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January 6, 1876.

Dr. J. DALTON HOOKER, C.B., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

I. "On the Expansion of Sea-water by Heat." By T. E. Thorpe, Ph.D., and A. W. Rücker, M.A. (Fellow of Brasenose College, Oxford), Professors of Chemistry and Physics in the Yorkshire College of Science. Communicated by W. B. Carpenter, M.D., LL.D., F.R.S. Received November 12, 1875.

(Abstract.)

The extensive contributions which have recently been made to the physical history of the ocean have shown the desirability of exact knowledge of the relations of sea-water to heat. We have accordingly thought it worth while to make observations in order to determine the law of the thermal expansion of sea-water.

The only attempt hitherto made to solve this problem which can lay any real claim to consideration is due to the late Professor Hubbard, of the United States National Observatory. The results of his investigation are contained in Maury's 'Sailing Directions,' 1858, vol. i. p. 237.

Muncke, nearly fifty years ago, determined the expansion of an artificial sea-water at various temperatures between 0° and 100° C.; but our confidence in the results as applicable to natural sea-water is affected by the circumstance that the solution was prepared from data furnished by the imperfect analyses of Vogel and Bouillon La Grange.

The observations of Despretz were confined to temperatures below 13°·27, as the main object of his inquiry was the determination of the point of maximum density of sea-water. The subsequent investigations of Neumann and Rossetti were equally limited, as they were undertaken with the same view.

The water used in our observations was collected from the Atlantic, in lat. 50° 48′ N. and long. 31° 14′ W.; and its specific gravity at 0° C., compared with distilled water at the same temperature, was found by the bottle to be 1·02867.

The method of experiment was precisely the same as that already employed by one of us in determining the expansion of the liquid chlorides of phosphorus. It was essentially that already used by Kopp and Pierre; *i. e.* the expansion was observed in thermometer-shaped vessels (so-called dilatometers), graduated and accurately calibrated.

Three of these instruments and two sets of thermometers were employed. The latter were made by Casella; the length of a degree in different instruments varied between 9 and 13 millims.; they had been compared (the one set directly, the other indirectly) with Kew Standards.

Three perfectly independent sets of observations were made with the water in the state in which it was collected; and as Mr. Buchanan, of H.M.S. 'Challenger,' has found that the specific gravities of different sea-waters lie between the extreme values 1·0278 and 1·0240, and since, in order to be of value in the investigation of the physical condition of the ocean, the observations on their values and the formulæ of reduction ought to be correct to the fourth decimal place, we diluted quantities of our sea-water with distilled water, so as to have specimens of approximately the specific gravities of 1·020 and 1·025; and we concentrated a third quantity by evaporation until its specific gravity was increased to 1·033, and made two series of independent observations on the expansion of each solution.

As we wished to confine ourselves to circumstances to which sea-water s naturally exposed, we did not carry on our experiments at temperatures higher than 40° C.

Empirical formulæ were calculated to express the results of each series of observations; and in the original paper full details of the observations are given, together with Tables showing the agreement between the

calculated and observed results, and also (after the necessary corrections and reductions have been made) between the volumes calculated from the formulæ from different series of observations on the same solutions.

Finally, a general formula of the form

$$v=\phi(t)+\psi(t)f(s)$$

was found, giving the relation between the volume (v), temperature (t), and specific gravity at 0° C. (s) of any solution of the same composition of sea-water the specific gravity of which at 0° C. lies between 1·020 and 1·033, the volume at the same temperature being taken as unity; in which expression

$$\phi(t)=1+\cdot00008097t+\cdot0000049036t^2-\cdot000000012289t^3,$$
$$\psi(t)=-10^{-5}(\cdot5509t-\cdot020198t^2-\cdot00033276t^3),$$
and $$f(s)=11\cdot95-940(s-1\cdot02)^*.$$

In the original paper we show that if σ be the specific gravity at any temperature t of a solution the specific gravity of which at 0° C. is s, we may without sensible error assume $\frac{d\sigma}{ds}$ to be constant; whence, by means of the above formula, we are able to give in the following Table all the data necessary for calculating the specific gravity of sea-water of any degree of salinity at any temperature between 0° and 36°. Column II. contains the specific gravities at the temperatures given in Column I. of a solution the specific gravity of which at 0° C. is 1·02000; Column III. contains the numbers which must be subtracted from those in Column I. for each increase of 0°·1 over the temperatures opposite to which they are placed; and Column IV. the numbers which must be added for each increase of ·00001 of the specific gravity of the solutions at zero. At the heads of Columns III. and IV. are the numbers of ciphers which must be prefixed to the figures written in them in the unit place.

In order to facilitate the use of the Table, we subjoin directions for its application in the form of rules, and give a couple of examples.

I. Given the specific gravity of a sample of sea-water at any temperature t, to find it at 0° C.:—Look out in Column I. the figure giving the number of entire degrees of the temperature; multiply the corresponding number in III. by the fraction by which the observed temperature exceeds that number, and subtract the result from the corresponding number in Column II. Subtract the difference from the observed specific gravity, and divide the number so obtained by that corresponding to the observed temperature in Column IV. (without prefixing the ciphers at the top of the column); add the quotient to 1·02000, and the sum will be the specific gravity required.

Example I. Specific gravity observed at 18°·5 C.=1·02475. Number

* The numerical constants involved in the above formula are given in the forms in which they were, for facility of calculation, determined. The expression can of course be easily transformed to the simpler form, $v=F_1(t)+sF_2(t)$.

Temperature.	Specific gravity.	Proportional parts for 1° C.	Proportional parts for ·00001 increase in spec. grav.	Temperature.	Specific gravity.	Proportional parts for 1° C.	Proportional parts for ·00001 increase in spec. grav.
		·0000	·0000			·0000	·0000
0°	1·02000	3	1	19°	1·01740	25	0·944
1	1·01997	4	0·995	20	1·01715	25	0·943
2	1·01993	5	0·990	21	1·01690	26	0·941
3	1·01988	6	0·986	22	1·01664	27	0·940
4	1·01982	8	0·932	23	1·01637	28	0·938
5	1·01974	9	0·879	24	1·01609	29	0·937
6	1·01965	11	0·975	25	1·01580	29	0·935
7	1·01954	12	0·972	26	1·01551	30	0·934
8	1·01942	13	0·969	27	1·01521	30	0·932
9	1·01929	14	0·966	28	1·01491	31	0·930
10	1·01915	15	0·963	29	1·01460	32	0·928
11	1·01900	17	0·961	30	1·01428	32	0·925
12	1·01883	17	0·958	31	1·01396	32	0·922
13	1·01866	19	0·956	32	1·01364	33	0·919
14	1·01847	20	0·954	33	1·01331	33	0·915
15	1·01827	21	0·952	34	1·01298	33	0·912
16	1·01806	21	0·950	35	1·01265	34	0·908
17	1·01785	22	0·948	36	1·01231	34	0·903
18	1·01763	23	0·946				

opposite 18 in Column III. is ·00023, which multiplied by ·5 equals ·00011; and

$$1{\cdot}01763-{\cdot}00011=1{\cdot}01752.$$

Subtract this from the observed specific gravity,

$$1{\cdot}02475-1{\cdot}01752={\cdot}00723.$$

Divide by ·945 (the number corresponding to 18·5), and the quotient is ·00765, which added to 1·02000 gives 1·02765 as the specific gravity at 0° C.

Example II. Specific gravity observed at 15° C. =1·02570.

$$1{\cdot}02570-1{\cdot}01827={\cdot}00743,$$

and

$$\frac{{\cdot}00743}{{\cdot}952}={\cdot}00780.$$

Therefore specific gravity at 0° C. =1·02780.

We next discuss the discrepancies which occur between our own results and those of Professor Hubbard; and we point out various circumstances in the methods employed in making and reducing the latter observations which appear to us to explain in a great measure the divergences which exist.