and a current, whose intensity, C, was measured by the tangent galvanometer included in the circuit, was sent through the helix first in one direction and then in the other, and the plane of polarization observed. Half the difference of the readings was the rotation produced by the current.

If we call θ this rotation expressed in circular measure, and define Verdet's constant as the rotation which a unit current in a unit coil could produce in unit of length of distilled water, we have

$$\omega = \frac{\theta}{NC}.$$

The result of the series of experiments made was to obtain for ω the value

$$\omega = (10^{-7})\,4{\cdot}49 \text{ centimetre-gramme-seconds.}$$

Its dimensions obviously are the reciprocal of those of current, viz.

$$[\omega] = [L^{-\frac{1}{2}} M^{-\frac{1}{2}} T].$$

If we put our result in a slightly different form we may say that,

If plane polarized light passes through distilled water, and the magnetic potential of the water at any two points in the path of the ray differs by unity, then the plane of polarization will be rotated between those points $4\frac{1}{2}$ ten-millionths of a unit of circular measure.

Cavendish Laboratory, Cambridge,
April 30, 1875.

V. "On Rolling-Friction." By Professor Osborne Reynolds. Communicated by Dr. Balfour Stewart, F.R.S. Received May 24, 1875.

(Abstract.)

The motion of a roller or wheel on a surface is always attended with resistance. Coulomb made some experiments with wooden rollers on a wooden plane, from which he deduced two laws, viz. that the resistance is proportional to the weight of the roller, and inversely proportional to its diameter. These laws have since been found to apply to other substances, a different coefficient being used in each case. Beyond this, however, nothing appears hitherto to have been ascertained as regards the nature of this resistance to rolling. The source from which it springs does not appear to have been made the subject of investigation.

Some time ago it occurred to the author that it was probable that the deformation of the surface of the roller and of the plane, which must take place at the point of contact, would affect the distance which the roller would advance in turning through a certain angle*. The pressure of the roller on the plane causes a certain temporary indentation and

* The Engineer, 27th Nov., 1874.

lateral extension in the latter, so that in passing from one point to another the roller does in truth pass over a greater extent of surface than the distance between these points. A simple experiment was sufficient to verify the truth of this conclusion. An iron roller 18 inches in circumference was found to roll through something like $\frac{3}{4}$ inch less than a yard in two complete revolutions when rolling on a plate of india-rubber. The softness of the india-rubber suffered the roller to indent it considerably; and hence it might be expected that the effect would be much more apparent than when the roller was rolling on iron or any hard material. At the same time there is doubtless a certain amount of indentation in this latter case; and this will probably cause a similar alteration in the distance rolled through, although too small to allow its being measured.

This falling off from what may be called the geometrical distance, suggested an explanation of the resistance to rolling, namely, that the extension of the surface or surfaces at the point of contact causes the one surface to slide over the other; and this sliding is accomplished against friction. In this way we should expect to find the resistance to rolling greatest under those circumstances in which the sliding is greatest, *i. e.* where the indentation is greatest; and so far it is in accordance with Coulomb's laws. In the case of india-rubber, we find the slipping is very large; and hence we should expect the resistance to rolling to be large also; and accordingly we find it so, for it is more than ten times as great as when the roller is on an iron plane. This very great resistance which india-rubber causes to rolling appears not to have previously caught attention; and yet it is the natural explanation of the invariable failure which has attended the numerous endeavours which have been made to use this material for the tires of wheels.

This idea, that the resistance to rolling is due to the friction between the surfaces sliding at the point of contact, naturally leads to the conclusion that it must depend on the coefficient of friction between these surfaces, and that we might expect to diminish the resistance by using oil or any other means of reducing the coefficient of friction. This was the author's first impression. Experiments, however, showed that the effect of oiling the surface, although it did generally reduce the resistance, was very small; and sometimes it appeared to act in the reverse manner, and increase the resistance. This conclusion or surmise was therefore wrong; and the cause of the error was not far to seek. It consisted in having overlooked the fact that friction not only opposes the sliding of the one surface over the other, but also prevents it to a considerable extent, and thus modifies the deformation which would otherwise take place; so that any diminution in the coefficient of friction is attended with an increase in the extent of slipping, which tends to balance the advantage gained by the reduced coefficient.

The truth of this view derives independent support from a circumstance remotely connected with rolling-friction, of which it furnishes an

explanation. When the roller rests on a horizontal surface and is very slightly disturbed, it does not move off, but oscillates backwards and forwards. This happens on all kinds of elastic surfaces; on soft india-rubber the oscillations are both large and continue for some time. Now if the deformation in the surface of the rubber were complete, there would be no tendency to bring the roller back; but since, owing to friction, the india-rubber, under the advancing side of the roller, is prevented from extending while that under the other side is prevented from contracting, there will exist a state of constraint from which the surface is endeavouring to free itself by forcing the roller back.

Besides the relative softness of the materials, the curvature of the roller will affect the lateral extension both of the roller and the plane at the point of contact, so that if the roller and the plane were of the same material there would still be slipping. This would not be the case, however, between two wheels of the same diameter and material rolling in contact.

Such is a short sketch of the subject of the paper, a considerable part of which is devoted to the examination and illustration of the exact manner in which the deformation at the point of contact occurs, and the influence of friction upon it. The latter part of the paper contains an account of numerous experiments, and their results, which were undertaken as part of this investigation.

The first series of experiments relate to the resistance which an iron roller experiences on surfaces of different hardness. Cast iron, glass, brass, boxwood, and india-rubber were tried. Extreme care was taken to make the roller and the surfaces true; and this was so far successful that on cast iron the roller would roll in either direction when the surface had an inclination of one in five thousand, or, roughly, a foot in a mile. Comparing the different surfaces, we see that the resistance increases with the softness, although apparently not in the simple proportion; on boxwood the resistance is nearly double as great as on the harder surfaces, and on india-rubber from six to ten times as great.

The second series of experiments were to ascertain the actual extent of slipping on india-rubber, both with a cast-iron roller and also with an india-rubber tire glued on to the roller, and rolled on hard surfaces and on plates of india-rubber of different thicknesses.

These experiments bear out the arguments expressed in the first part of the paper; in fact the arguments were based on the experiments. There is no intention to imply that the whole of the resistance to rolling is in all cases due to the causes already mentioned. Under ordinary circumstances the irregularities of the surfaces and the crushing of the material beneath the roller are the chief causes. And, besides these, two other causes are discussed in the paper as having been brought to light by the experiment, viz. the communication of heat between the compressed material and that which surrounds it, which prevents the material im-

mediately expanding to the same volume as it previously occupied, and the viscosity of the material, which also renders it slow to expand. Both these causes are, however, rather connected with the effect of the speed of the roller on the resistance than with the residual resistance, which, so far as the surfaces are perfectly true and perfectly hard, appears to be due to the friction which accompanies the deformation, and is hence called *rolling-friction.*

No attempt has yet been made to investigate the laws of rolling-friction, although the author hopes to continue the investigation in this direction as soon as he has obtained the necessary apparatus.

At the end of the paper attention is called to certain phenomena connected with railway-wheels, which it is thought now, for the first time, receive an explanation. Thus the surprising superiority of steel rails over iron in point of durability is explained as being due as much to the fact that their hardness prevents the wearing-action, *i. e.* the slipping, as that it enables them better to withstand the wear. Also the slipping beneath the wheel explains the wear of the rails in places where brakes are not applied; and the severe lateral extension beneath the wheel is thought to explain the scaling of wrought-iron rails.

VI. "On Multiple Contact of Surfaces." By WILLIAM SPOTTISWOODE, M.A., Treas. R.S. Received May 24, 1875.

(Abstract.)

In a paper "On the Contact of Quadrics with other Surfaces," published in the Proceedings of the London Mathematical Society (May 14, 1874, p. 70), I have shown that it is not in general possible to draw a quadric surface V so as to touch a given surface U in more than two points, but that a condition must be fulfilled for every additional point. The equations expressing these conditions, being interpreted in one way, show that two points being taken arbitrarily, the third point of contact, if such there be, must lie on a curve, the equation whereof is there given. The same formulæ, interpreted in another way, serve to determine the conditions which the coefficients of the surface V must fulfil in order that the contact may be possible for three or more points taken arbitrarily upon it; and, in particular, the degrees of these conditions give the number of surfaces of different kinds which satisfy the problem.

In another paper, "Sur les Surfaces Osculatrices" (Comptes Rendus, 6 Juillet, 1874, p. 24), the corresponding conditions for the osculation of a quadric with a given surface are discussed.

In the present paper I have regarded the question in a more general way; and having shown how the formulæ for higher degrees of contact