IV. "On the Heating of a Disk by rapid Rotation *in vacuo*." By Balfour Stewart, M.A., F.R.S., Professor of Natural Philosophy in Owens College, Manchester, and P. G. Tait, M.A., Professor of Natural Philosophy in the University of Edinburgh. Received April 10, 1873.

26. In two previous communications (Proc. Roy. Soc. June 15, 1865, and No. 88, 1866) to this Society, we gave an account of some experiments which we had made upon the heating of a disk through rotation *in vacuo*. In these experiments the increase of radiation of the heated disk was observed by means of a delicate thermopile and galvanometer. Three aluminium disks of various thicknesses and one ebonite disk were used; and the results derived from the experiments were as follows:—

(1) The heating effect observed appeared to be independent of the density, and of the chemical constitution, of the residual air and vapour surrounding the disks.

(2) The quantity of heat developed under similar circumstances of rotation in three aluminium disks ·05, ·0375, ·025 of an inch in thickness respectively appeared to be the same, inasmuch as the relative thermometric effect for these disks varied inversely as their thickness.

(3) Besides the heating effect alluded to in (1) and (2), there was found to be, when the vacuum had been recently made, a strictly temporary effect, sometimes in the direction of heat, sometimes in that of cold, owing probably to the condensation or evaporation of small quantities of aqueous vapour; but this effect was only noticeable during rotation, disappearing the moment the motion was stopped.

27. The experiments described in these communications were resumed in 1870. In the interval an addition had been made to the apparatus, in virtue of which an ordinary carbonic-acid vacuum might be subjected to the influence of a vessel containing potash allowed to open in it, and thus to absorb as much as possible of the remaining gas.

On May 4 a carbonic-acid vacuum was obtained by this means (pressure 0·05 in.). A disk made of cartridge-paper, when made to rotate in this vacuum, gave a very perceptible result. Carbonic acid was then allowed to enter the vacuum until the pressure became 0·65 in. The consequence of this increased pressure was in this instance an increase in the effect, which was probably of a permanent nature, inasmuch as it remained after three days. Unfortunately the exact increase was not noted; but it is believed that the heat-indication became about three times as great in consequence of the additional pressure.

28. On May 11 another carbonic-acid vacuum was made (pressure 0·12 in.), and at the suggestion of Professor Maxwell a sulphuric-acid gauge was placed in the receiver. A rotation was then made; and the result of the rotation was a hardly perceptible rise in the sulphuric-acid

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gauge. We may therefore imagine that the residual air was not greatly heated.

29. On May 16 another carbonic-acid vacuum was obtained (pressure 0·08 in.), and with an ebonite disk of about $\frac{1}{20}$ inch thickness, the heat-indication was 16. Carbonic acid was introduced until the pressure was 1·6 in.; but the indication was only 18. This result is in conformity with our previous experiments, in which an increase of pressure of the residual air produced little or no effect.

30. The ebonite disk was likewise tried in a carbonic-acid vacuum, pressure 0·04 in.; and also in one, pressure 0·02 in., which was the lowest we could obtain. The result of these experiments appeared to show that in all probability the cartridge-paper disk radiated more than twice as much as the thin ebonite disk. The experiments were put a stop to by a collapse of the glass receiver during rotation, fortunately without injuring any one present.

31. In June 1871 the experiments were resumed. In the mean time Mr. Beckley had fitted the apparatus with an arrangement working through a barometer-tube, by means of which, instead of trusting to radiation, the disk itself might, after rotation, be tapped by means of the pile, which could be brought up to it and then withdrawn. By this means a much larger effect might be obtained; and it became possible, by varying the adjustment, to find according to what law the heat effect varies with the distance from the centre.

32. These experiments were conducted in the following manner:—The disk was first of all tapped before rotation several times; at each tapping the momentary swing of the needle was recorded, and the mean of the readings was regarded as indicating the state of the disk with respect to heat.

The disk was next tapped after rotation, and the difference between the readings before and after was taken as indicating the change in the state of the disk produced by rotation. In the later experiments the disk before rotation was kept in slight motion in order to equalize any tendency to unequal heating of its various parts; but this was not done in the experiments of June 1871. In these experiments (June 1871) the disk was of brown paper, and the results obtained were as follows:—For a carbonic-acid vacuum (pressure 0·065 in.) a swing of 307 divisions was recorded, while for a hydrogen vacuum (pressure 0·150 in.) a swing of 281 divisions was recorded. Each result was the mean of three rotations.

33. The next experiments were made in January 1872. The galvanometer was one of Thomson's, but more adapted for battery currents, and hence not in a very delicate state for these experiments; the time of vibration of the needle was 3 seconds. The disk used was of ebonite (thickness about $\frac{1}{20}$ in.). The ebonite of this disk was completely black; and in this respect, as well as in being thinner, it differed from the ebonite disk first used in the radiation-experiments (art. 17). In the pre-

sent experiments the centre of the pile was made to tap the disk at a distance of 1·5 in. from the rim. The amount and velocity of rotation were represented by 30 turns of the handle, or 3750 turns of the disk, in about 40 seconds. The following results were obtained, a result representing on an average somewhat more than four rotations:—

	Pressure of residual gas, in inches.	Heat-indication for		
		Dry hydrogen.	Dry air.	Dry carbonic acid.
(A)	$\frac{9}{20}$	11·5	35·0	33·0
(B)	$\frac{3}{20}$	7·5	15·0	15·0

It would thus appear that the results derived by tapping are very different from the radiation-results, inasmuch as in the former the effect of the pressure and quality of the residual air is very apparent, while in the radiation-results it is hardly perceptible. A probable explanation of this will be given afterwards (art. 46); but in the mean time, in view of these results, it will be expedient to discuss them quite independently and by themselves, with the view of ascertaining whether they can best be explained by a gas-effect alone, or whether they likewise indicate a residual effect independent of gas.

34. With this object let us take $\frac{(A)+(B)}{2}$ as representing the *whole effect* at a pressure of $\frac{6}{20}$ in., due to whatever cause or causes. We thus obtain

	Dry hydrogen.	Dry air.	Dry carbonic acid.
Whole effect at $\frac{6}{20}$ ..	9·5	25·0	24·0

Again, let us suppose that (A)−(B) denotes the *gas-effect* for $\frac{6}{20}$ in., and we obtain

	Dry hydrogen.	Dry air.	Dry carbonic acid.
Gas-effect at $\frac{6}{20}$	4·0	20·0	18·0

Finally, let us regard as *unknown residual effect* the difference between the *whole effect* and the *gas-effect*, and we obtain

	Dry hydrogen.	Dry air.	Dry carbonic acid.
Residual effect	5·5	5·0	6·0

35. Similar experiments with the same galvanometer were made with a disk of cartridge-paper, of which the pores were filled with solid paraffin. And here we may mention that in all experiments with paper disks a small wooden attachment was placed at some little distance behind the disk and at the height of the pile, and against this the disk was pressed during tapping—care being taken that the disk did not touch it during motion, but only when it was pressed against it by bringing up the pile.

With this paper disk the amount and speed of motion were represented by 30 turns of the handle, or 3750 turns of the disk, in 30 seconds. The

results were as follows, a result representing on an average somewhat less than four rotations :—

	Pressure of residual gas, in inches.	Heat-indication for Dry hydrogen.	Dry air.	Dry carbonic acid.
(A)	$\frac{9}{20}$	27·0	55·0	55·0
(B)	$\frac{3}{20}$	23·0	35·0	32·0

Treating these results in the same manner as those of the ebonite disk, we obtain :—

	Dry hydrogen.	Dry air.	Dry carbonic acid.
Whole effect ($\frac{6}{20}$)..	25·0	45·0	43·5
Gas-effect ($\frac{6}{20}$)	4·0	20·0	23·0
Residual effect	21·0	25·0	20·5

36. Now, if we suppose that there is only one effect due to gas, it follows :—

(*α*) That the proportion between the effects due to the various gases experimented on (and all of the same pressure) is nevertheless different for the two disks.

(*β*) That the proportion (for the same disk) between the effects due to the various gases experimented on is different according to the pressure.

If, however, we suppose that there are two effects, one of which is independent of the residual gas, we find :—

(*α*) That, as regards the *gas-effect*, the proportion between that due to the various gases is nearly the same for both disks. Thus in the ebonite disk we have 4, 20, 18, while in the paper disk we have 4, 20, 23 as representing the gas-effect for the various gases.

(*β*) That the *residual effect* in either disk is nearly the same for the various gases. Thus in the ebonite disk we have 5·5, 5·0, 6·0, while in the paper disk we have 21·0, 25·0, 20·5 as representing the residual effect for the various gases.

The results are thus much more simple on the hypothesis of two effects, one of these being independent of the residual gas, than on the hypothesis of only one effect.

37. It was next endeavoured to ascertain whether these two effects were differently influenced by a blind.

A linen blind was first used before the thin ebonite disk, which it cloaked during rotation, falling down afterwards. During rotation the blind did not appear to touch the disk. A delicate galvanomer was used for this experiment; and the time of vibration of the needle was 2 seconds. The rotation consisted of 30 turns of the handle in 40 seconds, as before. The following result was obtained (each number representing the mean of two observations) :—

	Without blind.	With blind.
$\frac{3}{20}$ of dry hydrogen	165	127
$\frac{3}{20}$ hydrogen + $\frac{6}{20}$ air	477	248

Thus the proportion between the two effects is greatly altered by the blind; for while the hydrogen effect is not much diminished, the other is reduced to little more than one half.

38. A chamois-leather blind was next used, which lay close to the disk during rotation. For all these experiments the time of vibration of the galvanometer-needle was 20 to 30 seconds, while the speed was the same as before. Also the disk was tapped immediately after rotation, and then again at the interval of one minute; and in order to obtain as many individual results as possible, the mean of these two tappings has been taken in the following comparison:—

	Without blind.		With blind.	
$\frac{3}{20}$ of dry hydrogen .	53	mean of 76 individual observations.	52	mean of 22 individual observations.
$\frac{3}{20}$ hydrogen + $\frac{6}{20}$ air	217	mean of 68 individual observations.	140	mean of 16 individual observations.

Here also we find that the proportion between the two effects is greatly altered by the blind; so that while the hydrogen-effect is not much stopped, the other is diminished very considerably.

39. A set of experiments of 8 observations, each similar to those of arts. 33, 34, 35, were made with the blind on for the two gases, dry hydrogen and dry air. From these we obtain the following:—

	Pressure of residual gas, in inches.	Heat-indication for Dry hydrogen.	Dry air.
Blind on	$\frac{9}{20}$	83	178
	$\frac{3}{20}$	52	82

From which we deduce:—

Residual effect, blind on	36·5	34

Again, from certain experiments with the blind off (each result being the mean of 6 observations), we obtain the following effect:—

For dry hydrogen....	$(\frac{9}{20})$	88
	$(\frac{3}{20})$	50

and hence we find residual effect $=31$.

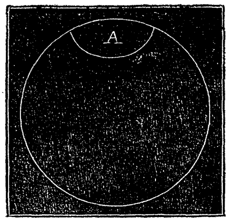
From all these results, allowing for errors of experiment, we may suppose that the residual effect is not much altered by a chamois-leather blind.

We ought to mention that the numbers of this article, similarly to those of art. 38, represent the mean of two tappings, the one taken immediatly after rotation, and the other after the interval of one minute.

40. It was suggested to us by Prof. Helmholtz that it would be desirable to ascertain whether any difference was produced in the results by loading the disk on one side; for if these results were due to vibration, it might be supposed that they would be affected by this means.

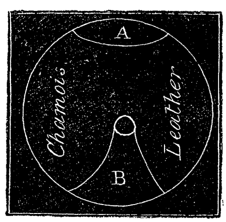
It will be seen by art. 39 that the residual effect obtained from a disk covered with chamois leather is approximately the same as that from an uncovered disk; this would appear to us to be against the vibration hypothesis.

In the following experiment the disk was covered with a chamois-leather blind with a segment cut out as underneath—



A mean of two sets of experiments (consisting of 16 separate observations, and embracing results immediately after rotation, as well as results one minute after) gave as the result of tapping in A with $\frac{3}{20}$ dry hydrogen 47, while a mean of 24 similar observations with the same atmosphere gave as the result of tapping an uncovered disk 53. Allowing for errors of experiment, we may conclude that this arrangement does not much influence the results.

41. The disk was next treated in the following manner:—



It was covered with a chamois leather blind tied into holes drilled in the disk, and having the above shape. The mean of 3 sets of experiments similar to those in art. 40, and consisting of 18 separate observations, gave as follows for an atmosphere of $\frac{3}{20}$ dry hydrogen, tapping in A 54, tapping in B 56, which two results are approximately equal to one another, the mean being 55.

The mean of 3 sets of experiments, consisting of 24 separate observations for an atmosphere of $\frac{3}{20}$ hydrogen, but with an uncovered disk, gave 53 as the heat-result. All these experiments apparently combine to prove that the result is not due to vibration.

42. When the atmosphere was $\frac{3}{20}$ hyd. + $\frac{6}{20}$ air, we also found A and B approximately the same, A giving 181 while B gave 182; we have therefore not hesitated to combine together the results of tapping in this atmosphere, whether at A, or at B, or with an uncovered disk, in the comparison given in art. 38.

43. Our next experiments were made with the view of testing whether or not the two effects, the residual and the gas-effect, were resident in the same particles of the disk; and for this purpose the experiments made immediately after rotation were compared with those made one minute afterwards.

The experiments available for this purpose are so numerous that they can bear splitting into two portions, in each of which the same result is seen.

Thus we have for an atmosphere of $\frac{3}{20}$ dry hydrogen, and as the mean of 30 individual comparisons,

Effect at first : Effect one minute after : : 1·30 : 1;

also, as the mean of 22 individual comparisons, we obtain the proportion of 1·19 : 1, while as the mean of the whole we obtain 1·25 : 1.

Treating in a similar manner the observations made with an atmosphere of $\frac{3}{20}$ hyd. + $\frac{6}{20}$ air, we obtain

As the mean of 25 comparisons .... 1·47 : 1,
As the mean of 21 comparisons .... 1·41 : 1,

while as the mean of the whole we obtain 1·44 : 1.

We therefore conclude that the residual effect is less diminished during the interval of one minute than the gas-effect.

44. We next made experiments with two aluminium disks ·05 and ·025 of an inch in thickness respectively. These disks were covered on both sides with a coating of lampblack applied by negative photographic varnish.

In both disks we found that with a vacuum of $\frac{3}{20}$ hyd. the effect one minute after rotation was greater than the effect immediately after rotation. Thus we have:—

For thick disk (mean of 4 observations)	Effect at first	:	Effect one minute after	: :	·85 : 1;
For thin disk (mean of 2 experiments, embracing 7 observations)	do.	:	do.	: :	·86 : 1.

But in cases where the gas-effect was considerable, this peculiarity disappeared. Thus we have for an atmosphere of $\frac{3}{20}$ hyd. + $\frac{6}{20}$ air:—

For thick disk (mean of 2 observations)	Effect at first	:	Effect one minute after	: :	1·05 : 1.
For thin disk (mean of 2 observations)	do.	:	do.	: :	1·45 : 1.

Also, for an air-vacuum of $\frac{9}{20}$, we have

For thin disk (mean of 3 observations)	Effect at first	:	Effect one minute after	: :	1·49 : 1.

In all the experiments with aluminium disks the needle of the galvanometer vibrated 20 times in 20 seconds, and the rotation consisted of 30 turns of the handle in 40 seconds.

Without pretending to explain all these experiments*, it must, we think, be concluded from them that there are two effects which are differently distributed over the particles of the disk.

45. The relation between the effect for $\frac{3}{20}$ hyd. and that for $\frac{3}{20}$ hyd. + $\frac{6}{20}$ air for the thin aluminium disk (galv. 20 in 20 seconds) is given as follows:—

$$\left.\begin{array}{l}\text{Effect for } \tfrac{3}{20} \text{ hyd. (mean}\\ \text{of 22 observations)}\end{array}\right\} : \left\{\begin{array}{c}\text{Effect for } \tfrac{3}{20} \text{ hyd.} + \tfrac{6}{20} \text{ air}\\ \text{(mean of 10 observations)}\end{array}\right\} :: 48 : 228.$$

In the preceding experiments the centre of the pile was at a distance (along the radius) of 1·5 in. from the rim of the disk. In the following experiment it was adjusted so as to be at a radial distance of 2·9 in. from the rim of the same disk. With this alteration and with the thin aluminium disk we obtained

$$\left.\begin{array}{l}\text{Effect for } \tfrac{3}{20} \text{ hyd. (mean}\\ \text{of 7 observations)}\end{array}\right\} : \left\{\begin{array}{c}\text{Effect for } \tfrac{3}{20} \text{ hyd.} + \tfrac{6}{20} \text{ air}\\ \text{(mean of 9 observations)}\end{array}\right\} :: 37 : 175.$$

We thus see that the effect for $\frac{3}{20}$ hyd. (which may be supposed to represent the residual effect) and that for $\frac{3}{20}$ hyd. + $\frac{6}{20}$ air (which may be supposed to represent the gas-effect) are both diminished in very nearly the same proportion, namely 100 : 77, by the above transference of the pile to a position nearer the centre of the disk. It would thus appear that the difference in distribution among the particles of the two effects brought to light in art. 44 cannot be explained by a difference from centre to rim, but would rather seem to be due to a difference in depth. If this appear to be improbable, it must be remembered that these experiments were not made on a naked metallic disk. These experiments would therefore appear to show that in an aluminium disk covered with varnish, as well as in a disk of ebonite, we may imagine the residual effect to be more deeply seated than the gas-effect.

46. We venture on the following as what appears to us to be the most probable explanation of the whole body of experiments, including those with radiation.

(1) There is a temporary heat or cold effect (art. 26) which may be supposed to arise in particles very slightly attached to the disk; this is radiated off chiefly during rotation, and probably does not greatly affect the disk afterwards.

(2) There is a surface gas-effect, which in an aluminium and even in an ebonite disk is conducted into the interior as it arises, so that it does

* May not this peculiarity of the aluminium disk in the hydrogen vacuum be due to some hygrometric surface-effect resident in the varnish? In one experiment, but only one, the peculiarity was absent.

not greatly radiate during rotation of the disk (art. 26). In a paper disk, however, which is formed of a badly conducting material loosely put together, part of the effect does escape as radiation during rotation (art. 27).

(3) There is a residual effect, which is more deeply seated than the gas-effect. And inasmuch as radiation takes place from a perceptible depth, this effect is much more influential than the gas-effect in increasing radiation after rotation. In the case of a paper disk, this deeply seated effect will be less diminished by radiation during rotation than the gas-effect, and therefore after rotation in such a disk we might expect the gas-effect to be peculiarly small (art. 35).

47. In the course of these experiments we have endeavoured to prove that this residual effect is not caused by vibration. The radiation-experiments with aluminium disks of three different thicknesses went, on the other hand, to show that it was of the nature of a surface-effect. This is confirmed by the results derived from tapping; for, in the first place, the experiments of art. 45 show that the two effects (the residual and the gas-effect) are probably distributed in the same proportion, going from the centre to the circumference of the disk. Again, taking the two disks of thickness ·05 and ·025 of an inch, we obtain the following results:—

	Effect for $\frac{3}{20}$ hyd.	Effect for $\frac{3}{20}$ hyd. + $\frac{6}{20}$ air.
Thin disk	48 (22 observations).	228 (10 observations).
Thick disk......	29 (20 observations).	108 (10 observations).

Now, allowing for errors of experiment, we see that the residual, as well as the gas-effect, is reduced to about one half for the thick disk.

Again, an experiment of a similar nature gave the effect for $\frac{3}{20}$ hyd. in an ebonite disk of $\frac{1}{10}$ in. in thickness $=33$, against a result $=55$ for the thin ebonite disk. Unfortunately it was omitted to make a comparison with these two disks for the gas-effect; nevertheless these results are all in favour of the residual effect being a surface-effect.

48. It might be well to make one remark regarding these experiments. They are not like the radiation-experiments, which required an extremely delicate instrument in order to give a sensible effect. But, on the other hand, the effect obtained by tapping being that due to the mere surface of the disk, is liable to be altered by any thing which affects the surface of the disk. We have come to the conclusion that in such experiments it is unadvisable to use a porous hygrometric surface, such as that of paper, not having its pores filled with paraffin or some other similar substance. It is likewise desirable that all parts of the apparatus should be as nearly as possible of the same temperature; indeed we suspect that some experiments made during some very peculiar summer weather were influenced by a want of temperature-equilibrium between the various parts of the apparatus, the result appearing to be that the gas-effect for the ebonite disk was abnormally large.

49. Our conclusion from the evidence before us is, that the residual effect is a surface-effect more deeply seated than the gas-effect, but distributed outwards from the centre to the circumference, very much in the same manner as the gas-effect. The residual effect likewise appears able to penetrate a chamois-leather blind without any perceptible diminution. We regard these conclusions as preliminary, and shall endeavour in our future experiments to procure additional evidence of these properties of the residual effect, as well as to obtain new facts regarding it. In the mean time, as the subject is one of interest, and has been already too long delayed, we have not hesitated to bring these results before the notice of the Royal Society.

In concluding we would desire to express our thanks to Mr. F. Kingdon for his assistance to us in many of these experiments.

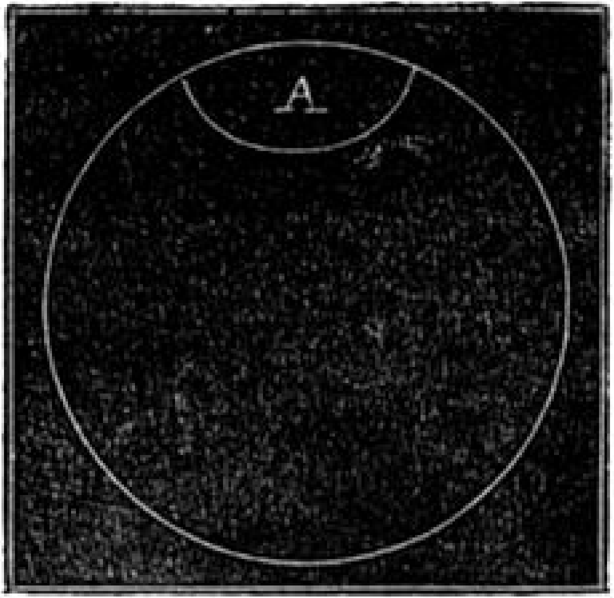
V. "On the Extension of the Numerical Value of π." By William Shanks, Houghton-le-Spring, Durham. Communicated by Prof. G. G. Stokes, Sec. R.S. Received April 16, 1873.

In the 'Messenger of Mathematics' for Dec. 1872, J. W. L. Glaisher, Esq., has given some very interesting particulars regarding the calculation of π, in the justness of which the author generally concurs. He, however, differs from him as to the comparative merits of Van Ceulen, who, in the early part of the seventeenth century, calculated π to 36 decimals. Hutton's formula also, given in the 'Messenger,' appears, notwithstanding Hutton's own opinion, to be not so well adapted for extensive computation as Machin's, which the author has used on the present as well as former occasions, regarding it as the best yet found.

The values of $\tan^{-1}\frac{1}{5}$ and of $\tan^{-1}\frac{1}{239}$ are each given below to 709, and the value of π to 707 decimals. It will be observed that a few figures in the values of $\tan^{-1}\frac{1}{5}$ and of π, published in 1853, were erroneous. The author detected the error quite recently, and has corrected it. The values of each term of the two series in $\frac{\pi}{4}=4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$, are far too bulky to be given *in extenso*: fortunately, but few would care to see them!

It may here be stated that Prof. Richter, of Elbing, found π to 500 decimals in the year 1853—all of which agree with the author's, published early in the same year.

The Society adjourned over Ascension Day to Thursday, May 29.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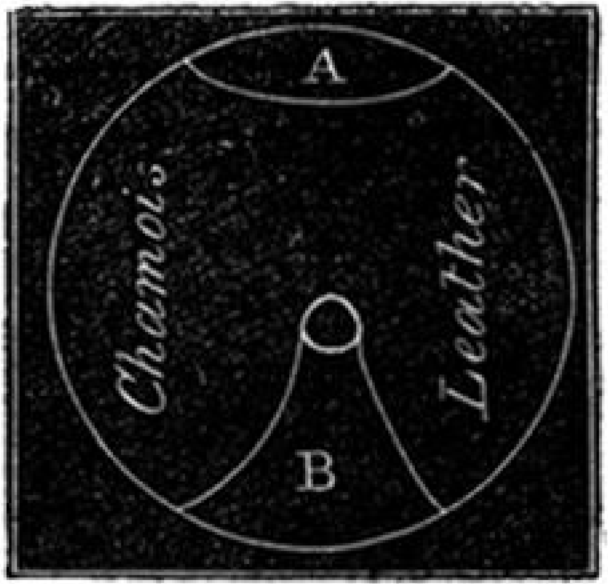
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