

are obtained, I have developed more in detail some special cases of interest.

For the convenience of the reader, I have in § 1 briefly recapitulated the principal parts of the two papers above quoted. In § 2 I have given, at all events, a first sketch of a general theory of multiple contact with quadrics; in § 3 the particular cases of three-, four-, five-, and six-pointic contact are discussed; and in § 4 some conditions for the existence of points of four-, five-, six-pointic single (*i. e.* not multiple) contact are established.

Thus far the investigation concerns the contact of quadrics only with other surfaces. The concluding part of the paper is concerned with the corresponding problem for cubics, in which case conditions of possibility do not arise for simple or two-pointic contact, but are first met with for three-pointic contact. The conditions in question, with some of their consequences, are here given; and their complexity will perhaps be sufficient justification for not pursuing the subject further in this direction.

VII. "On the Theory of the Solution of a System of Simultaneous Non-linear Partial Differential Equations of the First Order."

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(Abstract.)

Given an equation of the form

$$z = \phi(x_1, x_2, \dots, x_{n+m}, a_1, a_2, \dots, a_n),$$

we obtain by differentiation with respect to each of the $n+m$ independent variables x_1, x_2, \dots, x_{n+m} , and elimination of the n arbitrary constants a_1, a_2, \dots, a_n , a system of $m+1$ non-linear partial differential equations of the first order. Of this system the given equation may be said to be a "complete primitive."

Conversely, given a system of non-linear partial differential equations of the first order, it is proposed to determine the conditions which must be satisfied in order that the system may admit of a complete primitive, and also to examine what kind of solution, if any, exists when the conditions above referred to are not satisfied.

The late Professor Boole has given an elegant method of treating a system of linear partial differential equations of the first order; but the present memoir relates to a more general system, which appears not to have been hitherto considered, viz. to a non-linear system of partial differential equations. This is here discussed in the two cases—first, when the dependent variable z is not explicitly involved in the proposed system; and, secondly, when z is explicitly involved in the system, the solution of this last case being made to depend upon that of the first-mentioned one.