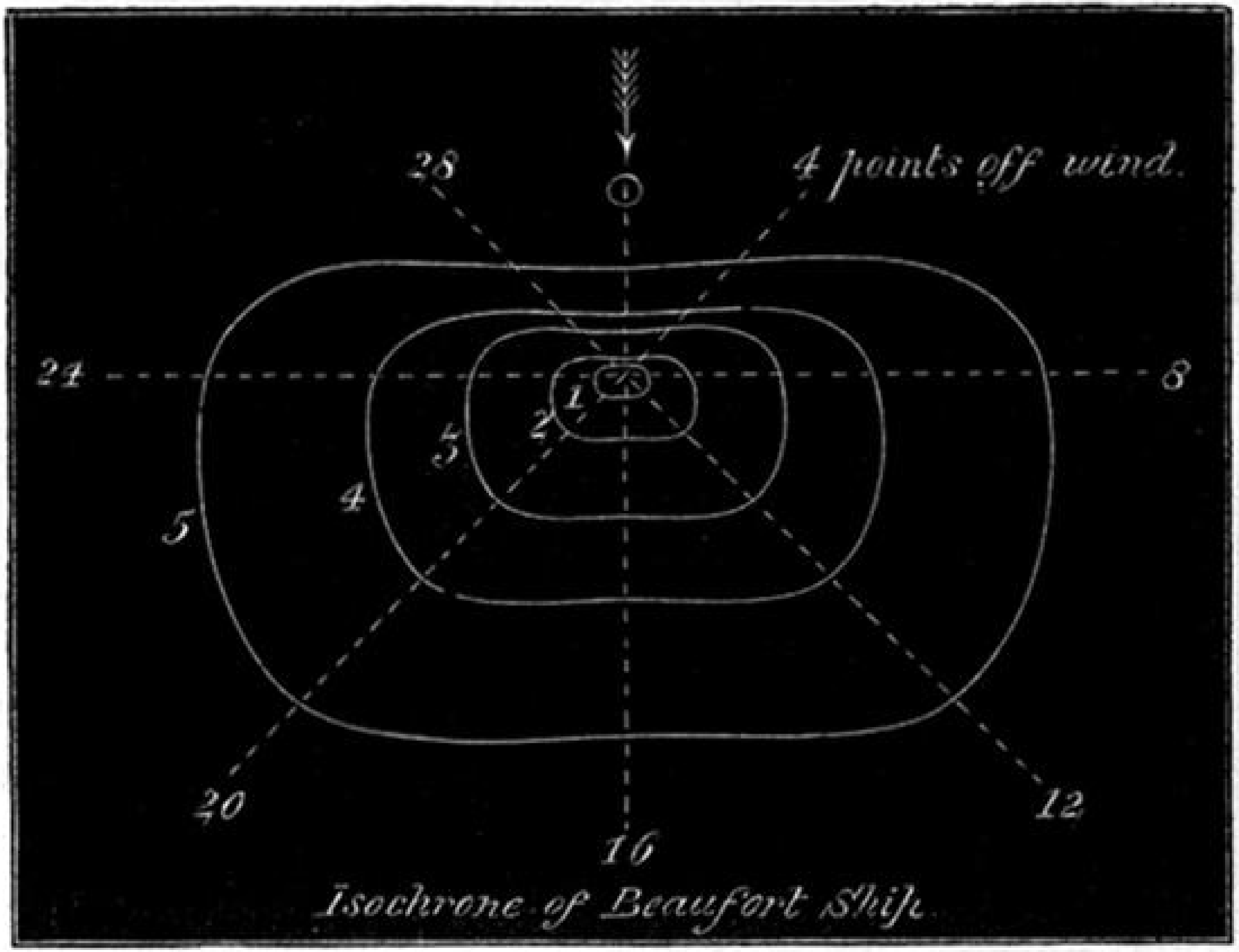
In pursuance of the Statutes, the names of the Candidates recommended for election into the Society were read from the Chair as follows:—

William Aitken, M.D.
Sir Alexander Armstrong, M.D., K.C.B.
Robert Stawell Ball, LL.D.
John Beddoe, M.D.
Frederick Joseph Bramwell, C.E.
Edward Killwick Calver, Capt. R.N.
Robert Lewis John Ellery, F.R.A.S.
James Augustus Grant, Lieut.-Col., C.B.
Clements Robert Markham, C.B.
George Edward Paget, M.D., D.C.L.
George West Royston-Pigott, M.D.
Osbert Salvin, M.A.
The Hon. John William Strutt, M.A.
Henry Woodward, F.G.S.
James Young, F.C.S.

Dr. Arthur Gamgee was admitted into the Society.

The following communications were read:—

Fig. 1.



4 points off wind.
28
24
8
1
2
3
4
5
20
12
16
Isochrone of Beaufort Ship.

Fig. 2.

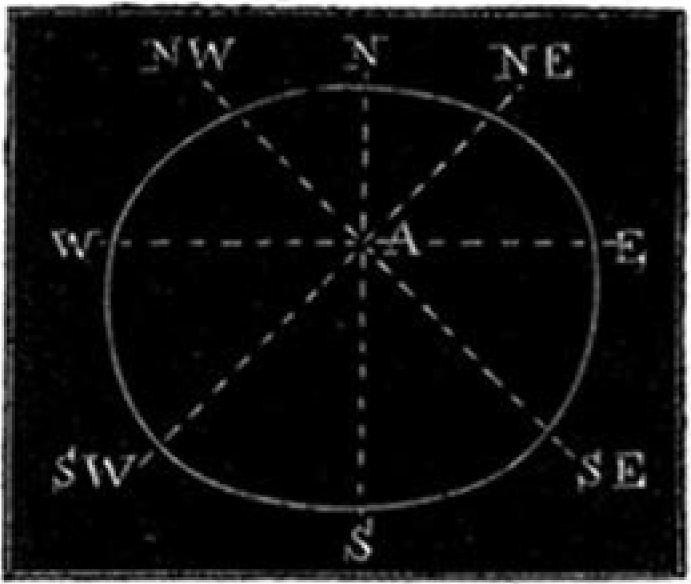


Fig. 3.

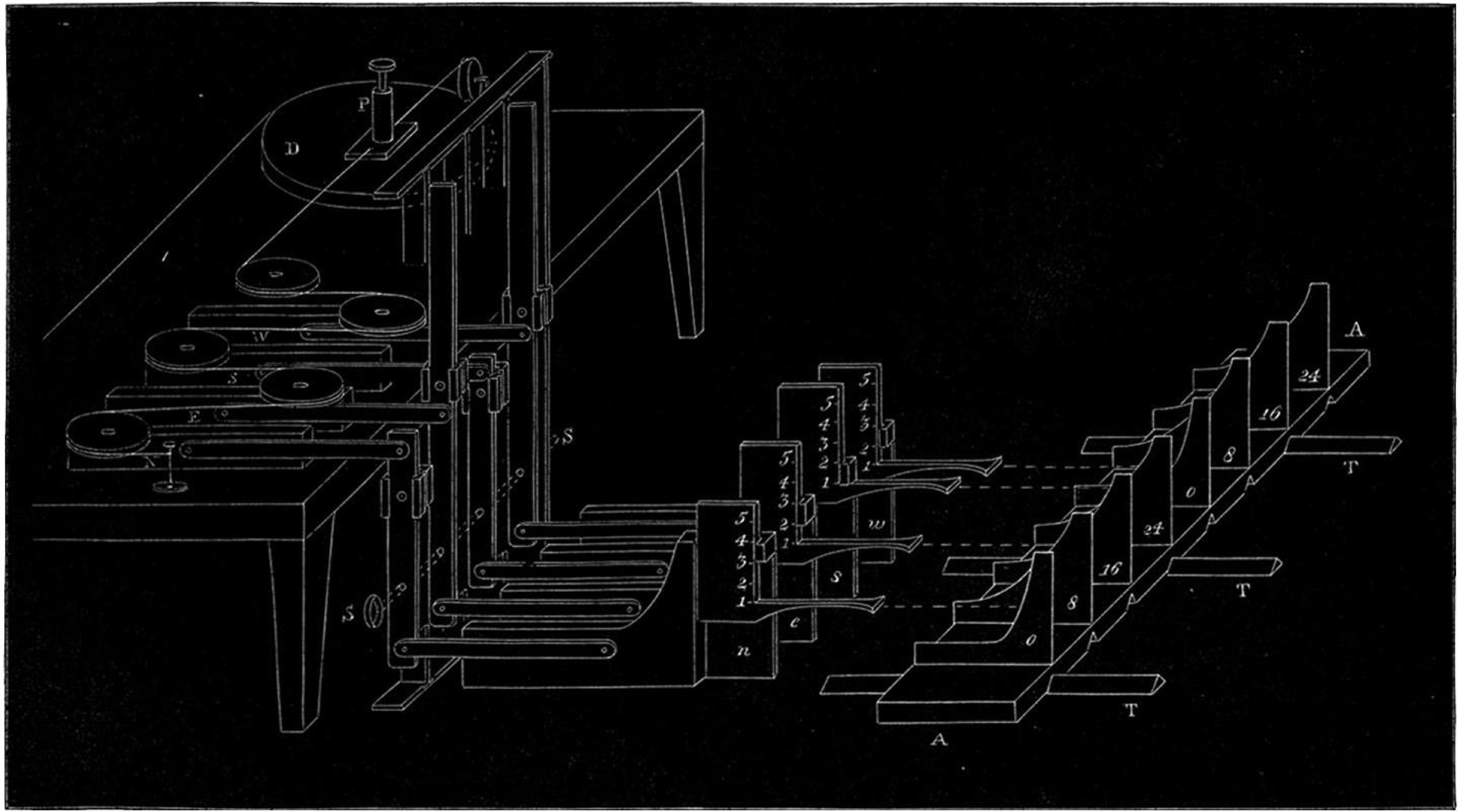


Fig. 4.

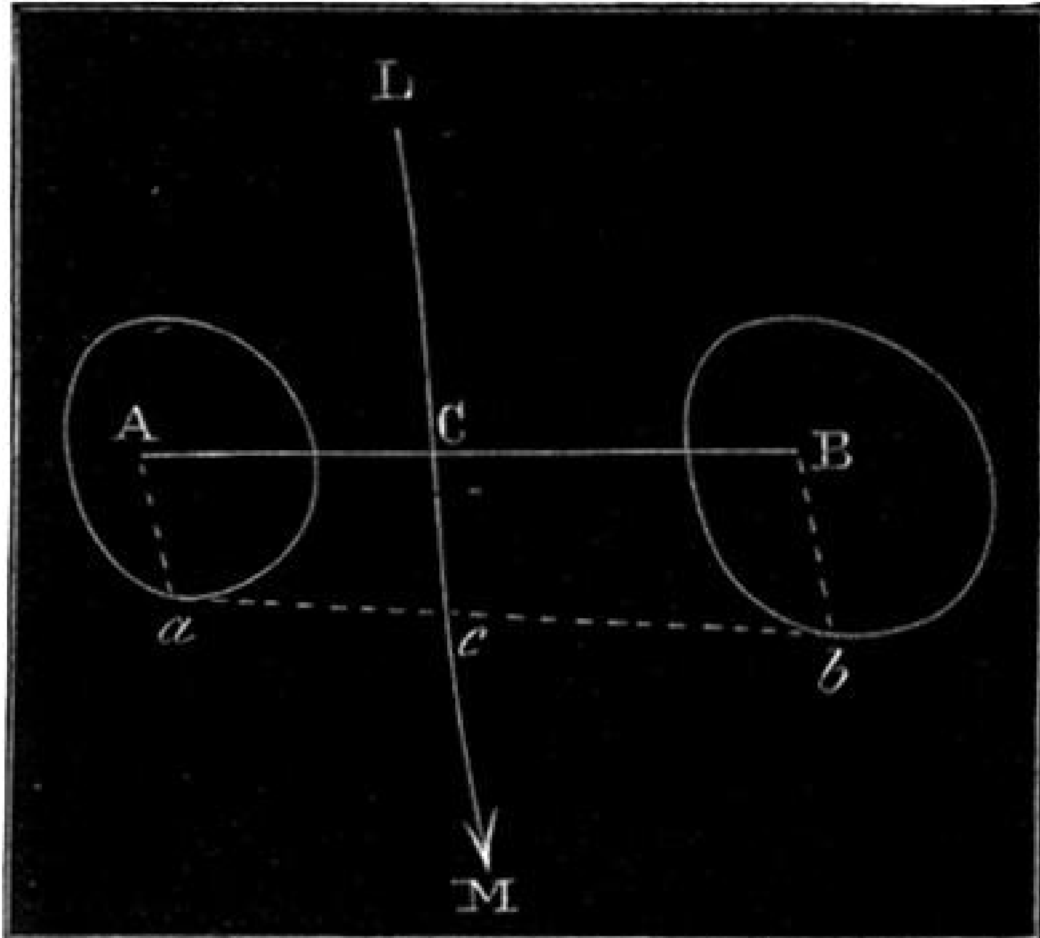


Fig. 5.

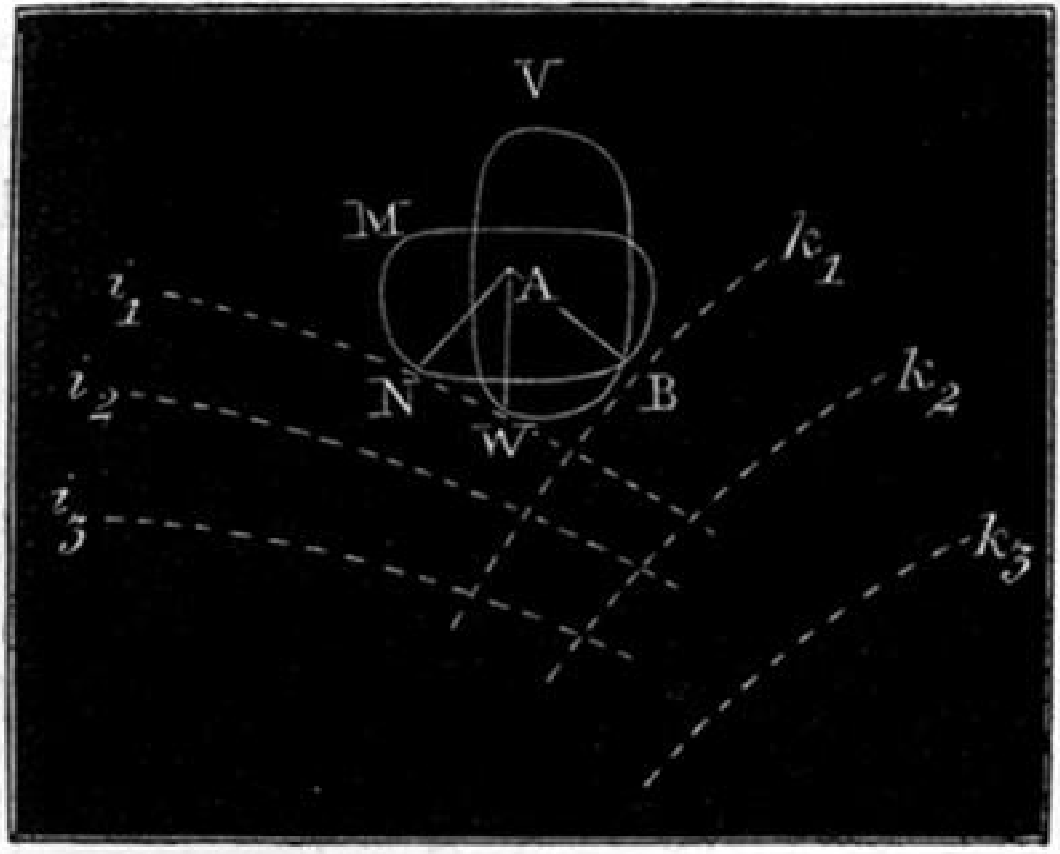


Fig. 6.