XXVIII. "On the Forms assumed by Drops of Liquids falling vertically on a horizontal Plate." By A. M. WORTHINGTON. Communicated by R. B. CLIFTON, M.A., F.R.S., Professor of Experimental Philosophy in the University of Oxford. Received May 17, 1876.

My attention was first drawn to the subject of this paper last spring, by Mr. H. F. Newall, of the Rugby School Natural-History Society, who showed me the mark made by drops of water and mercury falling on a smoked glass plate, the lampblack being swept away in concentric circles and radial striæ. The patterns thus left were generally symmetrical and beautiful, and varied with the height of fall of the drop. I have since sought to investigate the cause of these appearances in Prof. Helmholtz's laboratory in Berlin.

My first care was to obtain a series of what I will call "patterns" left by drops of various liquids, of various and measured diameters, falling from various and measured heights on horizontal smoked glass plates.

I experimented with water and mercury as types of liquids which do not wet the lampblack, and with alcohol as a type of those which do. Drops of a constant size were obtained with water and alcohol by allowing the liquid to fall, drop by drop, from the end of a vertical capillary tube. In the case of mercury, a narrow vertical glass tube was provided at its upper end with a closed caoutchouc tube; the pressure of the hand on this expelled the air, and a column of mercury, about 60 millims. in length, was drawn up by suction to be expelled when required, drop by drop. The caoutchouc tube was found preferable to a hollow caoutchouc ball, as by wrapping it round the finger the internal volume could be diminished more gradually and regularly than with the ball, and thus the danger of expelling more than the normal-sized drops, whose diameter should depend only on the internal diameter of the tube, was diminished. This danger was, however, only completely avoided in later experiments by using an inclined glass tube, to the lower end of which an open caoutchouc tube was attached, whose upper end could be raised or lowered at pleasure, and the mercury in the U-shaped tube thus formed brought to the level of the mouth of the glass tube, and made to fall over drop by drop. The drops thus obtained were found to be very constant in magnitude. The diameters of the drops were calculated from the weight of 10, 15, 20, or 30. The height of fall was taken as the distance between the plate and the bottom of the vertical or the lower edge of the inclined tube. The glass plates were smoked in the flame of a stearine candle: in cases where the height of fall was great, the adhesion of the smoke was increased by dipping the plate in petroleum or turpentine, and gently wiping before smoking. I thus obtained a large number of patterns; and examination of them showed the extreme difficulty of explaining

from them alone the movements of the liquid which gave rise to them, and the probable uselessness of seeking, while ignorant of these movements, a quantitative connexion in the case of any given liquid between the size or number of rings of the pattern and the constants at my disposal, viz. the height of fall and diameter of the drop.

I preferred to endeavour, by means of the electric spark, to see the forms through which the drop passed in the act of making its pattern. In this I have been tolerably successful.

The principle of the method was to make the drop fall in comparative darkness on the plate, and at the moment of incidence itself to break an electrical circuit, by which means a spark was produced in its neighbourhood sufficiently bright to illuminate the drop and enable it to be seen in the form which it had at that instant; to see the consecutive stages it was necessary to postpone the appearance of the spark for excessively short but increasing intervals of time after the first contact of the drop. The accompanying sketch of the apparatus will explain the details of the method.

Fig. 1.

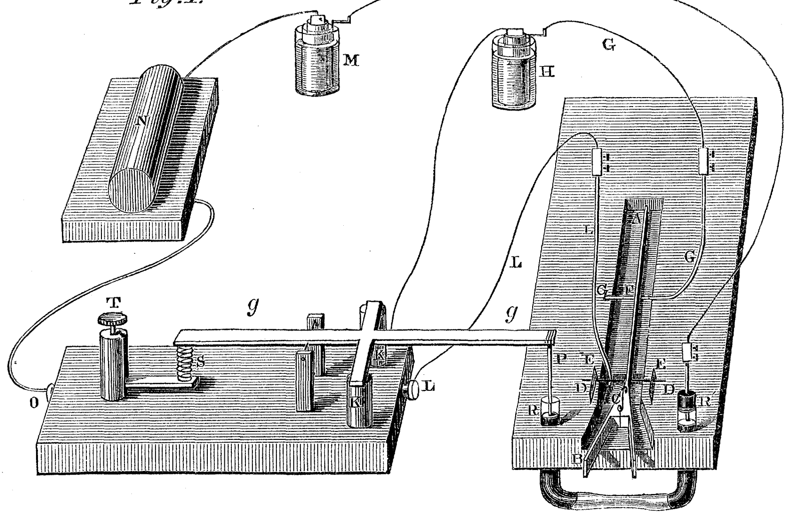
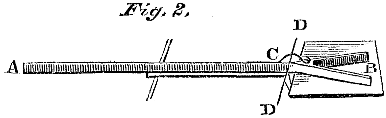


Fig. 2.



A B (fig. 1) is a light wand of cedar wood with a forked end; it is $13\frac{1}{2}$ millims. long, 1 millim. wide, 4 millims. deep. The end B bears the glass plate on which the drop falls, the plate being kept in its place by a spring C, as shown in fig. 2. This wand is fixed on a horizontal

axis D D, made of a fine sewing-needle working in small triangular holes cut in the copper plates E, E; underneath the wand and along it is bound to it a platinum wire, one end of which is bent vertically under the axis, and dips into mercury contained in a hollow in the deal board in which E, E are fixed. The other end of the platinum wire rests, when the wand is horizontal, on a strip of platinum foil F, wound round a copper wire G G. A Bunsen's element (H) sends a current through the coils K, K of a relay, along the wire L L to the mercury in the little trough, along the platinum wire, platinum foil, and wire G G back to H.

The current which passes through the relay comes from 1 or 2 Bunsen's elements, M, through the inducing spiral of a Ruhmkorff's coil, N, into the relay at O, down the platinum wire P, which, when the iron bar *g g* is held down by the magnet, dips into mercury contained in the **U** tube R R′, so that the current passes out of the mercury at R′ up a thick immersed copper wire and back to the pile M.

The plate of thin glass having been placed on B, the balance of the wand is so adjusted, with a small counterpoise at the other end A, that the slightest downward pressure, even the breath of the observer, is sufficient to raise the end A, and to break the connexion between the platinum wire and foil. Accordingly the moment the drop touches the plate, the current of the pile H is broken at F, the core of K K ceases to be a magnet, and the point of the platinum wire P is pulled out of the mercury at R by the force of the spring S, and the strong primary spark obtained at the surface of the mercury is sufficient to illuminate the drop on the plate. The stage at which it was required to see the drop could be altered at pleasure by altering the depth of immersion of the platinum wire at R, which was done by plunging the connecting wire at R′ more or less deeply into the mercury in that branch of the **U** tube. This gave a rough adjustment; a finer was obtained by regulating by means of the screw T the tension of the spring S, and so changing the rate of withdrawal of the platinum point.

To secure the complete illumination of the drop and plate, the end R of the **U** tube and the plate were surrounded with a white cardboard box, with slits to allow of the motion of the wand, the wire P, and the admission of the drop, and open in front, so that the plate could be seen. Complete darkness was found by no means necessary for the experiments; light just sufficient to allow the plate to be seen and the eye easily directed on it was found the most convenient. The results I have obtained have been with mercury and milk. Mercury, from its high reflecting power, is easy to see; water, from its transparency, even when coloured with indigo, very difficult to see; and I substituted milk, which has the advantages of appearing white on a black ground, of being semitransparent, and of showing blue or darkish where it is spread thinly over the black plate, and thus allowing an estimate to be made of the relative thickness of the drop in various places, and especially the advan-

tage of diffusing light through its mass. The mercury, on the contrary, allows no light to penetrate its interior, and for this reason the form of the drop is less easy to ascertain; for the contrast between the brightly illuminated raised or convex parts and the parts that are hollowed or in shadow is so great that it is often uncertain whether the dark portion of the figure is black plate or unilluminated mercury. This was at first a serious cause of error, as in some earlier arrangements which I tried, the light of the spark was reflected laterally from some distance by means of a concave mirror, when the deception was very great, as will be seen by reference to fig. IIα, Set 6. By means of the cardboard box, however, from the surface of which light was diffused from all directions on the plate, and by having the spark close to the plate, I became quite certain of the figures. Owing to the appreciable amount of time required for the demagnetization of the electromagnet, I was not able to obtain figures of the first portion of the spreading out of the drop.

Owing also to accidental causes difficult of control, such as the variations of the contact at F, the variations of the pressure with which the iron bar of the relay was initially pressed down by hand on the ends of the electromagnet, and to the oxidation and irregularities of the surface of the mercury at R, the time of appearance of the spark after the first contact of the drop varied slightly, so that the stages seen with a given depth of immersion of the wire P and a given tension of the spring were not always the same, but varied between narrow limits. But a little judgment enables the observer to tell whether the stage seen is before or after the mean stage most frequently seen with that arrangement of level of mercury and velocity of withdrawal of the platinum wire.

Sets 1, 2, and 4 are figures of milk;

Sets 5, 6, 7, 8, and 9 of mercury.

Explanatory notes are attached where necessary.

The figures suggest a few general observations.

The existence of the radial arms is a deviation from perfect symmetry of figure round the vertical axis; and some slight initial disturbance of symmetry must be required to determine the formation of arms.

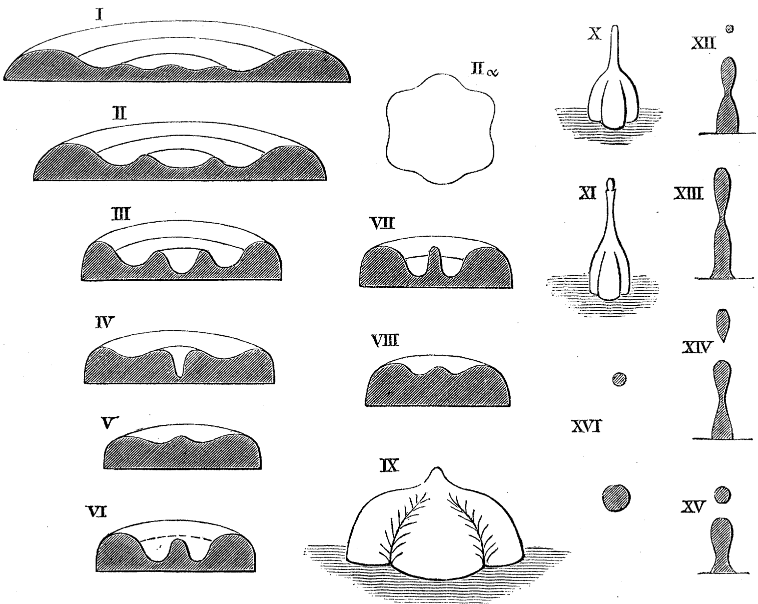
Such a disturbing cause is always at hand in the irregularities of the surface of the glass or of the layer of smoke, which allow the drop to spread with less frictional resistance in one direction than in another, and also in the oscillations of the drop about its mean figure while it falls through the air. The occurrence of such a figure as III, Set 6, confirms this view, as it is an approximation to perfect symmetry when the disturbing cause has been probably very small.

The fact, too, that a very slight dirtiness of the plate was sufficient to cause great irregularities of figure is confirmative of this, as is also the fact that the tendency to form radial arms increases with the height of fall.

In seeking the explanation of the fact that the arms which are first

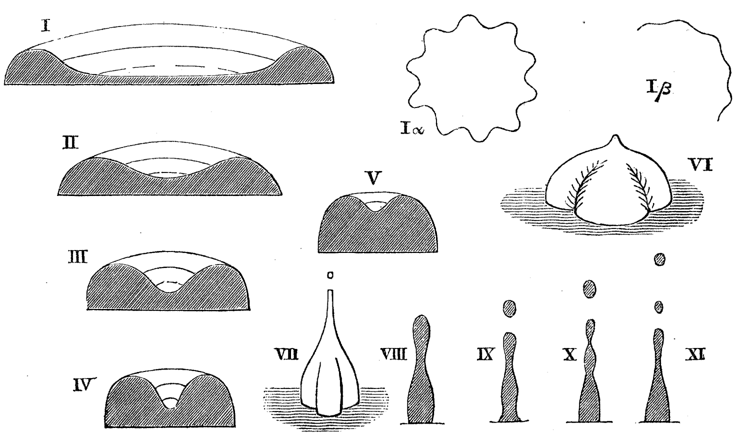
seen when the drop is nearly at its maximum spread contract more slowly than the central part which joins them, we may, I think, leave

Set 1.



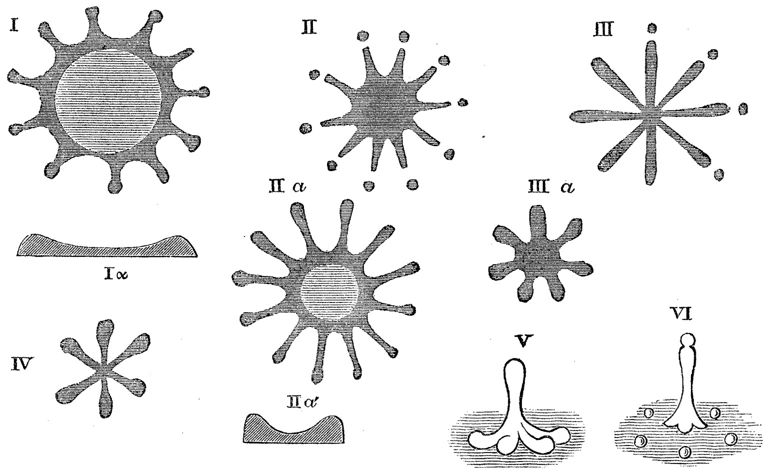
out of consideration the action of gravity on the liquid contained in them, as the duration of their existence is so short that this will not have time to produce an appreciable change of form. The arms, con-

Set 2.



sidered as free cylinders of liquid, will be in equilibrium till the length bears a certain proportion to their diameter, after which they will tend to split each into a row of drops.

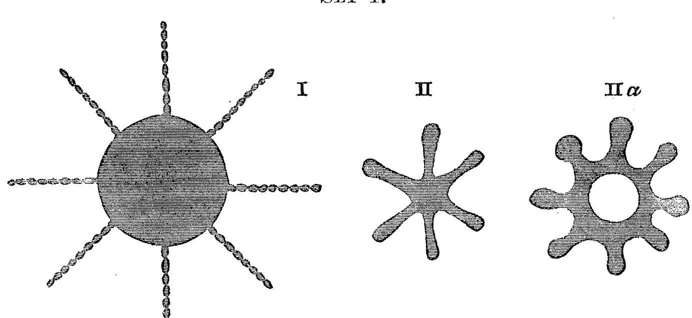
SET 3.



M. Plateau, in his 'Statique expérimentale des Liquides,' has shown that a cylinder of mercury lying on a horizontal plate breaks into drops, whose number depends on the friction between the liquid and the plate.

The pressure of the convex surface at the end of the cylinder will tend to drive the liquid into the cylinder and diminish its length while increasing its thickness, but only so long as the sum of the reciprocals of the principal radii of curvature at any point of the convexity are greater than the reciprocal of the radius of the cylinder.

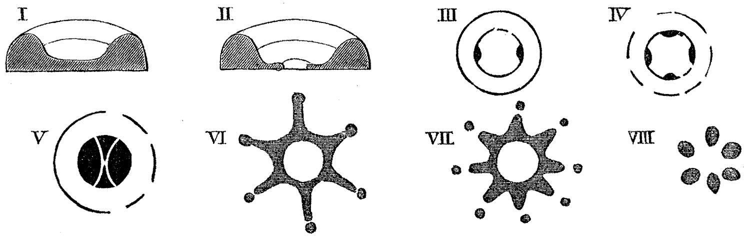
SET 4.



This may, however, always be the case, for the thickness and radius of the cylinder is continually increased by the supply of liquid from the

centre, which keeps contracting under the pressure due to the curvature of its limb.

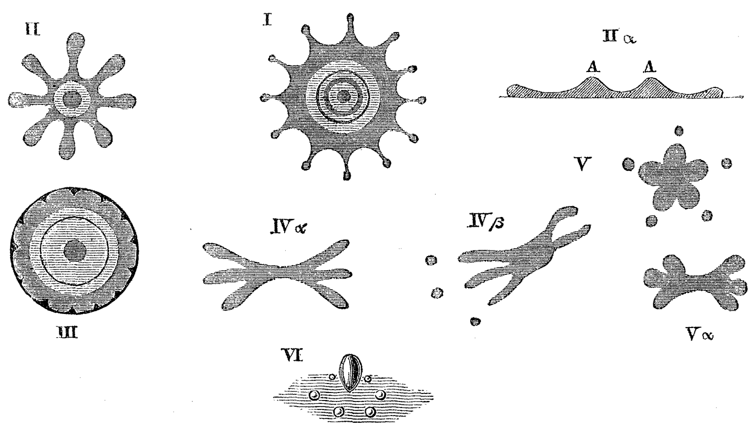
Set 5.



Thus as the centre contracts the arm increases in length.

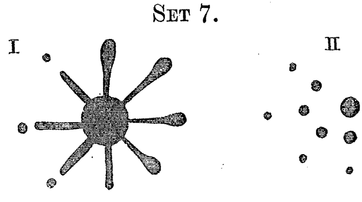
This increase in length causes, after a certain point, a tendency to split into drops. This tendency is counteracted by the thickening of the cylinder from the injection of liquid at the inner end.

Set 6.



This thickening gives efficiency to the curvature of the end of the cylinder, and the arm contracts slowly.

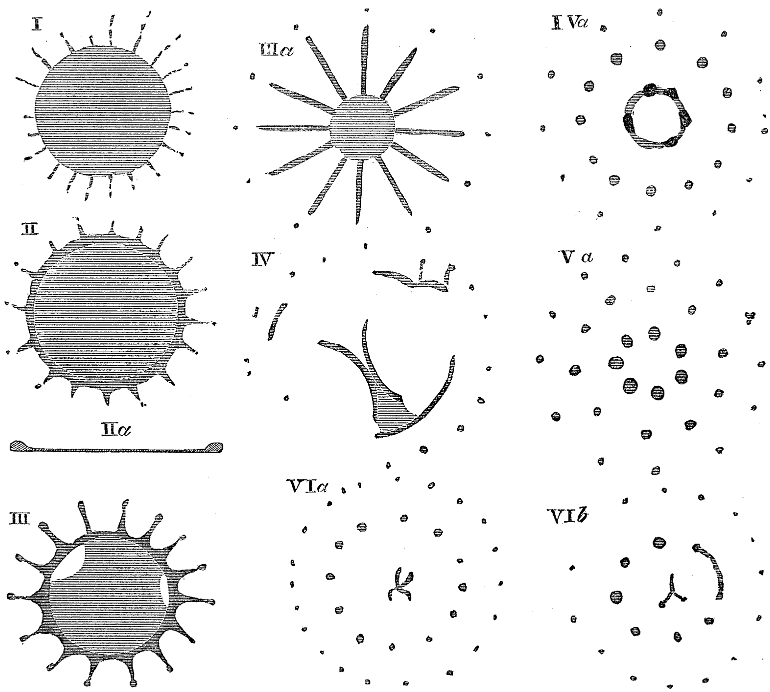
Set 7.



This accounts for our seeing arms whose length is very great compared with their diameter, but which do not succeed in splitting into drops.

When the thickening does not keep pace with the lengthening of the cylinder, drops will split off. That in such case the arms split at the ends rather than in the middle is, I think, to be attributed to the fact that the thickening spreads from the centre to the ends, which receive their additional liquid later.

SET 8.



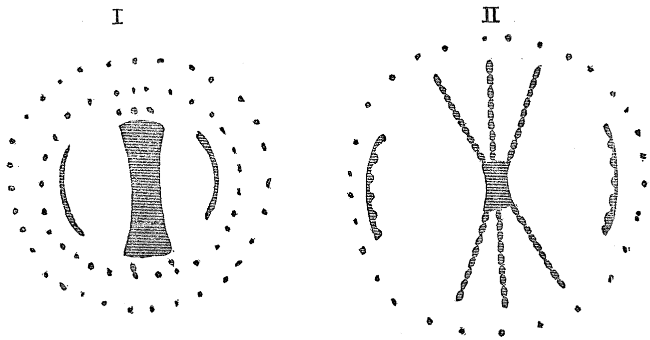
It is perhaps worthy of remark that the earliest stage I was able to see was nearly the maximum spread for every height of fall, for both milk and mercury. As my adjustment for seeing the first stage was nearly the same in all cases, I have reason to assume that the time which elapsed between the contact of the drop and the appearance of the spark was nearly the same. Hence the conclusion that the drop requires the same time to reach its maximum spread whatever the diameter of the spread; or, in other words, the oscillation follows the law of a simple elastic force. The limits of error are, however, here wide.

The difficulty of counting the arms was considerable. Most frequently there were 6, sometimes 8 or 12, sometimes more; and I am inclined to think their number was always even.

The number of the drops left from the first spreading out in the case of higher falls was hard to count. In the case of mercury on unsmoked glass, these were left on the plate in a more or less complete circle; and

their number, when the height of fall was 150 millims., was generally 24; but two or three having sometimes run together, made the estimate

SET 9.



uncertain. Incidental disturbing causes seem to alter the number of arms, and to determine the tearing, regularly, irregularly, or not at all, of the central patch in the case of higher falls.

It may be objected that any results with so variable a substance as milk must be unsatisfactory. My object, however, was to study the type of form before proceeding to quantitative measurements. For these I am not likely for some time to have time or opportunity, which is my excuse for presenting my paper as it is, in the hope that other and abler observers may be attracted to the investigation. To the kindness of Prof. Helmholtz, in giving me help and suggestions, much of the success of my experiments is due.

EXPLANATION OF THE FIGURES.

The shaded parts throughout are sections.

SET 1.

Milk on smoked glass. Height of fall 37 millims. Diameter of drop 6·012 millims. The shaded parts are vertical central sections, seen at an angle of about 30°.

I. Rises sometimes wavy, as in IIα.
III. Smaller; central hollow deeper; edge always wavy.
IV. The central hollow now fills up.
IX. The wavy edge visible in the form of lobes as the drop contracts.

SET 2.

Milk on smoked glass. Diam. of drop 6·012 millims. Height of fall 50 millims.

I. More spread out than from lower height. Edge wavy, as Iα or Iβ. Irregular on unevenly smoked glass.
VII. The small detached drop flies upwards, while the remainder rises and splits again as in the remaining consecutive phases.

SET 3.

Milk (boiled) on smoked glass. Diam. of drop 6·012 millims. Height of fall 100 millims.

I. Number of arms uncertain.

Iα. Vertical section of the same between the arms.

IIα & IIIα are phases of an alternative course, sometimes taken instead of II & III.

IV. This phase is consecutive on either III or IIIα.

V. The centre rises before the arms have come in.

SET 4.

Milk on smoked glass. Diam. of drop 6·012 millims. Height of fall 200 millims.

I. Very much spread out; arms beaded, tending to split into rows of drops.

IIα is an alternative phase sometimes seen instead of II; the centre having been torn, the arms contract into the ring thus formed.

Falling from a height of 280 millims. on a smoked glass plate, the same sized drop of milk went through phases similar to those of mercury from 150 millims. (see Set 8).

The succeeding figures represent the forms assumed by a drop of mercury 4·05 millims. in diameter falling on smoked (sometimes on unsmoked) glass from the heights stated.

The first height of fall was 34 millims.

The phases in this case were very similar to those of milk falling from 50 millims. (Set 2), the main difference being that the arms were not so long and did not split off into drops, while the centre was deeply hollowed as in fig. IV. Set 2; after which the central part filled up and rose before the arms came in, as in the case of milk.

From a height of 60 millims. on unsmoked glass the consecutive phases were very similar to those of 34 millims. fall. On smoked glass, however, the arms were longer, and the resemblance to the forms seen in the case of milk was closer than on unsmoked glass.

An alternative course was often taken by the drop, of which the phases are given on

SET 5.

I. The thickness of the central portion is very slight.

II. The thickness diminishes, till the central membrane of liquid tears in the centre and flows to the circumference; or more frequently the tearing takes place under the edge of the outer band, leaving such forms as III, IV, & V, where the white represents the mercury and the black the plate. After this the succeeding stages were as in VI and

VII, where the arms contract into the ring, which splits into drops, as in VIII. These run together. The final stages are like those of milk.

SET 6.

Mercury on unsmoked glass. Height of fall 100 millims.

I. Slight waves on the central patch.

II. This, when illuminated by reflection from a concave mirror at the side, looked like two concentric circles of drops, showing probably that the raised circular band A A seen in the section IIα was lobed where the arms joined it.

III. Sometimes the contour was almost circular, being very slightly lobed.

If the plate was in the least dirty, irregular forms were seen, as in IVα, IVβ, & Vα; otherwise the final stages were very similar to those of milk, such deviations as V and VI being seen.

The same height of fall on smoked glass gave a rather wider spreading out than in I; after which the phases differed only from those depicted in the arms being thinner in the necks, with bulby heads, and the drop finally breaking into three when it rose vertically.

SET 7.

Unsmoked glass. 150 millims.

I. The phase I was seen, after which drops disposed as in fig. II were left on the plate, indicating that the arms from which the small drops split, split a second time, while the rest did not. The figures were of much the same type as when the fall was 100 millims.

A later stage of II, Set 6, was seen, more contracted, and with a complete circle of small drops round it, left from the first spreading out.

SET 8.

Smoked glass. 150 millims.

I. Very much spread out, flat, and uniform, with tendency to irregular small drop-forming arms.

IIα is the central vertical section from rim to rim of II.

III. The central patch begins to tear.

IV. The ring splits off, and the torn central patch runs together into arms; or the alternative course indicated by the next five figures is taken.

IIIα. Sometimes the centre contracted till the arms met. (In the case of milk from 280 millims., whose forms, it has been remarked, were similar to these, the centre invariably contracted till the arms met; the arms were also beaded, as if tending to split into groups along their whole length.)

IVα. The central patch tears open into a ring, into which portions of the arms contract.

Vα. The ring splits into drops.

Later stages showed a general distribution of drops over the plate, rather hard to remember with certainty, even immediately after they were seen, with occasional small arms remaining somewhat as in the figs. VIα & VIβ.

From 255 millims. on unsmoked glass the forms were much the same as from 100 and 150 millims. The arms of fig. II, Set 6, were seen, 6, 8, and 12 in number, and rather longer than there drawn. Occasionally the centre tore, and concentric rings of drops were formed.

SET 9.

Smoked glass. 250 millims.

The phases were generally the same as from 150 millims. (Set 8), with the variations of figs. I & II.

Later, the whole mass of central arms, or thin layer of liquid, split up into fine drops, which rose in a splutter from the plate.

It is to be observed that while on unsmoked glass the type of forms hardly changes while the fall increases from 100 to 250 millims., the same increase of fall on smoked glass is accompanied by very marked alterations in the behaviour of the drop, and that generally the wider spreading out of the drop on smoked glass indicates much less friction than on unsmoked.

XXIX. "On the Behaviour of the Fixed Elements of the Connective-tissue of the Tongue in Inflammation." By George Dowdeswell, M.A. Cantab. Communicated by J. Burdon-Sanderson, F.R.S., Professor of Physiology in University College. Received June 14, 1876.

Presents received, June 15, 1876.

Transactions.

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Fig. 1.

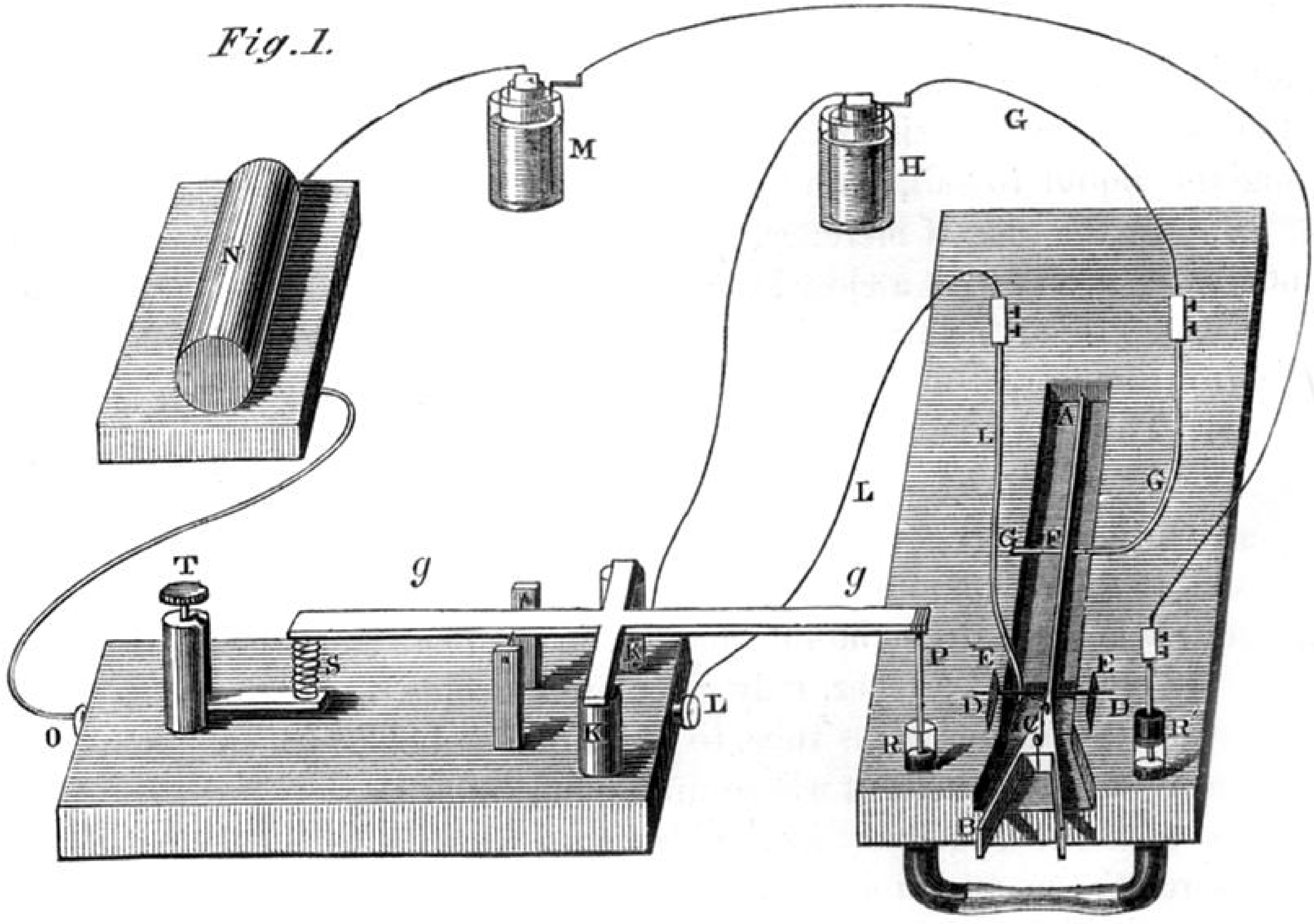
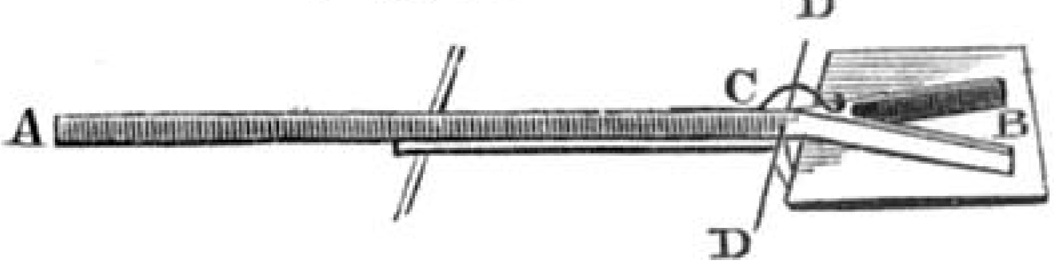


Fig. 2.



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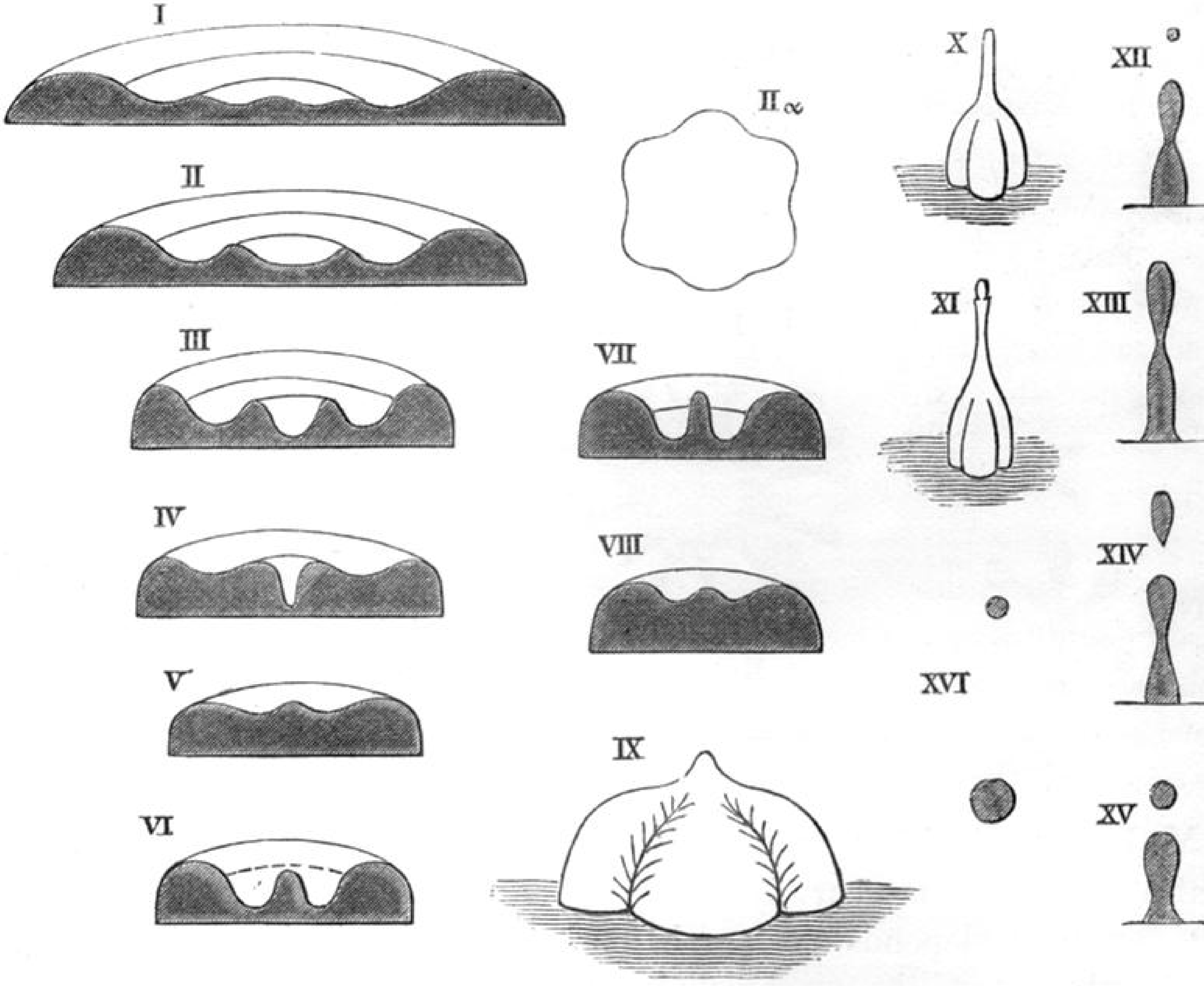

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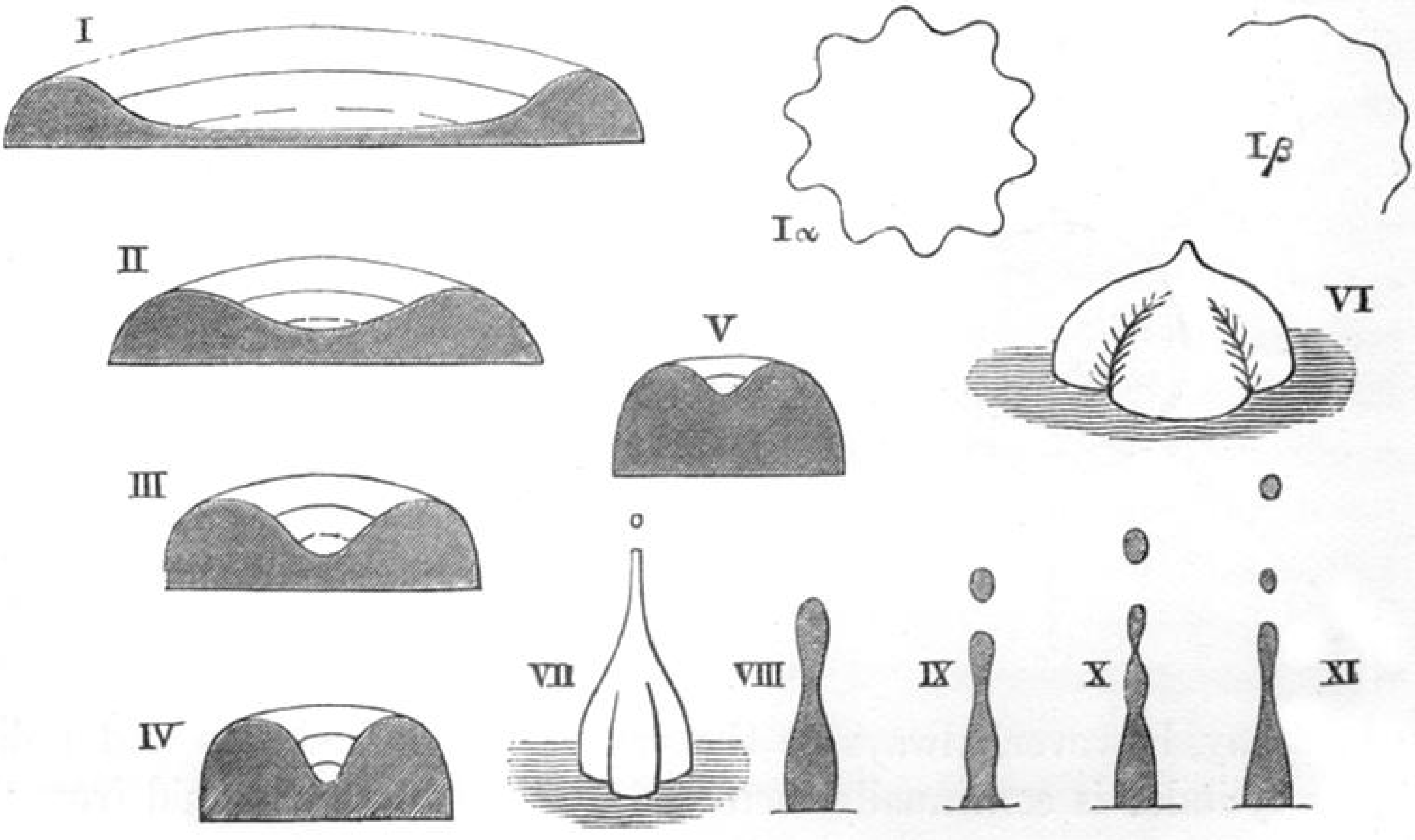

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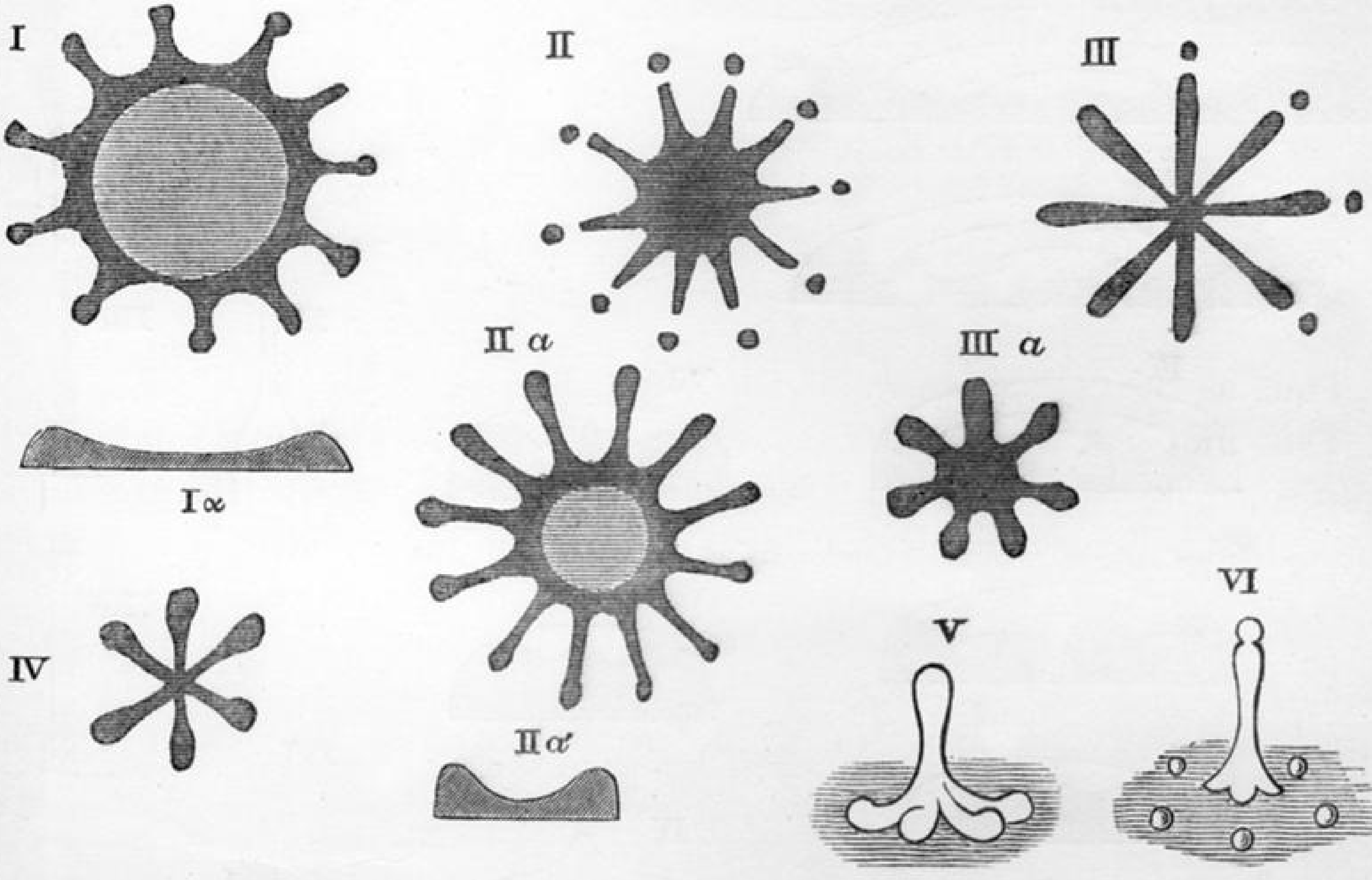

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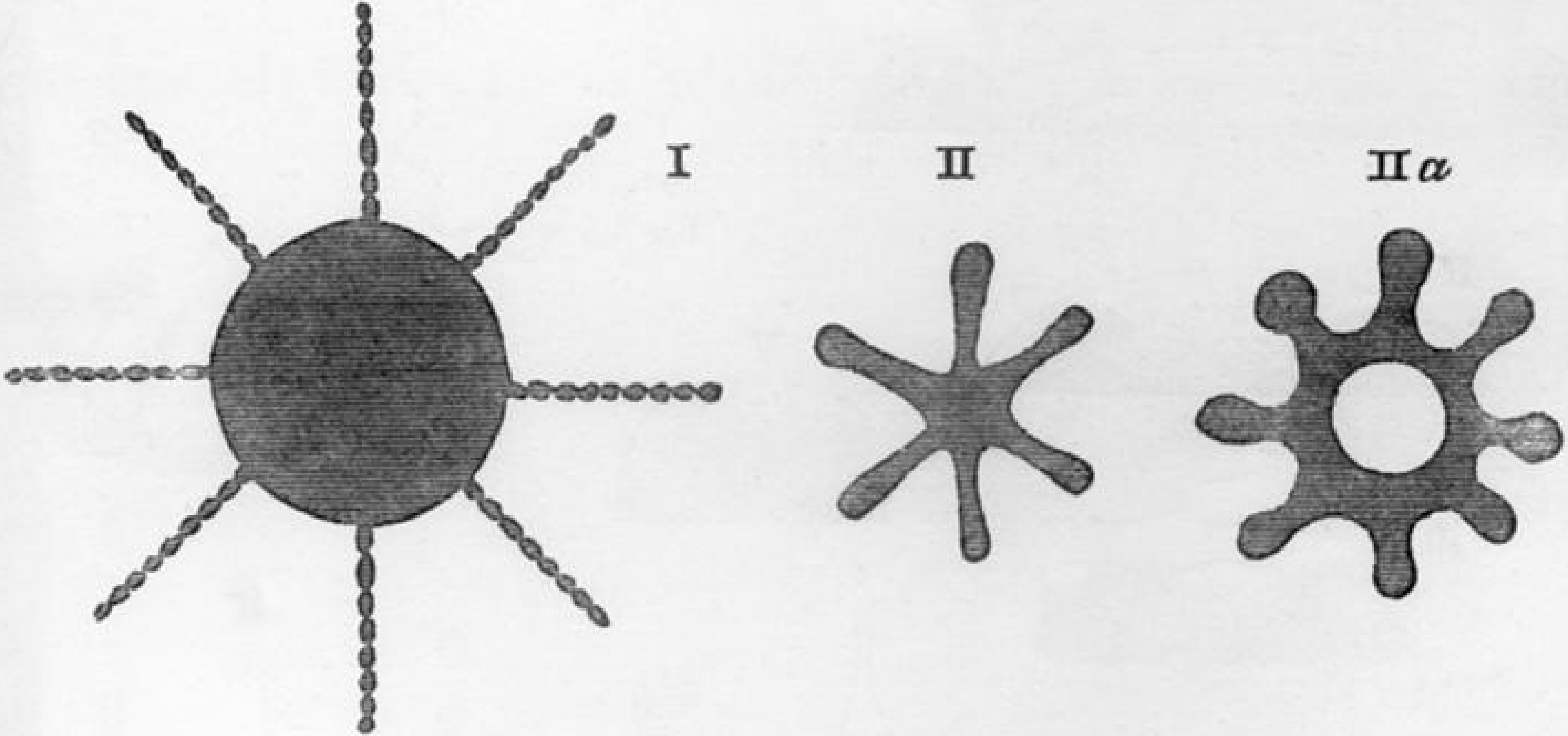

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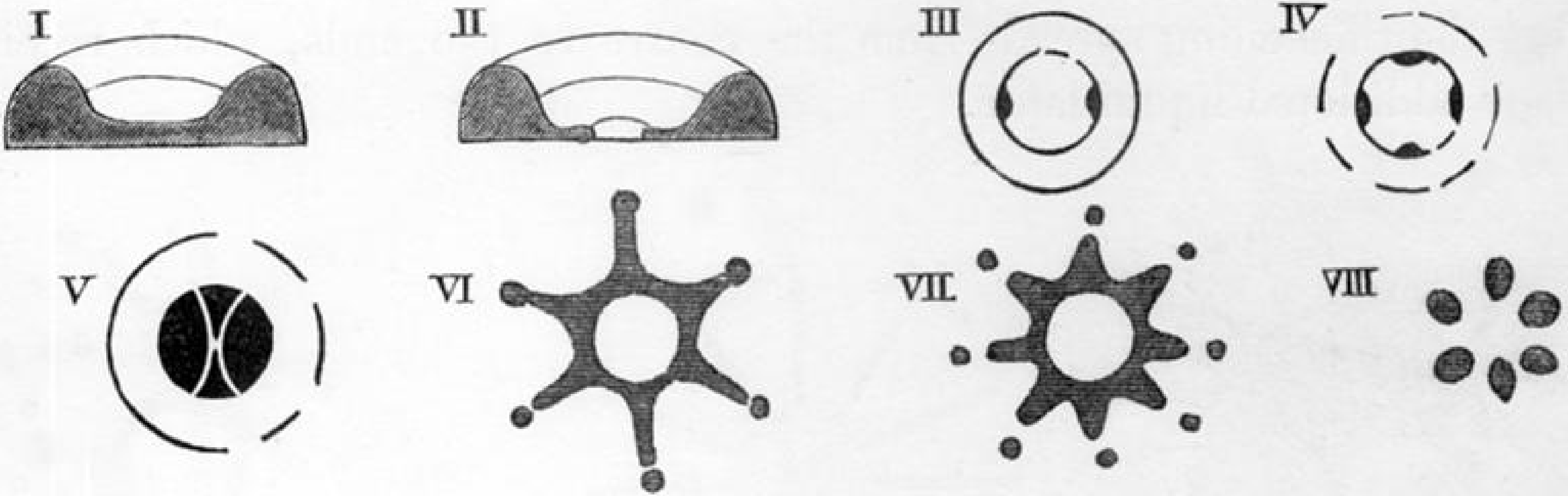

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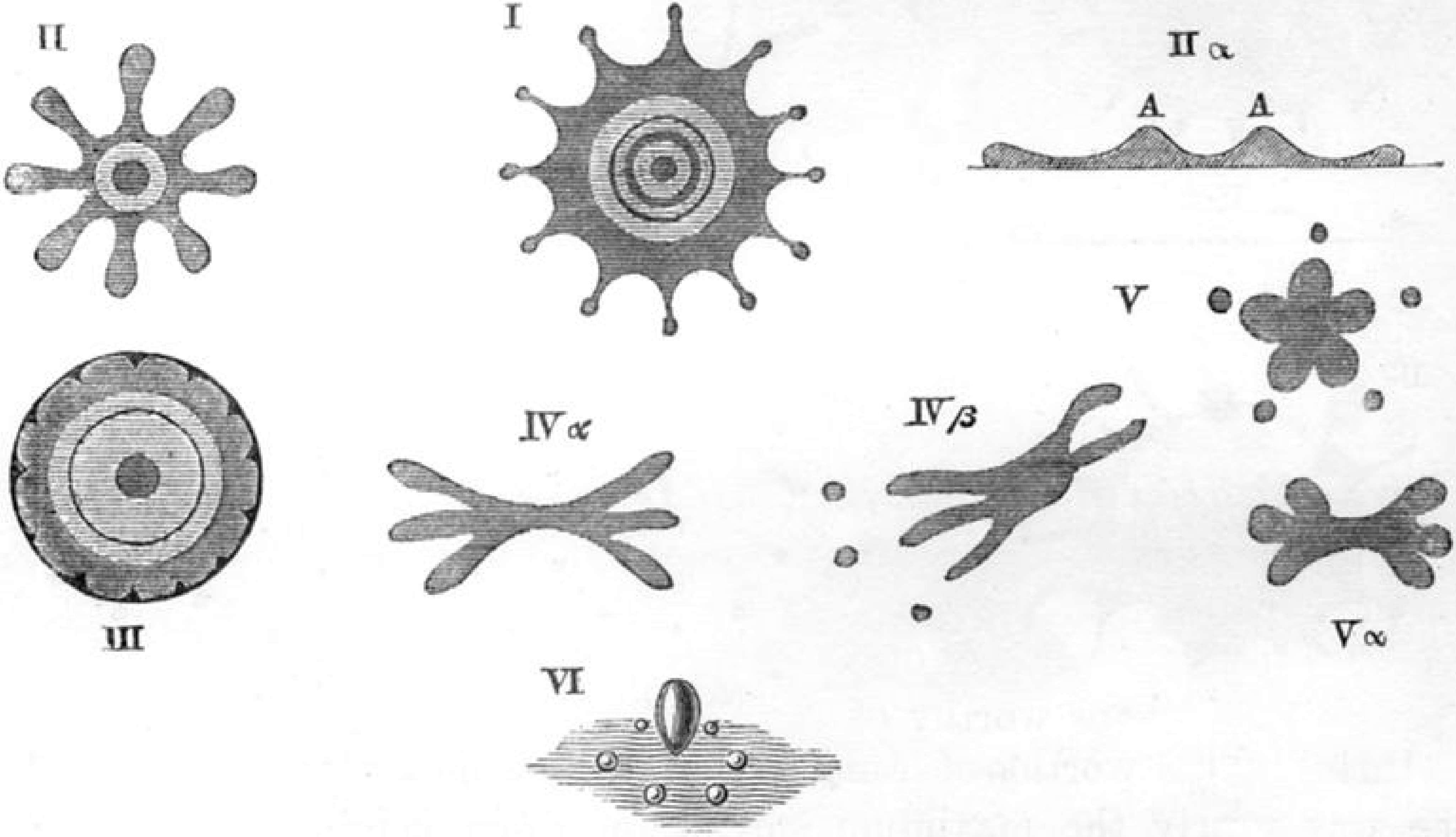

Set 7.

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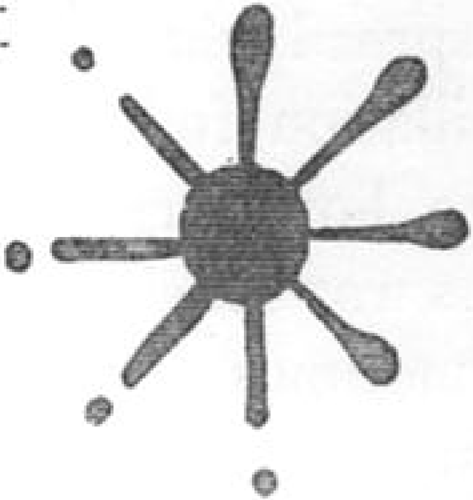

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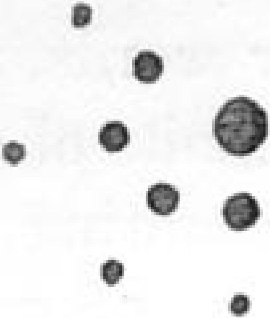

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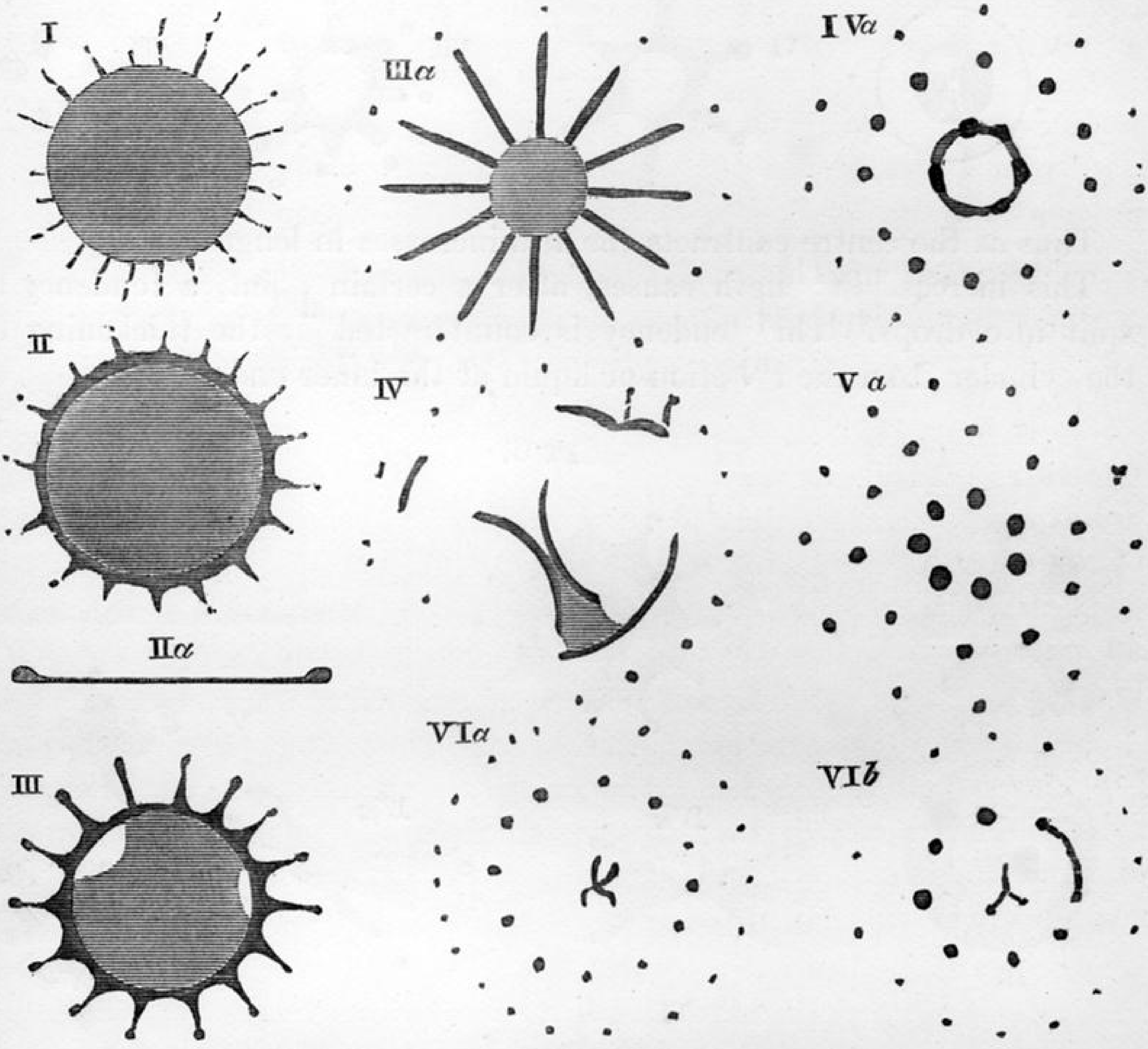

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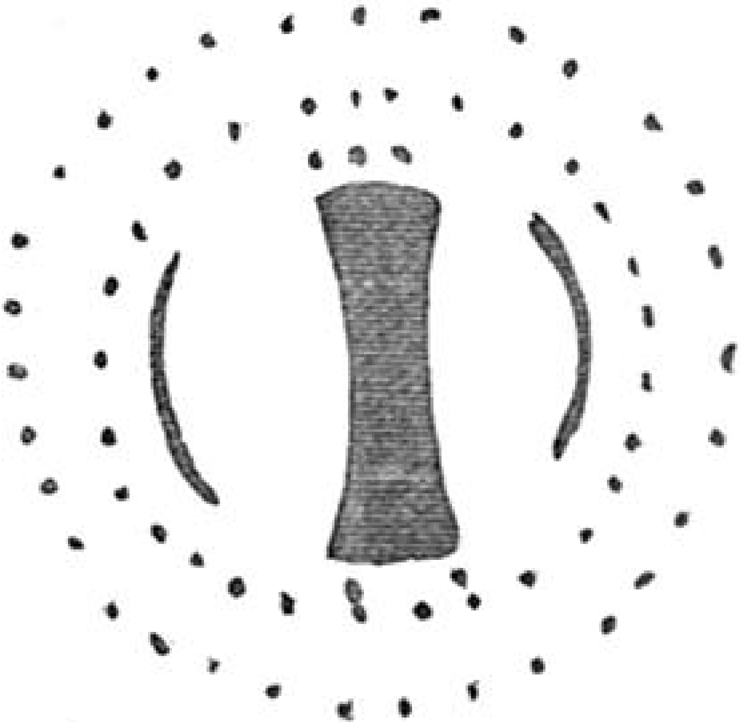

II