

strongly heated; they melt, boil, and yield a semisolid distillate. The distillation is carried on as far as possible without charring the residue, which consists of potassium benzoate (from which half the benzoic acid employed may be recovered). By addition of ammonia to the semisolid distillate any benzoic acid that has passed over is retained, whilst the nitrile distills with the water, from which it is afterwards separated, dried, and redistilled. A roughly carried out experiment yielded 50 per cent. of the theoretical quantity of nitrile in a perfectly pure condition; the loss owing to secondary reactions is amply compensated by the ease and rapidity of the operation.

Action of Cuminic Acid on Potassium Sulphocyanate.

An experiment with cuminic acid yielded very satisfactory results: cumonitrile was obtained in about the same proportions as the benzonitrile. The temperature at which the reaction commenced was here about 210° ; the nitrile was purified as in the preceding case.

Finally, an experiment was made with cinnamic acid; but although sulphuretted hydrogen was evolved, and the reaction appeared to proceed exactly as in the foregoing cases, no cinnamonnitrile was obtained in the liquid distillate. The cinnamic acid seemed to be decomposed into carbonic anhydride and cinnamol before being acted upon by the sulphocyanic acid.

The ease with which nitriles and amides are obtained in this manner, both in the aromatic and fatty series, induces me to hope that the new method may be of use in many cases, and perhaps, by its application to other series, give rise to bodies hitherto uninvestigated.

The tediousness of preparing the acid chloride and subsequent treatment with ammonia (for the amide), or of the amide with phosphoric anhydride (for the nitrile), in the ordinary method for producing these nitrogen compounds, is here replaced by simple digestion of the acid with sulphocyanate of potassium, bodies generally readily procurable.

- V. "Investigation of the Attraction of a Galvanic Coil on a small Magnetic Mass." By JAMES STUART, M.A., Fellow of Trinity College, Cambridge. Communicated by the President. Received July 26, 1872.

From investigations given by Ampère, we can deduce an expression for the potential U at an external point Q of a closed circular galvanic current carried by a wire of indefinitely small section. Let a be the radius of the circle; let the distance of Q from C , the centre of the circle, be r ; and let the line CQ make an angle θ with the normal to the plane of the circle. Then it can be shown that when r is less than a ,

$$U = 2\pi k \left\{ -1 + \frac{r}{a} P_1 - \frac{1}{2} \frac{r^3}{a^3} P_3 + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{r^5}{a^5} P_5 - \dots \right\};$$

and when r is greater than a ,

$$U = 2\pi k \left\{ -\frac{1}{2} \frac{a^2}{r^2} P_1 + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{a^4}{r^4} P_3 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{a^6}{r^6} P_5 + \dots \right\},$$

where k depends only on the intensity of the current, and where P_1, P_3, P_5 are defined by the equation

$$\frac{1}{\sqrt{1 - 2x \cos \theta + x^2}} = 1 + P_1 x + P_2 x^2 + P_3 x^3 + \dots$$

If, therefore, X represents the resolved part perpendicular to the plane of the circle and towards it of the force exerted by the current on a unit of magnetism placed at Q , and if Y represent the resolved part of that force parallel to the plane of the circle and directed from its centre outwards, then

$$X = \frac{dU}{r \cdot d\theta} \sin \theta - \frac{dU}{dr} \cos \theta,$$

$$Y = \frac{dU}{r \cdot d\theta} \cos \theta + \frac{dU}{dr} \sin \theta.$$

To calculate these quantities, we know that

$$P_1 = \cos \theta,$$

$$P_2 = \frac{5}{2} \left(\cos^3 \theta - \frac{3}{5} \cos \theta \right),$$

$$P_3 = \frac{63}{8} \left(\cos^5 \theta - \frac{10}{9} \cos^3 \theta + \frac{15}{63} \cos \theta \right).$$

We shall only consider the case of those points for which r is greater than a . Substituting these values in the expression which in such instances holds for U , we have

$$U = 2\pi k \left\{ -\frac{1}{2} \cdot \frac{a^2}{r^2} \cos \theta + \frac{15}{16} \cdot \frac{a^4}{r^4} \left(\cos^3 \theta - \frac{3}{5} \cos \theta \right) - \frac{315}{128} \cdot \frac{a^6}{r^6} \left(\cos^5 \theta - \frac{10}{9} \cos^3 \theta + \frac{15}{63} \cos \theta \right) + \dots \right\}.$$

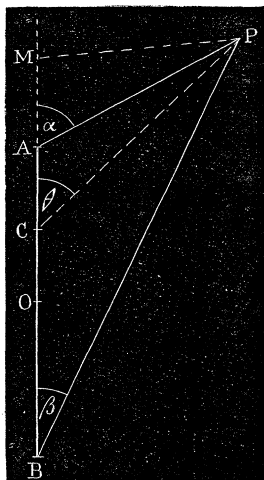
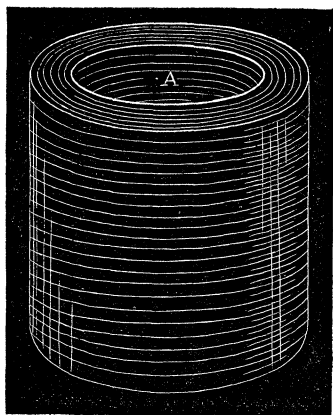
From which, after some reduction, we obtain

$$\begin{aligned} \frac{X}{2\pi k} = & -\frac{1}{2} (-1 + 3 \cos^2 \theta) \cdot \frac{a^2}{r^3} + \frac{1}{16} \cdot (9 - 90 \cos^2 \theta + 105 \cos^4 \theta) \frac{a^4}{r^5} \\ & - \frac{1}{128} (-75 + 1575 \cos^2 \theta - 4725 \cos^4 \theta + 3465 \cos^6 \theta) \frac{a^6}{r^7} \\ & + \dots, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1) \end{aligned}$$

$$\frac{Y}{2\pi k} = \sin \theta \cdot \left\{ + \frac{3}{2} \cos \theta \cdot \frac{a^2}{r^3} - \frac{1}{16} (-27 \cos \theta + 105 \cos^3 \theta) \frac{a^4}{r^5} \right. \\ \left. + \frac{1}{128} (525 \cos \theta - 3150 \cos^3 \theta + 3465 \cos^5 \theta) \frac{a^6}{r^7} \right. \\ \left. - \dots \dots \dots \right\} \dots \dots \dots (2)$$

Each of these expressions consists of a series of terms in ascending powers of $\frac{a}{r}$ which will be converging.

We shall now seek to find X and Y for a galvanic current traversing a wire coiled into the form of a hollow cylinder, of which the internal radius is b , the external radius $b+c$, and of which the length is $2f$. We shall suppose the individual turns of the wire to lie so close as that each may be regarded as an exact circle.



Let A B be the axis of the coil, so that A and B are the centres of its two faces; then $AB=2f$. Let O be the middle point of A B. Let P be the attracted point, P M its perpendicular distance p from A B. Let $\angle P A M=\alpha$, $\angle P B M=\beta$.

Let C be the centre of any turn of the wire regarded as a circle of radius a , $CP=r$, $\angle P C M=\theta$, $OC=x$; then it is readily seen that for the whole cylindrical bobbin the forces X, Y are given by

$$\frac{X}{\mu} = \int_{-f}^{+f} \int_b^{b+c} L dx da,$$

$$\frac{Y}{\mu} = \int_{-f}^{+f} \int_b^{b+c} M dx da,$$

where L and M stand for the expressions on the right-hand side of (1) and (2) respectively, and where μ depends on the strength of the current.

To perform the integrations for the length of the bobbin in these expressions, we have the formulæ

$$\begin{aligned} p &= r \cdot \sin \theta, \\ \delta x \cdot \sin \theta &= -r \cdot \delta \theta; \\ \therefore \delta x &= \frac{-p \delta \theta}{\sin^2 \theta}, \end{aligned}$$

and

$$r = \frac{p}{\sin \theta}.$$

Making these substitutions for δx and r , the integrals with respect to x become integrals with respect to θ , which can be easily evaluated by a continued application of the method of integration by parts, the limits being from $\theta = \alpha$ to $\theta = \beta$. If we then integrate the result thus obtained with respect to a , from the limit b to the limit $b+c$, we finally obtain

$$\begin{aligned} \frac{X}{\mu} &= \frac{\overline{b+c}^3 - b^3}{6p^2} \{ -(\cos \beta - \cos \alpha) + (\cos^3 \beta - \cos^3 \alpha) \} \\ &+ \frac{\overline{b+c}^5 - b^5}{80p^4} \{ -9(\cos \beta - \cos \alpha) + 33(\cos^3 \beta - \cos^3 \alpha) \\ &\quad - 39(\cos^5 \beta - \cos^5 \alpha) + 15(\cos^7 \beta - \cos^7 \alpha) \} \\ &+ \frac{\overline{b+c}^7 - b^7}{896p^6} \{ -75(\cos \beta - \cos \alpha) + 575(\cos^3 \beta - \cos^3 \alpha) \\ &\quad - 1590(\cos^5 \beta - \cos^5 \alpha) + 2070(\cos^7 \beta - \cos^7 \alpha) \\ &\quad - 1295(\cos^9 \beta - \cos^9 \alpha) + 315(\cos^{11} \beta - \cos^{11} \alpha) \} \\ &+ \dots \dots \dots \\ \frac{Y}{\mu} &= \frac{\overline{b+c}^3 - b^3}{6p^2} \{ +(\sin^3 \beta - \sin^3 \alpha) \} \\ &+ \frac{\overline{b+c}^5 - b^5}{80p^4} \{ -12(\sin^5 \beta - \sin^5 \alpha) + 15(\sin^7 \beta - \sin^7 \alpha) \} \\ &+ \frac{\overline{b+c}^7 - b^7}{896p^6} \{ +120(\sin^7 \beta - \sin^7 \alpha) - 420(\sin^9 \beta - \sin^9 \alpha) \\ &\quad + 315(\sin^{11} \beta - \sin^{11} \alpha) \} \\ &+ \dots \dots \dots \end{aligned}$$

These expressions for X and Y will be converging for all points situated at a greater distance than $b+c$ from any point of the axis A B, inasmuch as they are composed by adding together corresponding terms of series which are then all convergent. Among other points, these expressions hold for such as are situated on the axis external to the bobbin, and not nearer A or B than by the distance $(b+c)$. For such points, however, the expressions become illusory, assuming the form $\frac{0}{0}$. They may, how-

ever, be evaluated by the methods for the evaluation of vanishing fractions. Y is clearly zero. X may be more readily obtained directly from the expression for U . From that expression we find that for a single circular current the attraction on such points is

$$X = 2\pi k \left\{ + \frac{a^2}{r^3} - \frac{3}{2} \frac{a^4}{r^5} + \frac{15}{8} \frac{a^6}{r^7} - \dots \right\}.$$

Hence, in the case of a bobbin, if x be the distance of the attracted point from O , the middle point of the axis of the bobbin, we have

$$\begin{aligned} \frac{X}{\mu} &= \int_{x+f}^{x-f} \int_b^{b+c} dr da \left(+ \frac{a^2}{r^3} - \frac{3}{2} \frac{a^4}{r^5} + \frac{15}{8} \frac{a^6}{r^7} - \dots \right) \\ &= - \frac{\overline{b+c}^3 - b^3}{6(x^2 - f^2)^2} (\overline{x+f^2} - \overline{x-f^2}) \\ &\quad + 3 \frac{\overline{b+c}^5 - b^5}{40(x^2 - f^2)^4} (\overline{x+f^4} - \overline{x-f^4}) \\ &\quad - 5 \frac{\overline{b+c}^7 - b^7}{112(x^2 - f^2)^6} (\overline{x+f^6} - \overline{x-f^6}) \\ &\quad + \dots, \end{aligned}$$

which gives X for points situated on the axis for which x is not less than $(b+c+f)$.

December 12, 1872.

WILLIAM SPOTTISWOODE, M.A., Treasurer and Vice-President, in the Chair.

Announcement was made from the Chair that the President had appointed Dr. Sharpey a Vice-President.

The Presents received were laid on the Table, and thanks ordered for them.

The following communications were read :—

- I. "A Contribution to the Knowledge of *Hæmoglobin*." By E. RAY LANKESTER, M.A. Oxon., Fellow of Exeter College, and Radcliffe Travelling Fellow of the University. Communicated by Prof. HUXLEY. Received July 29, 1872.

The fact that exceedingly small quantities of *Hæmoglobin* can be detected with great facility by means of the microspectroscope, has rendered it possible to trace the distribution of this important body among organisms of various classes, and by comparing its absence from certain animals or

