

III. "On the Number of Figures in the Period of the Reciprocal of every Prime Number below 20,000." By WILLIAM SHANKS. Communicated by the Rev. GEORGE SALMON. Received December 2, 1873\*.

The following Table, in reality the joint production of the Rev. George Salmon, F.R.S., and myself, was commenced, and indeed nearly completed, before either calculator was distinctly aware that Burckhardt, Jacobi, or Desmarest had written or published any thing on the same subject. This fact is perhaps to be regretted; but it has led to the independent recalculation, by two different methods, both of Burckhardt's (Jacobi's Table is professedly a reprint of Burckhardt's) and of Desmarest's Table, and has resulted in the detection of several errors, which have, as far as I know, never before been pointed out. These errors, in the first place regarded as discrepancies, have been carefully examined; in fact every case has been reworked by me, with the view of either proving or disproving the accuracy of such numbers as differ from those in our Table. The result is, that such discrepancies are found to be errors both in Burckhardt and Desmarest. The two lists of errors are given below.

I now proceed to give the theorems used, and some account of the means employed by me in forming the Table.

Let  $P$  be any prime number, except 2 and 5. Then, from Fermat's theorem, we have  $\frac{10^{P-1}}{P} \equiv 1$ ; or, adopting the usual notation,  $10^{P-1} \equiv 1$ .

Again, since the number of figures in the period of the reciprocal of all primes is not  $P-1$  (or, in other words, since 10 is not a primitive root of all primes),

Let  $10^{\frac{P-1}{n}} \equiv 1$ , where  $n$  is even or odd, not less than 2, and not greater than  $\frac{P-1}{2}$ . Then we have

(1) The number of figures in the period of the reciprocal of  $P$  is either  $P-1$  or a submultiple of  $P-1$ .

(2) Let  $a$  and  $b$  be integers, and let  $m$  be the remainder from  $\frac{10^a}{P}$ ; that is, let  $10^a \equiv m$ ; then  $10^{ab} \equiv m^b$ .

In practice  $b$  is never greater than 2, at least little or no advantage is gained by putting  $b$  higher. Also  $ab$  need not be greater than  $\frac{P-1}{2}$ .

Cor. When  $m$  is greater than  $\frac{P-1}{2}$  we may obviously use  $P-m$ , or simply  $-m$ ; for  $(P-m)^2 = P^2 - 2Pm + m^2 \equiv m^2$ , or, because  $(-m)^2 = m^2$ ,  $b$  being 2.

(3) Let  $10^a \equiv m$ , and  $10^b \equiv n$ ; then  $10^{a+b} \equiv mn$ .

In practice  $a+b$  is never greater than  $P-1$ .

Cor. 1. When  $m$  and  $n$  are each of them less than  $P$ , we may with advantage use  $-m$  and  $-n$ ; that is, we may subtract  $m$  and  $n$  severally from  $P$ ; for  $(P-m)(P-n) = P^2 - P(m+n) + mn = (-m)(-n)$ .

Cor. 2. When  $m$  is  $> \frac{P-1}{2}$ , and  $n$  is  $< \frac{P-1}{2}$ , or *vice versa*, we may use

\* The part from 17,000 to 20,000 was received January 8, 1874.

— $m$  and  $n$ , or *vice versâ*, obtaining a negative result, which becomes positive by being subtracted from  $P$ .

(4) Let  $2c$  and  $3c$ , not greater than  $\frac{P-1}{2}$ , be submultiples of  $P-1$ ; and let  $10^c \equiv \pm S$ , and  $10^{2c} \equiv S-1$ ; then  $10^{3c} \equiv \mp 1$ . This is evident from (2) and (3).

From (1) we have  $10^{\frac{P-1}{2}} \equiv \pm 1$ , according as the submultiple of  $P-1$  is even or odd.

On these theorems and adjuncts my calculations have been based. They enable us to find the remainder either from  $\frac{10^{P-1}}{P}$ , or from any submultiple, such as  $\frac{10^{\frac{P-1}{n}}}{P}$ , or from any figure in  $\frac{10^{P-1}}{P}$ , and, if required, the *figure* itself. Compared with other methods, such for instance as Dr. Salmon's\*, mine may seem tedious, requiring as it does much multiplication and division. All I can say is, I did not find it so, though I am free to admit that the calculation of such a Table as ours demands very considerable labour.

It would be foreign to my purpose to enter upon the consideration of primitive roots, or even of prime numbers. If we have found 10 to be a primitive root of a great many prime numbers between 10,000 and 20,000, we have contributed something, as far as I know, quite new. In addition to this we have found the number of figures in the period of each of the other primes between 10,000 and 20,000, and have corrected upwards of 70 errors in Burckhardt's and Desmarest's Tables.

I beg to refer to the works of Euler, Lagrange, Legendre, Gauss; Poincot, Cauchy, and Jacobi (mentioned by Desmarest), and to Desmarest himself, for valuable information touching prime numbers and primitive roots.

I cannot, however, refrain from quoting from Desmarest's '*Théorie des Nombres*' the view of Euler as to prime numbers and primitive roots:—"On ne peut saisir entre un nombre premier et les racines primitives qui lui appartiennent, aucune relation d'où l'on puisse déduire *une seule* de ces racines, de sorte que la loi qui règne entre elles paraît aussi profondément cachée que celle qui existe entre les nombres premiers eux-mêmes."

Not discouraged by Euler's remark, Desmarest thus writes:—"Car pourquoi nous serait-il défendu d'ajouter que nous croyons que l'intelligence humaine n'a pas, sur ce point, dit son dernier mot, et que les opérations nombreuses que nous avons dû faire sur les nombres, ne nous ont pas convaincu de l'impossibilité de saisir, sinon l'ensemble, du moins quelques-uns des anneaux de la chaîne mystérieuse qui unit les racines primitives aux nombres premiers."

\* *Note by Dr. Salmon.*—The method here referred to is explained, '*Messenger of Mathematics*' (1872), p. 49. It is founded on the remark that if we have  $10^a \equiv 2^p$ ,  $10^b \equiv 2^q$ , we may deduce  $10^{aq-bp} \equiv 1$ . Thus, let the prime be 251, we can at once write down the equations  $10^3 \equiv -2^2$ ,  $2^q \equiv 10^1$ , whence immediately  $10^{25} \equiv -1$ ,  $10^{50} \equiv 1$ . In like manner from the equations  $10^a \equiv 2^{3p}$ ,  $10^b \equiv 2^{m3q}$ ,  $10^c \equiv 2^{n3r}$ , we deduce that the number of figures in the period of the reciprocal of the prime is

$$a(nr-nq)+b(np-rt)+c(lq-mp).$$

By the application of these principles I calculated the results obtained in the following Table as far as 18500. For the primes above that number Mr. Shanks is solely responsible; but my experience of his accuracy gives me confidence in his results.

TABLE I. List of Errors in Desmarest's Table.

Primes.	No. of figures in period of reciprocal.		No. of figures in period of reciprocal.	Primes.	No. of figures in period of reciprocal.		No. of figures in period of reciprocal.
3 omitted }	omitted	should be	1	5557	1389	should be	926
277	138	"	69	5779	5778	"	2889
317	158	"	79	5827	582	"	2913
397	198	"	99	6101	3050	"	1220
449	224	"	32	6277	3138	"	1569
787	786	"	393	6287	3143	"	6286
1409	1408	"	32	8421	} should be		
1657	276	"	552	6421			
1733	433	"	866	6781		6780	1356
1889	59	"	118	6997		3498	1749
2087	$\frac{P-1}{8} \& \frac{P-1}{9}$	"	298	7001		3500	1750
3253	271	"	542	7127		509	1018
3373	562	"	843	7481		3740	748
3413	853	"	1706	7561		3780	1890
3517	1758	"	879	7717		3858	1929
3541	60	"	20	7741		2580	860
3547	3546	"	1773	7841		392	56
3637	3636	"	909	7853	} omitted	omitted	3926
3677	919	"	1838	8011			
3769	942	"	1884	8087		1335	2670
3821	1910	"	3820	8093		4043	8086
3911	3910	"	1955	8092		8092	4046
4049	1012	"	2024	8101		8100	1620
4167	} should be			8219		4109	8218
4157				8423	{ 8422 4211 }		8422
4397				8521		355	710
4621	157	"	314	8609		538	1076
4651	4620	"	924	8681		4340	868
5871	2325	"	4650	8893		4446	2223
4871	} should be			8999		8998	4499
4943				9067		9066	4533
5081				9187		9186	4593
5107	2471	"	4942	9397		4698	81
5407	2540	"	1270	9521		952	595
5479	5106	"	2553	9629		4814	9628
5519	901	"	1802	9649		1206	603
	5478	"	2739	9941		9940	1988
	5518	"	2759				

N.B. There are 64 errors, 3 misprints, 2 omissions, viz. 3 and 7853

TABLE II. List of Errors in Burckhardt's and Jacobi's Tables.

	Primes.	No. of figures in period of reciprocal.		No. of figures in period of reciprocal.
Burckhardt and Jacobi ...	911	450	should be	455
Burckhardt .....	1979	1976	"	1978
Jacobi .....	3462	} should be		
	3467			
	1213			
	1597	1212	"	202
	1831	266	"	133
	1951	915	"	305
	1993	390	"	195
	2311	1992	"	664
	2437	462	"	231
	367	2436	"	1218
		3466	"	1733

N.B. There are 3 misprints and 8 errors,

In the left-hand columns of Table III. are primes; in the right-hand columns, immediately opposite, is the number of figures in the period of the reciprocal of each prime.

TABLE III.

3	1	311	155	691	230	1109	1108	1567	1566
7	6	313	312	701	700	1117	558	1571	1570
11	2	317	79	709	708	1123	561	1579	1578
13	6	331	110	719	719	1129	564	1583	1582
17	16	337	336	727	359	1151	575	1597	1597
19	18	347	173	733	61	1153	1152	1601	200
23	22	349	116	739	246	1163	581	1607	1606
29	28	353	32	743	742	1171	1170	1609	201
31	15	359	179	751	125	1181	1180	1613	403
37	3	367	366	757	27	1187	593	1619	1618
41	5	373	186	761	380	1193	1192	1621	1620
43	21	379	378	769	192	1201	200	1627	271
47	46	383	382	773	193	1213	202	1637	409
53	13	389	388	787	393	1217	1216	1657	552
59	58	397	99	797	199	1223	1222	1663	1662
61	60	401	200	809	202	1229	1228	1667	833
67	33	409	204	811	810	1231	41	1669	556
71	35	419	418	821	820	1237	206	1693	423
73	8	421	140	823	822	1249	208	1697	1696
79	13	431	215	827	413	1259	1258	1699	566
83	41	433	432	829	276	1277	638	1709	1708
89	44	439	219	839	419	1279	639	1721	430
97	96	443	221	853	213	1283	641	1723	287
101	4	449	32	857	856	1289	92	1733	866
103	34	457	152	859	26	1291	1290	1741	1740
107	53	461	460	863	862	1297	1296	1747	291
109	108	463	154	877	438	1301	1300	1753	584
113	112	467	233	881	440	1303	1302	1759	879
127	42	479	239	883	441	1307	653	1777	1776
131	130	487	486	887	886	1319	659	1783	1782
137	8	491	490	907	151	1321	55	1787	893
139	46	499	498	911	455	1327	1326	1789	1788
149	148	503	502	919	459	1361	680	1801	900
151	75	509	508	929	464	1367	1366	1811	1810
157	78	521	52	937	936	1373	686	1823	1822
163	81	523	261	941	940	1381	1380	1831	305
167	166	541	540	947	473	1399	699	1847	1846
173	43	547	91	953	952	1409	32	1861	1860
179	178	557	278	967	322	1423	158	1867	933
181	180	563	281	971	970	1427	713	1871	935
191	95	569	284	977	976	1429	1428	1873	1872
193	192	571	570	983	982	1433	1432	1877	938
197	98	577	576	991	495	1439	719	1879	313
199	99	587	293	997	166	1447	1446	1889	118
211	30	593	592	1009	252	1451	290	1901	380
223	222	599	299	1013	253	1453	726	1907	953
227	113	601	300	1019	1018	1459	162	1913	1912
229	228	607	202	1021	1020	1471	735	1931	386
233	232	613	51	1031	103	1481	740	1933	21
239	7	617	88	1033	1032	1483	247	1949	1948
241	30	619	618	1039	519	1487	1486	1951	195
251	50	631	315	1049	524	1489	248	1973	986
257	256	641	32	1051	1050	1493	373	1979	1978
263	262	643	107	1061	212	1499	214	1987	331
269	268	647	646	1063	1062	1511	755	1993	664
271	5	653	326	1069	1068	1523	761	1997	998
277	69	659	658	1087	1086	1531	1530	1999	999
281	28	661	220	1091	1090	1543	1542	2003	1001
283	141	673	224	1093	273	1549	1548	2011	670
293	146	677	338	1097	1096	1553	1552	2017	2016
307	153	683	341	1103	1102	1559	779	2027	1013

TABLE III. (*continued*).

2029	2028	2539	2538	3023	3022	3547	1773	4057	4056
2039	1019	2543	2542	3037	253	3557	254	4073	4072
2053	342	2549	2548	3041	380	3559	1779	4079	2039
2063	2062	2551	425	3049	508	3571	3570	4091	4090
2069	2068	2557	639	3061	204	3581	3580	4093	22
2081	1040	2579	2578	3067	1533	3583	1194	4099	4098
2083	1041	2591	259	3079	1539	3593	3592	4111	2055
2087	298	2593	2592	3083	1541	3607	3606	4127	4126
2089	1044	2609	1304	3089	1544	3613	602	4129	2064
2099	2098	2617	2616	3109	148	3617	3616	4133	1033
2111	1055	2621	2620	3119	1559	3623	3622	4139	4138
2113	2112	2633	2632	3121	156	3631	1815	4153	4152
2129	532	2647	882	3137	3136	3637	909	4157	2078
2131	710	2657	2656	3163	1581	3643	1821	4159	693
2137	2136	2659	886	3167	3166	3659	3658	4177	4176
2141	2140	2663	2662	3169	72	3671	367	4201	75
2143	2142	2670	1335	3181	636	3673	3672	4211	4210
2153	2152	2677	223	3187	177	3677	1838	4217	4216
2161	30	2683	447	3191	29	3691	1230	4219	4218
2179	2178	2687	2686	3203	1601	3697	1232	4229	4228
2203	1101	2689	42	3209	1604	3701	3700	4231	2115
2207	2206	2693	1346	3217	1072	3709	3708	4241	1060
2213	553	2699	2698	3221	3220	3719	1859	4243	2121
2221	2220	2707	1353	3229	1076	3727	3726	4253	1063
2237	1118	2711	1355	3251	3250	3733	933	4259	4258
2239	1119	2713	2712	3253	542	3739	1246	4261	4260
2243	1121	2719	1359	3257	3256	3761	1880	4271	2135
2251	2250	2729	682	3259	3258	3767	3766	4273	1424
2267	1133	2731	2730	3271	1635	3769	1884	4283	2141
2269	2268	2741	2740	3299	3298	3779	3778	4289	2144
2273	2272	2749	916	3301	3300	3793	1264	4297	1432
2281	228	2753	2752	3307	1653	3797	949	4327	4326
2287	762	2767	2766	3313	3312	3803	1901	4337	4336
2293	1146	2777	2776	3319	553	3821	3820	4339	4338
2297	2296	2789	2788	3323	1661	3823	1274	4349	4348
2309	2308	2791	31	3329	832	3833	3832	4357	242
2311	231	2797	699	3331	3330	3847	3846	4363	2181
2333	583	2801	1400	3343	3342	3851	770	4373	1093
2339	2338	2803	1401	3347	1673	3853	963	4391	2195
2341	2340	2819	2818	3359	1679	3863	3862	4397	314
2347	1173	2833	2832	3361	1680	3877	969	4409	551
2351	1175	2837	709	3371	3370	3881	1940	4421	4420
2357	1178	2843	1421	3373	843	3889	1944	4423	4422
2371	2370	2851	2850	3389	3388	3907	1953	4441	2220
2377	264	2857	408	3391	1695	3911	1955	4447	4446
2381	476	2861	2860	3407	3406	3917	1958	4451	4450
2383	2382	2879	1439	3413	1706	3919	653	4457	4456
2389	2388	2887	2886	3433	3432	3923	1961	4463	4462
2393	184	2897	2896	3449	431	3929	491	4481	2240
2399	1199	2903	2902	3457	384	3931	1310	4483	249
2411	2410	2909	2908	3461	3460	3943	3942	4493	1123
2417	2416	2917	1458	3463	3462	3947	1973	4507	751
2423	2422	2927	2926	3467	1733	3967	3966	4513	1504
2437	1218	2939	2938	3469	3468	3989	3988	4517	2058
2441	305	2953	984	3491	698	4001	500	4519	753
2447	2446	2957	1478	3499	318	4003	87	4523	2261
2459	2458	2963	1481	3511	1755	4007	4006	4547	2273
2467	137	2969	371	3517	879	4013	34	4549	1516
2473	2472	2971	2970	3527	3526	4019	4018	4561	2280
2477	619	2999	1499	3529	1764	4021	268	4567	4566
2503	278	3001	1500	3533	1766	4027	2013	4583	4582
2521	630	3011	3010	3539	3538	4049	2024	4591	2295
2531	46	3019	3018	3541	20	4051	4050	4597	2298

TABLE III. (continued).

4603	2301	5147	2573	5689	316	6247	6246	6803	3401
4621	924	5153	5152	5693	1423	6257	6256	6823	6822
4637	61	5167	5166	5701	5700	6263	6262	6827	3413
4639	2319	5171	110	5711	571	6269	6268	6829	6828
4643	2321	5179	5178	5717	1429	6271	1045	6833	6832
4649	7	5189	5188	5737	5736	6277	1569	6841	855
4651	4650	5197	433	5741	5740	6287	6286	6857	6856
4657	1552	5209	372	5743	5742	6299	94	6863	6862
4663	222	5227	2613	5749	5748	6301	6300	6869	6868
4673	4672	5231	2615	5779	2889	6317	3158	6871	3435
4679	2339	5233	5232	5783	5782	6323	3161	6883	3441
4691	4690	5237	77	5791	965	6329	3164	6899	6898
4703	4702	5261	1052	5801	1450	6337	6336	6907	1151
4721	2360	5273	5272	5807	5806	6343	6342	6911	3455
4723	2361	5279	2639	5813	2906	6353	6352	6947	3473
4729	1182	5281	2640	5821	5820	6359	3179	6949	6948
4733	1183	5297	5296	5827	2913	6361	1590	6959	3479
4751	2375	5303	5302	5839	2919	6367	6366	6961	3480
4759	2379	5309	5308	5843	2921	6373	1062	6967	6966
4783	4782	5323	2661	5849	1462	6379	2126	6971	6970
4787	2393	5333	1333	5851	1950	6389	6388	6977	6976
4789	228	5347	2673	5857	5856	6397	78	6983	6982
4793	4792	5351	2675	5861	5860	6421	2140	6991	3495
4799	2399	5381	5380	5867	2933	6427	1071	6997	1749
4801	800	5387	2693	5869	5868	6449	1612	7001	1750
4813	802	5393	5392	5879	2939	6451	2150	7013	3506
4817	4816	5399	2699	5881	2940	6469	924	7019	7018
4831	805	5407	1802	5897	5896	6473	6472	7027	1171
4861	972	5413	2706	5903	5902	6481	270	7039	391
4871	2435	5417	5416	5923	2961	6491	1298	7043	503
4877	1219	5419	5418	5927	5926	6521	815	7057	7056
4889	2444	5431	2715	5939	5938	6529	1088	7069	7068
4903	1634	5437	1359	5953	1984	6547	1091	7079	3539
4909	1636	5441	2720	5981	5980	6551	3275	7103	7102
4919	2459	5443	907	5987	2993	6553	6552	7109	7108
4931	4930	5449	2724	6007	858	6563	3281	7121	3560
4933	2466	5471	547	6011	6010	6569	1642	7127	1018
4937	4936	5477	1369	6029	6028	6571	6570	7129	594
4943	4942	5479	2739	6037	3018	6577	2192	7151	275
4951	2475	5483	2741	6043	3021	6581	1316	7159	3579
4957	413	5501	5500	6047	6046	6599	3299	7177	7176
4967	4966	5503	5502	6053	3026	6607	2202	7187	3593
4969	828	5507	2753	6067	3033	6619	6618	7193	7192
4973	226	5519	2759	6073	6072	6637	474	7207	7206
4987	2493	5521	345	6079	1013	6653	3326	7211	1030
4993	1664	5527	5526	6089	761	6659	6658	7213	1803
4999	357	5531	5530	6091	2030	6661	6660	7219	7218
5003	2501	5557	926	6101	1220	6673	6672	7229	7228
5009	626	5563	2781	6113	6112	6679	3339	7237	402
5011	1670	5569	1392	6121	3060	6689	1672	7243	3621
5021	5020	5573	2786	6131	6130	6691	6690	7247	7246
5023	1674	5581	5580	6133	1533	6701	6700	7253	74
5039	2519	5591	2795	6143	6142	6703	6702	7283	3641
5051	50	5623	5622	6151	1025	6709	6708	7297	2432
5059	5058	5639	2819	6163	79	6719	3359	7307	3653
5077	2538	5641	470	6173	3086	6733	3366	7309	7308
5081	1270	5647	1882	6197	3098	6737	6736	7321	3660
5087	5086	5651	5650	9199	3099	6761	1690	7331	1466
5099	5098	5653	2826	6203	443	6763	161	7333	611
5101	1700	5657	5656	6211	6210	6779	6778	7349	7348
5107	2553	5659	5658	6217	6216	6781	1356	7351	1225
5113	1704	5669	5668	6221	6220	6791	679	7369	1842
5119	853	5683	2841	6229	2076	6793	6792	7393	7392

TABLE III. (continued).

7411	7410	7937	7936	8543	8542	9103	9102	9649	603
7417	2472	7949	7948	8563	4281	9109	9108	9661	1380
7433	7432	7951	3975	8573	4286	9127	3042	9677	2419
7451	7450	7963	3981	8581	2860	9133	1522	9679	1613
7457	7456	7993	2664	8597	2149	9137	9136	9689	346
7459	7458	8009	2002	8599	1433	9151	1525	9697	9696
7477	3738	8011	2670	8609	1076	9157	4573	9719	4859
7481	748	8017	8016	8623	8622	9161	229	9721	4860
7487	7486	8039	4019	8627	4313	9173	4586	9733	2433
7489	1872	8053	4026	8629	2876	9181	3060	9739	9738
7499	7498	8059	8058	8641	4320	9187	4593	9743	9742
7507	3753	8069	8068	8647	8646	9199	4599	9749	9748
7517	3758	8081	2020	8663	8662	9203	4601	9767	9766
7523	3761	8087	8086	8669	8668	9209	2302	9769	4884
7529	1882	8089	1348	8677	723	9221	9220	9781	9780
7537	2512	8093	4046	8681	868	9227	4613	9787	4893
7541	7540	8101	1620	8689	2172	9239	4619	9791	4895
7547	3773	8111	811	8693	4346	9241	4620	9803	4901
7549	2516	8117	2029	8699	8698	9257	9256	9811	9810
7559	3779	8123	4061	8707	4353	9277	4638	9817	9816
7561	1890	8147	4073	8713	8712	9281	928	9829	9828
7573	631	8161	1020	8719	4359	9283	1547	9833	9832
7577	7576	8167	2722	8731	8730	9293	2323	9839	4919
7583	7582	8170	8170	8737	2912	9311	4655	9851	9850
7589	1084	8179	8178	8741	8740	9319	4659	9857	9856
7591	3795	8191	1365	8747	4373	9323	4661	9859	3286
7603	1267	8209	4104	8753	8752	9337	3112	9871	4935
7607	7606	8219	8218	8761	876	9341	9340	9883	4941
7621	508	8221	2740	8779	22	9343	9342	9887	9886
7639	3819	8231	4115	8783	8782	9349	3116	9901	12
7643	3821	8233	8232	8803	1467	9371	9370	9907	4953
7649	1912	8237	4118	8807	8806	9377	9376	9923	4961
7660	284	8243	4121	8819	8818	9391	4695	9929	1241
7673	7672	8263	8262	8821	8820	9397	81	9931	9930
7681	1920	8269	8268	8831	4415	9403	1567	9941	1988
7687	7686	8273	8272	8837	4418	9413	4706	9949	9948
7691	7690	8287	8286	8839	4419	9419	554	9967	9966
7699	7698	8291	8290	8849	553	9421	9420	9973	554
7703	7702	8293	2073	8861	8860	9431	4715	10007	10006
7717	1929	8297	8296	8863	8862	9433	1048	10009	5004
7723	1287	8311	4155	8867	4433	9437	4718	10037	386
7727	7726	8317	462	8887	8886	9439	1573	10039	5019
7741	860	8329	1041	8893	2223	9461	9460	10061	10060
7753	7752	8353	8352	8923	1487	9463	3154	10067	5033
7757	1939	8363	4181	8929	144	9467	4733	10069	10068
7759	3879	8369	4184	8933	2233	9473	9472	10079	5039
7789	2596	8377	8376	8941	2980	9479	4739	10091	10090
7793	7792	8387	599	8951	4475	9491	9490	10093	2523
7817	7816	8389	8388	8963	4481	9497	9496	10099	3366
7823	7822	8419	2806	8969	4484	9511	1585	10103	10102
7829	7828	8423	8422	8971	8970	9521	595	10111	5055
7841	56	8429	8428	8999	4499	9533	2383	10133	2533
7853	3926	8431	4215	9001	1125	9539	9538	10139	10138
7867	3933	8443	4221	9007	3002	9547	4773	10141	10140
7873	7872	8447	8446	9011	9010	9551	955	10151	5075
7877	3938	8461	2820	9013	2253	9587	4793	10159	5079
7879	3939	8467	4233	9029	9028	9601	4800	10163	5081
7883	3941	8501	8500	9041	1130	9613	267	10169	5084
7901	7900	8513	8512	9043	4521	9619	3206	10177	10176
7907	3953	8521	710	9049	4524	9623	9622	10181	10180
7919	3959	8527	2842	9059	9058	9629	9628	10193	10192
7927	7926	8537	8536	9067	4533	9631	4815	10211	10210
7933	3966	8539	2846	9091	10	9643	4821	10223	10222

TABLE III. (continued).

10243	569	10847	10846	11447	11446	12049	6024	12613	6306
10247	10246	10853	5426	11467	5733	12071	355	12619	4206
10253	2563	10859	10858	11471	5735	12073	12072	12637	3159
10259	10258	10861	10860	11483	5741	12097	4032	12641	3160
10267	5133	10867	1811	11489	2872	12101	12100	12647	12646
10271	79	10883	5441	11491	766	12107	6053	12653	6326
10273	10272	10889	2722	11497	11496	12109	4036	12659	12658
10289	5144	10891	1210	11503	11502	12113	12112	12671	181
10301	10300	10903	10902	11519	5759	12119	6059	12689	793
10303	3434	10909	1212	11527	3842	12143	12142	12697	12696
10313	10312	10937	10936	11549	11548	12149	12148	12703	4234
10321	2580	10939	10938	11551	1925	12157	2026	12713	12712
10331	2066	10949	10948	11579	11578	12161	6080	12721	2120
10333	5166	10957	2739	11587	1931	12163	6081	12739	4246
10337	10336	10973	2743	11593	11592	12197	3049	12743	12742
10343	10342	10979	10978	11597	5798	12203	6101	12757	2126
10357	5178	10987	5493	11617	11616	12211	4070	12763	709
10369	2592	10993	10992	11621	11620	12227	6113	12781	12780
10391	5195	11003	5501	11633	11632	12239	6119	12791	6395
10399	1733	11027	5513	11657	11656	12241	6120	12799	2133
10427	5213	11047	11046	11677	5838	12251	12250	12809	6404
10429	948	11057	11056	11681	5840	12253	3063	12821	12820
10433	10432	11059	11058	11689	487	12263	12262	12823	12822
10453	5226	11069	11068	11699	11698	12269	12268	12829	4276
10457	10456	11071	615	11701	11700	12277	3069	12841	6420
10459	10458	11083	1847	11717	2929	12281	6140	12853	459
10463	10462	11087	482	11719	5859	12289	384	12889	3222
10477	1746	11093	2773	11731	11730	12301	2460	12893	3223
10487	10486	11113	3704	11743	11742	12323	6161	12899	12898
10499	10498	11117	2779	11777	11776	12329	3082	12907	6453
10501	3500	11119	5559	11779	3926	12343	4114	12911	6455
10513	10512	11131	11130	11783	11782	12347	6173	12917	6458
10529	5264	11149	11148	11789	11788	12373	6186	12919	2153
10531	10530	11159	5579	11801	2950	12377	12376	12923	6461
10559	5279	11161	310	11807	11806	12379	12378	12941	12940
10567	10566	11171	11170	11813	5906	12391	6195	12953	12952
10589	10588	11173	5586	11821	11820	12401	12400	12959	6479
10597	5298	11177	11176	11827	5913	12409	6204	12967	4322
10601	10600	11197	2799	11831	169	12413	6206	12973	2162
10607	10606	11213	2803	11833	11832	12421	12420	12979	12978
10613	758	11239	5619	11839	5919	12433	4144	12983	12982
10627	5313	11243	5621	11863	11862	12437	6218	13001	1625
10631	5315	11251	2250	11867	5933	12451	12450	13003	6501
10639	5319	11257	11256	11887	11886	12457	12456	13007	13006
10651	10650	11261	2252	11897	11896	12473	12472	13009	2168
10657	10656	11273	11272	11903	11902	12479	6239	13033	4344
10663	10662	11279	5639	11909	11908	12487	12486	13037	6518
10667	5333	11287	11286	11923	5961	12491	12490	13043	6521
10687	10686	11299	11298	11927	11926	12497	12496	13049	3262
10691	10690	11311	377	11933	5966	12503	12502	13063	13062
10709	10708	11317	943	11939	11938	12511	2085	13093	2182
10711	595	11321	1132	11941	11940	12517	149	13099	13098
10723	5361	11329	1888	11953	11952	12527	12526	13103	13102
10729	596	11351	5675	11959	5979	12539	12538	13109	13108
10733	2683	11353	11352	11969	352	12541	4180	13121	6560
10739	10738	11369	812	11971	11970	12547	6273	13127	13126
10753	3584	11383	11382	11981	11980	12553	12552	13147	939
10771	2154	11393	11392	11987	5993	12569	6284	13151	1315
10781	10780	11399	5699	12007	4002	12577	12576	13159	2193
10789	10788	11411	2282	12011	12010	12583	12582	13163	6581
10799	5399	11423	11422	12037	3009	12589	12588	13171	4390
10831	5415	11437	2859	12041	6020	12601	6300	13177	13176
10837	63	11443	1907	12043	2007	12611	12610	13183	4394



TABLE III. (continued).

13187	6593	13807	13806	14461	4820	15061	5020	15629	15628
13217	13216	13829	13828	14479	7239	15073	15072	15641	391
13219	4406	13831	2305	14489	7244	15077	7538	15643	7821
13229	13228	13841	6920	14503	14502	15083	7541	15647	15646
13241	1655	13859	13858	14519	7259	15091	3018	15649	489
13249	288	13873	13872	14533	519	15101	604	15661	5220
13259	1894	13877	3469	14537	14536	15107	7553	15667	7833
13267	737	13879	6939	14543	14542	15121	7560	15671	1567
13291	13290	13883	6941	14549	14548	15131	3026	15679	2613
13297	13296	13901	13900	14551	485	11137	15136	15683	7841
13309	4436	13903	13902	14557	2426	15139	15138	15727	15726
13313	13312	13907	409	14561	7280	15149	15148	15731	15730
13327	4442	13913	13912	14563	809	15161	3790	15733	2622
13331	13330	13921	696	14591	7295	15173	7586	15737	15736
13337	13336	13931	13930	14593	14592	15187	7593	15739	15738
13339	13338	13933	6966	14621	14620	15193	15192	15749	15748
13367	13366	13963	6981	14627	7313	15199	7599	15761	394
13381	13380	13967	13966	14629	14628	15217	15216	15767	15766
13397	6698	13997	6998	14633	14632	15227	7613	15773	7886
13399	957	13999	2333	14639	7319	15233	15232	15787	7893
13411	13410	14009	3502	14653	3663	15241	3810	15791	7895
13417	4472	14011	14010	14657	14656	15259	15258	15797	3949
13421	13420	14029	14028	14669	14668	15263	15262	15803	7901
13441	6720	14033	2812	14683	2447	15269	1388	15809	3952
13451	13450	14051	2810	14699	14698	15271	7635	15817	5272
13457	13456	14057	14056	14713	14712	15277	3819	15823	15822
13463	13462	14071	7035	14717	7358	15287	15286	15859	15858
13469	13468	14081	1760	14723	7361	15289	1274	15877	567
13477	6738	14083	7041	14731	14730	15299	15298	15881	7940
13487	13486	14087	14086	14737	14736	15307	7653	15887	15886
13499	13498	14107	2351	14741	14740	15313	5104	15889	3972
13513	4504	14143	14142	14747	7373	15319	7659	15901	15900
13523	6761	14149	524	14753	14752	15329	7664	15907	7953
13537	13536	14153	14152	14759	7379	15331	5110	15913	15912
13553	1936	14159	7079	14767	14766	15349	15348	15919	7959
13567	4522	14173	3543	14771	2110	15359	7679	15923	7961
13577	13576	14177	14176	14779	14778	15361	256	15937	5312
13591	1359	14197	91	14783	14782	15373	1281	15959	7979
13597	3399	14207	14206	14797	3699	15377	15376	15971	15970
13613	6806	14221	2844	14813	7406	15383	15382	15973	121
13619	1238	14243	7121	14821	4940	15391	7695	15991	7995
13627	6813	14249	7124	14827	2471	15401	275	16001	2000
13633	4544	14251	14250	14831	1483	15413	7706	16007	16006
13649	853	14281	1190	14843	7421	15427	7713	16033	16032
13669	13668	14293	1191	14851	990	15439	7719	16057	1784
13679	6839	14303	14302	14867	7433	15443	7721	16061	3212
13681	3420	14321	3580	14869	4956	15451	5150	16063	5354
13687	4562	14323	1023	14879	7439	15461	15460	16067	8033
13691	13690	14327	14326	14887	14886	15467	7733	16069	5356
13693	326	14341	14340	14891	14890	15473	15472	16073	16072
13697	13696	14347	7173	14897	14896	15493	7746	16087	5362
13709	13708	14369	449	14923	7461	15497	1192	16091	16090
13711	6855	14387	7193	14929	1866	15511	7755	16097	16096
13721	6860	14389	4796	14939	14938	15527	15526	16103	16102
13723	6861	14401	3600	14947	7473	15541	740	16111	1611
13729	3432	14407	686	14951	1495	15551	7755	16127	16126
13751	6875	14411	2882	14957	7478	15559	2593	16139	16138
13757	362	14419	4806	14969	7484	15569	7784	16141	3228
13759	6879	14423	14422	14983	4994	15581	15580	16183	16182
13763	6881	14431	555	15013	3753	15583	15582	16187	8093
13781	13780	14437	218	15017	15016	15601	390	16189	852
13789	13788	14447	14446	15031	7515	15607	15606	16193	16192
13799	6899	14449	3612	15053	3763	15619	15618	16217	16216

TABLE III. (continued).

16223	16222	16889	8444	17483	8741	18097	6032	18719	9359
16229	16228	16901	3380	17489	2186	18119	9059	18731	3746
16231	8115	16903	5634	17491	17490	18121	2265	18743	18742
16249	1354	16921	423	17497	17496	18127	18126	18749	18748
16253	8126	16927	5642	17509	5836	18131	490	18757	3126
16267	8133	16931	16930	17519	8759	18133	9066	18773	9386
16273	16272	16937	16936	17539	5846	18143	18142	18787	9393
16301	3260	16943	16942	17551	2925	18149	1396	18793	18792
16319	8159	16963	8481	17569	2928	18169	9084	18797	127
16333	4083	16979	16978	17573	4393	18181	6060	18803	9401
16339	16338	16981	16980	17579	17578	18191	9095	18839	9419
16349	16348	16987	8493	17581	17580	18199	9099	18859	18858
16361	1636	16993	16992	17597	4399	18211	18210	18869	18868
16363	8181	17011	17010	17599	2933	18217	18216	18899	18898
16369	264	17021	3404	17609	8804	18223	6074	18911	9455
16381	5460	17027	8513	17623	1958	18229	18228	18913	18912
16411	16410	17029	17028	17627	8813	18233	424	18917	4729
16417	16416	17033	17032	17657	17656	18251	18250	18919	9459
16421	16420	17041	2840	17659	5886	18253	9126	18947	9473
16427	8213	17047	5682	17669	17668	18257	2608	18959	9479
16433	16432	17053	1421	17681	1105	18269	18268	18973	153
16447	16446	17077	2846	17683	8841	18287	18286	18979	18978
16451	16450	17093	4273	17707	8853	18289	9144	19001	2375
16453	2742	17099	17098	17713	17712	18301	18300	19009	4752
16477	4119	17107	8553	17729	1108	18307	3051	19013	4753
16481	2060	17117	4279	17737	17736	18311	9155	19031	9515
16487	16486	17123	8561	17747	8873	18313	18312	19037	9518
16493	4123	17137	5712	17749	17748	18329	4582	19051	6350
16519	2753	17159	8579	17761	8880	18341	18340	19069	6356
16529	8264	17167	17166	17783	17782	18353	18352	19073	19072
16547	8273	17183	17182	17789	17788	18367	6122	19079	9539
16553	16552	17189	17188	17791	8895	18371	3674	19081	795
16561	8280	17191	8595	17807	17806	18379	18378	19087	6362
16567	16566	17203	2867	17827	8913	18397	9198	19121	9560
16573	8286	17207	17206	17837	91	18401	4600	19139	19138
16603	8301	17209	4302	17839	8919	18413	4603	19141	19140
16607	16606	17231	8615	17851	714	18427	3071	19157	4789
16619	16618	17239	8619	17863	17862	18433	6144	19163	9581
16631	8315	17257	17256	17881	2235	18439	9219	19181	19180
16633	5544	17291	17290	17891	17890	18443	9221	19183	6394
16649	8324	17293	1441	17903	17902	18451	6150	19207	19206
16651	3330	17299	1922	17909	17908	18457	18456	19211	3842
16657	16656	17317	1443	17911	2985	18461	18460	19213	9606
16661	16660	17321	8660	17921	8960	18481	1320	19219	19218
16673	16672	17327	17326	17923	8961	18493	4623	19231	9615
16691	16690	17333	8666	17929	8964	18503	18502	19237	4809
16693	2782	17341	5780	17939	17938	18517	3086	19249	3208
16699	16698	17351	8675	17957	8978	18521	9260	19259	19258
16703	16702	17359	2893	17959	8979	18523	9261	19267	9633
16729	697	17377	17376	17971	17970	18539	18538	19273	19272
16741	16740	17383	17382	17977	856	18541	3708	19289	4822
16747	8373	17387	8693	17981	17980	18553	6184	19301	3860
16759	931	17389	17388	17987	8993	18583	18582	19309	19308
16763	29	17393	17392	17989	5996	18587	9293	19319	9659
16787	8393	17401	4350	18013	9006	18593	18592	19333	4833
16811	3362	17417	17416	18041	220	18617	1432	19373	4843
16823	16822	17419	17418	18043	291	18637	9318	19379	19378
16829	16828	17431	8715	18047	2578	18661	18660	19381	1292
16831	8415	17443	8721	18049	1128	18671	1867	19387	3231
16843	401	17449	4362	18059	18058	18679	3113	19391	1385
16871	1205	17467	8733	18061	18060	18691	18690	19403	9701
16879	2813	17471	8735	18077	4519	18701	3740	19417	19416
16883	8441	17477	8738	18089	9044	18713	18712	19421	19420

TABLE III. (*continued*).

19423	3237	19501	780	19681	9840	19777	6952	19913	19912
19427	9713	19507	9753	19687	19686	19793	19792	19919	9959
19429	19428	19531	6510	19697	19696	19801	9900	19927	19926
19433	19432	19541	19540	19699	19698	19813	4953	19937	2848
19441	1620	19543	19542	19709	19708	19819	19818	19949	19948
19447	19446	19559	9779	19717	9858	19841	1984	19961	9980
19457	19456	19571	19570	19727	19726	19843	3307	19963	3327
19463	19462	19577	19576	19739	19738	19853	9926	19973	9986
19469	19468	19583	19582	19751	9875	19861	6620	19979	19978
19471	9735	19597	4899	19753	19752	19867	6622	19991	9995
19477	3246	19603	3267	19759	9879	19889	9944	19993	19992
19483	9741	19609	9804	19763	9881	19891	6630	19997	9998
19489	406	19661	19660						

February 26, 1874.

JOSEPH DALTON HOOKER, C.B., President, in the Chair.

The following Papers were read :—

- I. “The Winds of Northern India, in relation to the Temperature and Vapour-constituent of the Atmosphere.” By HENRY F. BLANFORD, F.G.S., Meteorological Reporter to the Government of Bengal. Communicated by Major-General STRACHEY, R.E. Received May 25, 1873.

(Abstract.)

The object of this paper is to describe the normal wind-currents of Northern India, and their annual variation, and to trace out their origin and causes, so far as these can be discovered in the local physical changes of the atmosphere. After referring to the data on which his conclusions are based, the author goes on to describe the winds of the principal geographical regions of North India in detail.

#### PART I. *Description of Winds.*

1. *The Punjab.*—As a rule currents from the westward predominate on an average throughout the year; and this is also found to be the case in other parts of North India. In the most northern part of the Punjab, westerly winds prevail in the cold and hot dry months, easterly in the rainy months. In the central districts northerly winds preponderate over southerly, having in the cold months a westerly tendency, but drawing round to the north-east as the hot weather comes on, while as the rainy season sets in the winds tend to east and south-east, returning to west after the rain ceases in September. In the southern part of the Punjab (and this is also the case in Sindh) easterly winds never prevail,