and computing the integral as extended over an area bounded by the four straight lines thus represented, we have

$$\int_{y_0}^{y_1}\int_{x_0}^{x_1} \mathrm{P}\,dx\,dy = \int_{\frac{x_0+y_0}{\sqrt{2}}}^{\frac{x_0+y_1}{\sqrt{2}}}\int_{y_0\sqrt{2}-\xi}^{\xi-x_0\sqrt{2}} \mathrm{P}\,d\eta\,d\xi + \int_{\frac{x_0+y_1}{\sqrt{2}}}^{\frac{x_1+y_0}{\sqrt{2}}}\int_{y_0\sqrt{2}-\xi}^{y_1\sqrt{2}-\xi} \mathrm{P}\,d\eta\,d\xi$$

$$+\int_{\frac{x_1+y_0}{\sqrt{2}}}^{\frac{x_1+y_1}{\sqrt{2}}}\int_{\xi-x_1\sqrt{2}}^{y_1\sqrt{2}-\xi} \mathrm{P}\,d\eta\,d\xi.$$

After I had discovered this formula, I found that it had been already given in a memoir by Dr. Winckler in the Vienna Transactions for 1862. This memoir treats of the transformation of double integrals between fixed limits, and seems to me one of great interest and importance. My present object is to give two formulæ for the multiplication of definite integrals which will not be found in Dr. Winckler's paper.

$$\int_{y_0}^{y_1}\epsilon^{y^2}dy\int_{x_0}^{x_1}\epsilon^{-x^2}dx = \frac{\epsilon^{-x_0^2}}{2}\int_{y_0}^{y_1}\frac{\epsilon^{z^2}dz}{z+x_0} - \frac{\epsilon^{-x_1^2}}{2}\int_{y_0}^{y_1}\frac{\epsilon^{z^2}dz}{z+x_1}$$

$$-\frac{\epsilon^{y_0^2}}{2}\int_{x_0}^{x_1}\frac{\epsilon^{-z^2}}{z+y_0} + \frac{\epsilon^{y_1^2}}{2}\int_{x_0}^{x_1}\frac{\epsilon^{-z^2}dx}{z+y_1}.$$

Also

$$\int_{y_0}^{y_1}\epsilon^{y^2}dy\int_{x_0}^{x_1}\epsilon^{x^2}dx = \frac{x_1\epsilon^{x_1^2}}{2}\int_{y_0}^{y_1}\frac{dz\,.\,\epsilon^{z^2}}{z^2+x_1^2} - \frac{x_0\epsilon^{x_0^2}}{2}\int_{y_0}^{y_1}\frac{dz\,.\,\epsilon^{z^2}}{z^2+x_0^2}$$

$$+\frac{y_1\epsilon^{y_1^2}}{2}\int_{x_0}^{x_1}\frac{dz\,.\,\epsilon^{z^2}}{z^2+y_1^2} - \frac{y_0\epsilon^{y_0^2}}{2}\int_{x_0}^{x_1}\frac{dz\,.\,\epsilon^{z^2}}{z^2+y_0^2}.$$

The use of these formulæ is easily seen.

December 17, 1874.

JOSEPH DALTON HOOKER, C.B., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers and Communications were read:—

I. "On Polishing the Specula of Reflecting Telescopes." By W. Lassell, F.R.S., V.P.R.A.S. Received November 11, 1874.

(Abstract.)

The object of this paper is to describe a method of giving a high lustre and true parabolic curve with ease and certainty, by appropriate machinery, to the surfaces of the specula of large reflecting telescopes.

It may be remembered that many years ago Mr. Lassell invented, and described in the eighteenth volume of the Memoirs of the Royal Astronomical Society, a machine for polishing specula. It is no part of the object of this paper to disparage or supersede that machine, as with it he has polished many specula sensibly perfect, some of which are now in his possession, in whose surfaces he can find no imperfection whatever, and which he should vainly attempt to improve; but it possesses scarcely power enough for polishing a two-foot speculum, though the specula belonging to the telescope of that size which he took out to Malta in 1852 were polished with that machine. Indeed the first surfaces on the four-foot specula of the telescope taken out to Malta on his last expedition were obtained in the same way; but it was with great difficulty, and ultimately the machine broke down hopelessly, the result of which was the construction of the present one described in the paper now presented to the Society. But reference must be had to the paper itself, and to the drawings which accompany it, for an adequate description.

In this machine there may possibly be found nothing very essentially new; it contains parts adopted from others and modified, the principal novelty being a method of giving a regular and gently controlled axial motion to the polisher while it is undergoing the various other motions proper to the machine.

Mr. Lassell attempts in this paper to describe the processes with sufficient precision to enable persons of ordinary intelligence and some mechanical aptitude to obtain with ease and certainty surfaces on specula (taking a two-foot speculum as an example) which shall be sensibly perfect in figure and polish; and this, as his words imply, without the tedious trial-and-error process, which amateurs have had too frequently to experience. Another object he has had also especially in view is to render the process interesting and pleasurable throughout, by devising new modes of performing the most disagreeable parts of the operation, such as the formation of the pitch-tool, which in large surfaces is apt to be very troublesome and annoying. This is accomplished by simply studying the properties of pitch and adapting its treatment, so to speak, to its peculiar unaccommodating humour. A further aim has been to simplify to the utmost the mode of action in every particular, leaving out every thing which long experience has shown to be unnecessary—for instance, polishing with the speculum partly immersed in water, straining pitch through muslin, &c., processes tedious and vexatious enough without being required.

Rules are given in the paper for the rates of motion of both the polisher and speculum, as well as for the lengths of the strokes of the crank-arms, which have an immediate and powerful influence in the production of the required curve. The mode of making the polisher-base, and covering it with pitch in squares approximately to fit the speculum at once, and the mode of keeping the polisher for a considerable time and

through considerable changes of temperature, and renewing the surface for repeated polishings, are also described. The mode of construction of a bed of hones for bringing the curve of the speculum back to the sphere, if it should happen to have gone beyond the parabola in the polishing, without reverting to the emery-grinder, is also explained; and a word or two is added respecting the treatment of the speculum when finished.

II. "Note on the Vertical Distribution of Temperature in the Ocean." By J. Y. Buchanan, Chemist on board H.M.S. 'Challenger.' Communicated by Prof. A. W. Williamson, For. Sec. R.S. Received November 11, 1874.

From newspapers and other reports which have been received by late mails, it appears that the distribution of temperature in the ocean is occupying the attention of a certain portion of the scientific public, and even giving rise to considerable discussion. The observations made on board this ship, and more especially in the Atlantic, have furnished the greater part of the material on which the various speculations have been founded. It appears to me that one point suggested by these observations has not received sufficient attention from those who have written and spoken on the subject: I mean, the effect of the changing seasons on sea-water. Consider the state of the water at and near the surface of the ocean, somewhere not in the tropics. To be more precise, let us suppose that we have taken up our position in the middle of the North Atlantic, somewhere about the 30th parallel. This part of the ocean is not vexed with currents, and affords the best possible field for the observation of the phenomenon in question. The whole ocean enclosed by the 20th and 40th parallels of north latitude and the meridians of 30° and 60° west longitude forms one oceanic lake, not affected by the perturbing influence of currents or of land, and where, therefore, the true effect of differences of atmospheric temperature on the waters of the ocean may be most advantageously studied. Let us assume the winter temperature of the surface-water to be 60° F. and the summer temperature to be 70° F. If we start from midwinter, we find that, as summer approaches, the surface-water must get gradually warmer, and that the temperature of the layers below the surface must decrease at a very rapid rate, until the stratum of winter temperature, or 60° F., is reached; in the language of the isothermal charts, the isothermal line for degrees between 70° F. (if we suppose that we have arrived at midsummer) and 60° F. open out or increase their distance from each other as the depth increases. Let us now consider the conditions after the summer heat has begun to waver. During the whole period of heating, the water, from its increasing temperature, has been always becoming lighter, so that heat communication by convection with the water below has been entirely suspended during the whole period. The heating of