III. "On Combinations of Colour by means of Polarized Light." By W. Spottiswoode, M.A., Treas. & V.P.R.S. Received April 8, 1874.

The results of combining two or more colours of the spectrum have been studied by Helmholtz, Clerk Maxwell, Lord Rayleigh, and others; and the combinations have been effected sometimes by causing two spectra at right angles to one another to overlap, and sometimes by bringing images of various parts of a spectrum simultaneously upon the retina. Latterly also W. v. Bezold has successfully applied the method of binocular combination to the same problem (Poggendorff, Jubelband, p. 585). Some effects, approximating more or less to these, may be produced by chromatic polarization.

Complementary Colours.—First as regards complementary colours. If we use a Nicol's prism, N, as polarized, a plate of quartz, Q, cut perpendicularly to the axis, and a double-image prism, P, as analyzer, we shall, as is well known, obtain two images whose colours are complementary. If we analyze these images with a prism, we shall find, when the quartz is of suitable thickness, that each spectrum contains a dark band, indicating the extinction of a certain narrow portion of its length; these bands will simultaneously shift their position when the Nicol N is turned round. Now, since the colours remaining in each spectrum are complementary to those in the other, and the portion of the spectrum extinguished in each is complementary to that which remains, it follows that the portion extinguished in one spectrum is complementary to that extinguished in the other; and in order to determine what portion of the spectrum is complementary, the portion suppressed by a band in any position we please, we have only to turn the Nicol N until the band in one spectrum occupies the position in question, and then to observe the position of the band in the other spectrum. The combinations considered in former experiments are those of simple colours; the present combinations are those of mixed tints, viz. of the parts of the spectrum suppressed in the bands. But the mixture consists of a prevailing colour, corresponding to the centre of the band, together with a slight admixture of the spectral colours immediately adjacent to it on each side.

The following results, given by Helmholtz, may be approximately verified:—

Complementary	Colours.
Red,	Green-blue;
Orange,	Cyanic blue;
Yellow,	Indigo-blue.
Yellow-green,	Violet.

When in one spectrum the band enters the green, in the other a band will be seen on the outer margin of the red and a second at the opposite

end of the violet—showing that to the green there does not correspond one complementary colour, but a mixture of violet and red, *i. e.* a reddish purple.

Combination of two Colours.—Next as to the combination of two parts of the spectrum, or of the tints which represent those parts. If, in addition to the apparatus described above, we use a second quartz plate, Q, and a second double-image prism, P_1, we shall form four images, say O O, O E, E O, E E; and if A, A′ be the complementary tints extinguished by the first combination Q P alone, and B, B′ those extinguished by the second Q_1 P_1 alone, then it will be found that the following pairs of tints are extinguished in the various images:—

Image.	Tints extinguished.
O O	B, A,
O E	B′, A′,
E O	B′, A,
E E	B, A′.

It is to be noticed that in the image O E the combination Q_1 P_1 has extinguished the tint B′ instead of B, because the vibrations in the image E were perpendicular to those in the image O formed by the combination Q P. A similar remark applies to the image E E.

The total number of tints which can be produced by this double combination Q P, Q_1 P_1 is as follows:—

	4 single images,
	6 overlaps of two,
	4 overlaps of three,
	1 overlap of four.
Total. .	15

Collateral Combinations.—The tints extinguished in the overlap O O+E O will be B, A, B′, A; but since B and B′ are complementary, their suppression will not affect the resulting tint except as to intensity, and the overlap will be effectively deprived of A alone; in other words, it will be of the same tint as the image O would be if the combination Q_1 P_1 were removed. Similarly the overlap O E+E E will be deprived effectually of A′ alone; in other words, it will be of the same tint as E, if Q_1 P_1 were removed. If therefore the Nicol N be turned round, these two overlaps will behave in respect of colour exactly as did the images O and E when Q P was alone used. We may, in fact, form a Table thus:—

Image.	Colours extinguished.
O O+E O	$B+A+B'+A=B+B'+A=A$
O E+E E	$B'+A'+B+A'=B+B'+A'=A'$.

And since the tints B, B′ have disappeared from each of these formulæ, it follows that the second analyzer P may be turned round in any direction without altering the tints of the overlaps in question.

In like manner we may form the Table

$$\begin{array}{ll} OO+EE & B+A+B+A'=B+A+A'=B \\ OE+EO & B'+A'+B'+A=B'+A+A'=B'. \end{array}$$

Hence if the Nicol N be turned round, these overlaps will retain their tints; while if the analyzer P_1 be turned, their tints will vary, although always remaining complementary to one another.

There remains the other pair of overlaps, viz.:—

$$\begin{array}{ll} OO+OE & B+A+B'+A' \\ EO+EE & B'+A+B+A'. \end{array}$$

Each of these is deprived of the pair of complementaries A, A', B, B'; and therefore each, as it would seem, ought to appear white of low illumination, *i. e.* grey. This effect, however, is partially masked by the fact that the dark bands are not sharply defined like the Fraunhofer lines, but have a core of minimum or zero illumination, and are shaded off gradually on either side until at a short distance from the core the colours appear in their full intensity. Suppose, for instance, that B' and A' were bright tints, the tint resulting from their suppression would be bright; on the other hand, the complementary tints A and B would be generally dim, and the image B+A bright, and the overlap B+A+B'+A' would have as its predominating tint that of B+A; and similarly in other cases.

There are two cases worth remarking in detail, viz., first, that in which

$$B=A',\ B'=A,$$

i. e. when the same tints are extinguished by the combination QP and by $Q_1 P_1$. This may be verified by either using two similar quartz plates $Q_1 Q_1$, or by so turning the prism P_1 that the combination $Q_1 P_1$ used alone shall give the same complementary tints as QP when used alone. In this case the images have for their formulæ the following:—

$$\begin{array}{cccc} OO & OEO & EO & EE \\ A+A' & A+A' & 2A & 2A'; \end{array}$$

in other words, OO and EO will show similar tints, and EO, EE complementary. A similar result will ensue if B=A, B'=A'.

Again, even when neither of the foregoing conditions are fulfilled, we may still, owing to the breadth of the interference-bands, have such an effect produced that sensibly to the eye

$$B+A=B'+A';$$

and in that case

$$\begin{aligned} B'+A &= B+A-A'+A \\ &= B+A'+2A-2A', \end{aligned}$$

which imply that the images OO and OE may have the same tint, but that EO and EE need not on that account be complementary. They will differ in tint in this, that EE, having lost the same tints as EO, will have lost also the tint A, and will have received besides the addition of two measures of the tint A'.

Effect of Combinations of two Colours.—A similar train of reasoning might be applied to the triple overlaps. But the main interest of these parts of the figure consists in this, that each of the triple overlaps is complementary to the fourth single image, since the recombination of all four must reproduce white light: hence the tint of each triple overlap is the same to the eye as the mixture of the two tints suppressed in the remaining image; and since by suitably turning the Nicol N or the prism P_1, or both, we can give any required position to the two bands of extinction, we have the means of exhibiting to the eye the result of the mixture of the tints due to any two bands at pleasure.

Effect of Combinations of three Colours.—A further step may be made in the combination of colours by using a third quartz, Q_2, and a third double-image prism, P_2, which will give rise to eight images; and if C C′ be the complementaries extinguished by the combination Q_2 P_2, the formulæ for the eight images may be thus written:—

O O O	$C + B + A.$
O O E	$C + B' + A'.$
O E O	$C' + B' + A.$
O E E	$C' + B + A'.$
E O O	$C' + B + A.$
E O E	$C' + B' + A'.$
E E O	$C + B' + A.$
E E E	$C + B + A'.$

The total number of combinations of tint given by the compartments of the complete figure will be:—

$\frac{8}{1}$	=	8 single images.		
$\frac{8 \cdot 7}{1 \cdot 2}$	=	28 overlaps of two.		
$\frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3}$	=	56	,,	three.
$\frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4}$	=	70	,,	four.
$\frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3}$	=	56	,,	five.
$\frac{8 \cdot 7}{1 \cdot 2}$	=	28	,,	six.
$\frac{8}{1}$	=	8	,,	seven.
1	=	1	,,	eight.
Total		255		

The most interesting features of the figure consist in this, that the subjoined pairs are complementary to one another, viz.:—

O O O	E O E
C + B + A	C′ + B′ + A′
E O O	O O E
C′ + B + A	C + B′ + A′
E E O	O E E
C + B′ + A	C′ + B + A′
E E E	O E O
C + B + A′	C′ + B′ + A

And if the prisms P, P_1, P_2 are so arranged that the separations due to them respectively are directed parallel to the sides of an equilateral triangle, the images will be disposed thus:—

	O E O	O O O	
E E O	E O O	O E E	O O E
	E E E	E O E	

The complementary pairs can then be read off, two horizontally and two vertically, by taking alternate pairs, one in each of the two vertical, and two in the one horizontal row; and each image will then represent the mixture of the three tints suppressed in the complementary image.

Low-tint Colours.—A slight modification of the arrangement above described furnishes an illustration of the conclusions stated by Helmholtz, viz. that the low-tint colours (couleurs degradées), such as russet, brown, olive-green, peacock-blue, &c., are the result of relatively low illumination. He mentioned that he obtained these effects by diminishing the intensity of the light in the colours to be examined, and by, at the same time, maintaining a brilliantly illuminated patch in an adjoining part of the field of view. If therefore we use the combination N, Q, P, P_1 (*i. e.* if we remove the second quartz plate), we can, by turning the prism P round, diminish to any required extent the intensity of the light in one pair of the complementary images, and at the same time increase that in the other pair. This is equivalent to the conditions of Helmholtz's experiments; and the tints in question will be found to be produced.