mediately expanding to the same volume as it previously occupied, and the viscosity of the material, which also renders it slow to expand. Both these causes are, however, rather connected with the effect of the speed of the roller on the resistance than with the residual resistance, which, so far as the surfaces are perfectly true and perfectly hard, appears to be due to the friction which accompanies the deformation, and is hence called *rolling-friction*.

No attempt has yet been made to investigate the laws of rolling-friction, although the author hopes to continue the investigation in this direction as soon as he has obtained the necessary apparatus.

At the end of the paper attention is called to certain phenomena connected with railway-wheels, which it is thought now, for the first time, receive an explanation. Thus the surprising superiority of steel rails over iron in point of durability is explained as being due as much to the fact that their hardness prevents the wearing-action, *i. e.* the slipping, as that it enables them better to withstand the wear. Also the slipping beneath the wheel explains the wear of the rails in places where brakes are not applied; and the severe lateral extension beneath the wheel is thought to explain the scaling of wrought-iron rails.

VI. "On Multiple Contact of Surfaces." By WILLIAM SPOTTISWOODE, M.A., Treas. R.S. Received May 24, 1875.

(Abstract.)

In a paper "On the Contact of Quadrics with other Surfaces," published in the Proceedings of the London Mathematical Society (May 14, 1874, p. 70), I have shown that it is not in general possible to draw a quadric surface V so as to touch a given surface U in more than two points, but that a condition must be fulfilled for every additional point. The equations expressing these conditions, being interpreted in one way, show that two points being taken arbitrarily, the third point of contact, if such there be, must lie on a curve, the equation whereof is there given. The same formulæ, interpreted in another way, serve to determine the conditions which the coefficients of the surface V must fulfil in order that the contact may be possible for three or more points taken arbitrarily upon it; and, in particular, the degrees of these conditions give the number of surfaces of different kinds which satisfy the problem.

In another paper, "Sur les Surfaces Osculatrices" (Comptes Rendus, 6 Juillet, 1874, p. 24), the corresponding conditions for the osculation of a quadric with a given surface are discussed.

In the present paper I have regarded the question in a more general way; and having shown how the formulæ for higher degrees of contact

are obtained, I have developed more in detail some special cases of interest.

For the convenience of the reader, I have in § 1 briefly recapitulated the principal parts of the two papers above quoted. In § 2 I have given, at all events, a first sketch of a general theory of multiple contact with quadrics; in § 3 the particular cases of three-, four-, five-, and six-pointic contact are discussed; and in § 4 some conditions for the existence of points of four-, five-, six-pointic single (*i. e.* not multiple) contact are established.

Thus far the investigation concerns the contact of quadrics only with other surfaces. The concluding part of the paper is concerned with the corresponding problem for cubics, in which case conditions of possibility do not arise for simple or two-pointic contact, but are first met with for three-pointic contact. The conditions in question, with some of their consequences, are here given; and their complexity will perhaps be sufficient justification for not pursuing the subject further in this direction.

VII. "On the Theory of the Solution of a System of Simultaneous Non-linear Partial Differential Equations of the First Order." By E. J. Nanson. Communicated by Prof. Cayley, F.R.S. Received June 5, 1875.

(Abstract.)

Given an equation of the form

$$z = \phi\,(x_1, x_2, \ldots . x_{n+m}, a_1, a_2, \ldots a_n),$$

we obtain by differentiation with respect to each of the $n+m$ independent variables $x_1, x_2, \ldots . x_{n+m}$, and elimination of the n arbitrary constants $a_1, a_2, \ldots a_n$, a system of $m+1$ non-linear partial differential equations of the first order. Of this system the given equation may be said to be a "complete primitive."

Conversely, given a system of non-linear partial differential equations of the first order, it is proposed to determine the conditions which must be satisfied in order that the system may admit of a complete primitive, and also to examine what kind of solution, if any, exists when the conditions above referred to are not satisfied.

The late Professor Boole has given an elegant method of treating a system of linear partial differential equations of the first order; but the present memoir relates to a more general system, which appears not to have been hitherto considered, viz. to a non-linear system of partial differential equations. This is here discussed in the two cases—first, when the dependent variable z is not explicitly involved in the proposed system; and, secondly, when z is explicitly involved in the system, the solution of this last case being made to depend upon that of the first-mentioned one.