

IV. "Notes on Physical Geology." By the Rev. SAMUEL HAUGHTON, M.D. Dublin, D.C.L. Oxon., F.R.S., Professor of Geology in the University of Dublin. Received February 15, 1877.

No. I. *Preliminary Formulæ relating to the internal change of position of the Earth's Axis, arising from Elevations and Depressions caused by Geological Changes.*

1. If the earth's surface be an ellipsoid of revolution, whose moments of inertia round the polar and equatorial axes are  $C$  and  $A$ , and if  $\mu$  be the mass of a mountain placed on any meridian, with coordinates  $z, x$ , it is required to find the change of position in the earth's axis caused by the addition of the mass  $\mu$  (supposed in the first instance to be placed upon the earth *ab extra*). If  $\lambda$  be the latitude on which  $\mu$  is placed, we have  $\tan \lambda = \frac{z}{x}$ ; and if  $\theta$  be the angle made with the earth's axis by any axis in the meridian of  $\mu$ , if  $I$  be the total moment of inertia round this axis, we have

$$I = A \sin^2 \theta + C \cos^2 \theta + \mu (x^2 \cos^2 \theta + z^2 \sin^2 \theta - 2xz \sin \theta \cos \theta). \quad (1)$$

The new axis of rotation is that which makes

$$I = \text{maximum, or } dI = 0,$$

from which we find, after some reduction,

$$-\tan 2\theta = \frac{2\mu xz}{(C-A) + \mu(x^2 - z^2)}. \quad (2)$$

If we make  $\theta = \text{maximum}$ , or  $d\theta = 0$ , we ascertain the position in which the mass  $\mu$  must be placed so as to produce the maximum shift in the position of the earth's axis.

Differentiating (2), we find

$$x\{(C-A) + \mu(x^2 + z^2)\}dz + z\{(C-A) - \mu(x^2 + z^2)\}dx = 0;$$

and from the equation of the ellipse

$$\frac{z^2}{c^2} + \frac{x^2}{a^2} = 1$$

we have

$$\frac{zdz}{c^2} + \frac{xdx}{a^2};$$



or, neglecting small quantities,

$$-\tan 2\theta = 935.6 \rho \sin 2\lambda. \quad (7)$$

This equation shows that the pole moves away from the mass  $\mu$ , and that this mass is most effective at the latitude of  $45^\circ$ .

3. In order to apply the preceding to the case of our actual continents and oceans, we integrate (7) along the meridian as follows,

$$\rho = \frac{\mu}{M} = \frac{r^2 t d\lambda}{M} \cdot \cos \lambda d\lambda,$$

where

$r$  = radius of earth,

$l$  = longitude,

$\lambda$  = latitude,

$t$  = height of continent or depth of sea above or below the zero plane.

Hence we have

$$\tan 2\theta = 2\theta = -935.6 \frac{r^2 t d\lambda}{M} \int_{\lambda}^0 \cos \lambda \sin 2\lambda d\lambda,$$

and finally

$$\theta = -935.6 \frac{r^2 t d\lambda}{M} (1 - \cos^3 \lambda). \quad (8)$$

The zero plane, from which  $t$  is measured, is the surface of the ellipsoid similar to the sea-surface, and containing the same volume as the total solid matter of the globe. It is thus found: assuming the mean height of the continents above the sea-level at about 1000 feet, and the mean depth of the ocean at about two miles, we have, in miles,

$$x = \frac{2 \cdot 2L}{W + L}. \quad (9)$$

where  $x$  is the height of the zero plane above the present mean sea-bottom, and  $L$ ,  $W$  are the areas of land and water:

$L$  = 52 millions of square miles.

$W$  = 145     „     „     „

Substituting in (9) we find

$$x = 0.58 \text{ mile.}$$

The zero plane, therefore, or original surface of the solid earth before it became wrinkled by geological forces, lies at a depth of 1.42 foot below the sea-level. In using equation (8) we must therefore write

$t = +1.62$  mile (continent).

$t = -0.58$  „ (ocean).



Long.	$1 - \cos^3 \lambda$	$2 \cos l (1 - \cos^3 \lambda)$
45°	1.00	1.42
40	1.00	1.52
35	1.00	1.64
30	1.00	1.74
25	1.00	1.80
20	1.00	1.88
15	1.00	1.92
10	1.00	1.96
5	1.00	1.98
0	1.00	2.00

Total . . . 17.86

The displacement of the pole, in miles, produced by this imaginary continent is, by equation (10),

$$r\theta = -14.11 \times 17.86 = -252 \text{ miles.}$$

No. II. *On the amount of shifting of the Earth's Axis, already caused by the elevation of the existing Continents.*

Having shown in the preceding note that the motion of the earth's axis caused by the geological wrinkling of the earth's surface depends (in consequence of the weight of the sea-water) only on the continents, it remains for me to calculate the numerical amount of change of axis produced by each of the existing continents.

For this purpose I select the following meridians for the coordinates Y and X of the motion :—

Greenwich . . . . .	0°	+Y
Rangoon . . . . .	90	—X
Behring's Strait . . . . .	180	—Y
Yucatan . . . . .	270	+X

Reckoning the longitudes eastward, round the whole circumference of the earth, the equation (10) generalized becomes

$$r\theta = -14.11 (\cos^3 \lambda' - \cos^3 \lambda), \quad . \quad . \quad . \quad (11)$$

in which the meridian of each 5° of longitude is used,  $\lambda'$  and  $\lambda$  being the lowest and highest degrees of latitude of the land on each meridian.

The expression  $\cos^3 \lambda' - \cos^3 \lambda$  is found by observation on the globe, and resolved into its components X and Y, regarding the North Pole as the axis moved. We thus find

## I. EUROPE AND ASIA.

Long.	$\cos^3 \lambda' - \cos \lambda$ .	X.	Y.	
350°	0.18	-0.03	-0.17	
355	0.35	-0.03	-0.34	
360	0.18	-0.00	-0.18	X = -0.06
5	0.23	+0.02	-0.23	
10	0.28	+0.05	-0.27	
15	0.28	+0.07	-0.27	
20	0.39	+0.13	-0.36	
25	0.49	+0.20	-0.44	
30	0.49	+0.24	-0.42	
35	0.76	+0.43	-0.62	
40	0.83	+0.53	-0.63	
45	0.84	+0.59	-0.59	
50	0.78	+0.60	-0.50	
55	0.76	+0.62	-0.43	
60	0.68	+0.59	-0.34	
65	0.68	+0.62	-0.29	
70	0.84	+0.79	-0.29	
75	0.89	+0.86	-0.23	
80	0.77	+0.76	-0.13	
85	0.68	+0.67	-0.05	
90	0.77	+0.77	0.00	Y = -6.78
95	0.89	+0.88	+0.08	
100	0.89	+0.87	+0.15	
105	0.89	+0.86	+0.23	
110	0.77	+0.72	+0.26	
115	0.77	+0.70	+0.32	
120	0.49	+0.42	+0.29	
125	0.49	+0.40	+0.28	
130	0.39	+0.30	+0.26	
135	0.39	+0.27	+0.27	
140	0.13	+0.08	+0.10	
145	0.06	+0.03	+0.05	
150	0.06	+0.03	+0.05	
155	0.11	+0.05	+0.10	
160	0.06	+0.02	+0.05	
165	0.06	+0.01	+0.06	
170	0.06	+0.01	+0.06	
175	0.01	0.00	+0.01	
180	0.00	0.00	0.00	

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X = +14.19    Y = +2.62

Hence we obtain, finally,

$$X = -0.06 + 14.19 = +14.13,$$

$$Y = -6.78 + 2.62 = -4.16.$$

Multiplying these results by 14.11, the coefficient of equation (11), we find the following displacements in miles:—

$$X \text{ (towards Yucatan)} = 199.4 \text{ miles,}$$

$$Y \text{ (towards Behring's Strait)} = 58.7 \text{ miles.}$$

Compounding these together we find

$$\sqrt{X^2 + Y^2} = 207.1 \text{ miles,}$$

$$\frac{X}{Y} = \tan \phi,$$

$$\phi = 73^\circ 35' \text{ W. of Greenwich.}$$

This resultant coincides with the meridian of the Andes.

## II. AFRICA.

### North Africa.

Long.	$\cos^3 \lambda - \cos^3 \lambda_0$	X.	Y.	
345°	0.21	-0.05	-0.20	
350	0.44	-0.07	-0.43	
355	0.44	-0.04	-0.43	X = -0.16
360	0.44	0.00	-0.44	
5	0.44	+0.04	-0.43	
10	0.32	+0.05	-0.31	
15	0.35	+0.09	-0.34	
20	0.35	+0.12	-0.33	
25	0.35	+0.15	-0.32	
30	0.35	+0.17	-0.30	
35	0.17	+0.10	-0.14	
40	0.05	+0.03	-0.04	X = +0.76
45	0.01	+0.01	-0.01	Y = -3.72

### South Africa.

10	0.17	-0.03	+0.17
15	0.45	-0.11	+0.43
20	0.45	-0.15	+0.38
25	0.35	-0.15	+0.32
30	0.26	-0.13	+0.22
35	0.17	-0.10	+0.14
40	0.00	0.00	0.00
45	0.21	-0.15	+0.15

$$X = -0.82 \quad Y = +1.81$$

Adding all together we obtain, finally,

$$X = -0.22 = 3.1 \text{ miles (towards Rangoon),}$$

$$Y = -1.91 = 26.9 \text{ miles (towards Behring's Strait).}$$

### III. NORTH AMERICA.

Long.	$\cos^3 \lambda' - \cos^3 \lambda$	X.	Y.
195 <sup>0</sup>	0.06	-0.01	+0.06
200	0.13	-0.04	+0.12
205	0.06	-0.02	+0.05
210	0.06	-0.03	+0.05
215	0.06	-0.03	+0.05
220	0.06	-0.04	+0.04
225	0.13	-0.09	+0.09
230	0.21	-0.16	+0.13
235	0.49	-0.40	+0.28
240	0.49	-0.42	+0.21
245	0.68	-0.61	+0.28
250	0.68	-0.64	+0.23
255	0.77	-0.74	+0.20
260	0.84	-0.83	+0.14
265	0.68	-0.61	+0.06
270	0.65	-0.65	0.00
275	0.46	-0.45	-0.04
280	0.59	-0.58	-0.10
285	0.38	-0.37	-0.10
290	0.23	-0.19	-0.08
295	0.08	-0.07	-0.04
300	0.14	-0.12	-0.07
305	0.07	-0.06	-0.04
310	0.12	-0.09	-0.08
315	0.12	-0.08	-0.08
320	0.07	-0.04	-0.05
325	0.07	-0.04	-0.06
330	0.06	-0.03	-0.05
335	0.11	-0.04	-0.10
340	0.02	0.00	-0.02
		<hr/> X = -7.48	<hr/> Y = -0.91



Hence, finally,

$$\begin{aligned} X &= -7.48, \\ Y &= +1.99 - 0.91 = +1.08; \\ \text{or } X &= -7.48 = 105.5 \text{ miles (towards Rangoon),} \\ Y &= +1.08 = 15.2 \text{ miles (towards Greenwich).} \end{aligned}$$

#### IV. SOUTH AMERICA.

North of Equator.

Long.	$\cos^3 \lambda' - \cos^3 \lambda$ .	X.	Y.
280	0.05	-0.04	-0.01
285	0.05	-0.04	-0.01
290	0.05	-0.04	-0.02
295	0.05	-0.04	-0.02
300	0.01	-0.01	0.00
305	0.01	-0.01	0.00
		<hr/>	<hr/>
		X = -0.18	Y = -0.06

South of Equator.

280	0.05	+0.05	+0.01
285	0.10	+0.09	+0.02
290	0.88	+0.82	+0.30
295	0.55	+0.50	+0.23
300	0.47	+0.41	+0.24
305	0.45	+0.37	+0.26
310	0.26	+0.20	+0.17
315	0.25	+0.17	+0.17
320	0.09	+0.06	+0.07
		<hr/>	<hr/>
		X = +2.67	Y = +1.47

Hence, finally,

$$\begin{aligned} X &= -0.18 + 2.67 = +2.49, \\ Y &= -0.06 + 1.47 = +1.41; \\ \text{or } X &= +2.49 = 35.1 \text{ miles (towards Yucatan),} \\ Y &= +1.41 = 19.9 \text{ miles (towards Greenwich).} \end{aligned}$$

## V. AUSTRALIA AND PACIFIC ISLANDS.

Long.	Islands.		
	$\cos^3 \lambda' - \cos^3 \lambda$ .	X.	Y.
100 <sup>0</sup>	0.01	-0.01	0.00
105	0.01	-0.01	0.00
110	0.16	-0.15	-0.05
115	0.33	-0.30	-0.14
120	0.29	-0.25	-0.15
125	0.35	-0.28	-0.20
130	0.25	-0.19	-0.16
135	0.29	-0.20	-0.20
140	0.54	-0.35	-0.41
145	0.48	-0.27	-0.39
150	0.10	-0.05	-0.08
155	0.00	0.00	0.00
160	0.00	0.00	0.00
165	0.14	-0.04	-0.13
170	0.20	-0.03	-0.19
175	0.05	-0.01	-0.04

$$X = -2.14 \quad Y = -2.14$$

$X = -2.14 = 30.2$  miles (towards Rangoon),

$Y = -2.14 = 30.2$  miles (towards Behring's Strait).

Collecting all the preceding results into one Table, we see the relative effects of the elevation of each of the existing continents upon the position of the pole.

Displacement of North Pole caused by each continent.

	Towards Greenwich. miles.	Towards Behring's Strait. miles.	Towards Yucatan. miles.	Towards Rangoon. miles.
Europe and Asia .....	—	58.7	199.4	—
Africa .....	....	26.9	—	3.1
North America .....	15.2	—	—	105.5
South America .....	19.9	—	35.1	—
Australia, &c. ....	—	30.2	—	30.2

The power of Europe and Asia in moving the pole is partly due to the extension of this continent along the parallel of  $45^\circ$ , which is the most effective latitude. The actual effect produced by Europe and Asia was not much less than that of our imaginary continent (Note I.), occupying one eighth part of the surface of the globe.

The foregoing results are positive, and the motions of the pole indicated must have actually occurred when the existing continents were formed. But simultaneously with these elevations depressions must have gone on elsewhere, continents disappearing beneath the sea and sinking to the zero plane, while other continents were rising. It is to

be noticed that although the excavation of the sea-bottom to its present depth below the zero plane, corrected for the weight of the ocean, produces no motion in the pole, yet that the depression of a continent down to the zero plane produces a motion of pole equal and opposite to that produced by its elevation. I have calculated the hypothetical effects of the depression of imaginary continents occupying the sites of the present Pacific Ocean, with the following results :—

## VI. NORTH PACIFIC OCEAN (depressed).

Long.	$\cos^3 \lambda' - \cos^3 \lambda$ .	X.	Y.	
100	0.10	-0.10	-0.01	
105	0.17	-0.16	-0.04	
110	0.26	-0.24	-0.09	
115	0.26	-0.23	-0.11	
120	0.45	-0.39	-0.22	
125	0.45	-0.38	-0.26	
130	0.73	-0.56	-0.49	
135	0.81	-0.57	-0.57	
140	0.88	-0.56	-0.67	
145	0.88	-0.50	-0.72	
150	0.88	-0.44	-0.76	
155	0.88	-0.37	-0.80	
160	0.88	-0.30	-0.83	
165	0.81	-0.21	-0.78	
170	1.00	-0.17	-0.98	
175	1.00	-0.09	-0.99	
180	1.00	0.00	-1.00	X = -5.27
185	0.97	+0.08	-0.96	
190	0.94	+0.16	-0.92	
195	0.94	+0.24	-0.91	
200	0.94	+0.33	-0.88	
205	0.94	+0.39	-0.85	
210	0.94	+0.47	-0.81	
215	0.90	+0.51	-0.74	
220	0.81	+0.52	-0.62	
225	0.70	+0.49	-0.49	
230	0.70	+0.54	-0.45	
235	0.50	+0.41	-0.29	
240	0.50	+0.43	-0.25	
245	0.26	+0.23	-0.11	
250	0.26	+0.24	-0.09	
255	0.17	+0.16	-0.04	
260	0.10	+0.10	-0.02	
265	0.09	+0.08	-0.01	
270	0.07	+0.07	0.00	
275	0.06	+0.06	0.00	
		X = +5.51	Y = -17.76	

Hence, finally,

$$X = -5.27 + 5.51 = +0.24,$$

$$Y = -17.76;$$

or  $X = +0.24 = 3.4$  miles (towards Yucatan),

$$Y = -17.76 = 250.6 \text{ miles (towards Behring's Strait).}$$

This Table shows (*inter alia*) the remarkable symmetry of the North Pacific Ocean east and west of the meridian of Behring's Strait.

#### VII. SOUTH PACIFIC OCEAN (depressed).

Long.	$\cos^3 \lambda' - \cos^3 \lambda$ .	X.	Y.	
140°	0.45	+0.29	+0.34	
145	0.45	+0.26	+0.37	
150	0.94	+0.47	+0.81	
155	0.94	+0.39	+0.85	
160	0.94	+0.32	+0.88	
165	0.94	+0.24	+0.91	
170	0.94	+0.16	+0.92	
175	0.94	+0.08	+0.94	
180	0.94	0.00	+0.94	X = +2.21
185	0.94	-0.08	+0.93	
190	0.94	-0.16	+0.92	
195	0.94	-0.24	+0.91	
200	0.94	-0.32	+0.88	
205	0.94	-0.39	+0.85	
210	0.94	-0.47	+0.81	
215	0.94	-0.54	+0.77	
220	0.94	-0.60	+0.72	
225	0.94	-0.66	+0.66	
230	0.94	-0.72	+0.60	
235	0.94	-0.77	+0.54	
240	0.94	-0.81	+0.47	
245	0.94	-0.85	+0.39	
250	0.94	-0.88	+0.32	
255	0.94	-0.91	+0.24	
260	0.94	-0.92	+0.16	
265	0.94	-0.93	+0.08	
270	0.94	-0.94	0.00	Y = +17.21
275	0.94	-0.93	-0.08	
280	0.90	-0.88	-0.15	
285	0.40	-0.38	-0.10	
		X = -13.28	Y = -0.33	

Hence, finally,

$$X = + 2 \cdot 21 - 13 \cdot 28 = -11 \cdot 07,$$

$$Y = + 17 \cdot 21 - 0 \cdot 33 = + 16 \cdot 88 ;$$

or  $X = -11 \cdot 07 = 156 \cdot 2$  miles (towards Rangoon).

$$Y = + 16 \cdot 88 = 238 \cdot 2 \text{ miles (towards Greenwich).}$$

The total effect of a continent equal to the North Pacific would be

$$\sqrt{X^2 + Y^2} = 250 \cdot 6 \text{ miles,}$$

$$\tan(\phi) \frac{X}{Y} = \phi = 0^\circ 47' \text{ E. of } 180^\circ.$$

The total effect of a continent equal to the South Pacific Ocean would be

$$\sqrt{X^2 + Y^2} = 201 \cdot 8 \text{ miles,}$$

$$\tan(\phi) \frac{X}{Y} = \phi = 23^\circ 17' \text{ E. of Greenwich.}$$

*March 15, 1877.*

Dr. J. DALTON HOOKER, C.B., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read :—

- I. "On the Tides of the Arctic Seas.—Part VII. Tides of Port Kennedy, in Bellot Strait." (Final Discussion.) By the Rev. SAMUEL HAUGHTON, M.D. Dublin, D.C.L. Oxon., F.R.S., Fellow of Trinity College, Dublin. Received February 17, 1877.

(Abstract.)

The tidal observations at Port Kennedy were made hourly for 23 days ; and in my former discussion of these tides (Part VI.) I used only the observations made in the neighbourhood of H. W. and L. W., obtaining the following results for the Tidal Coefficients :—

<i>Diurnal Tide.</i>	<i>Semidiurnal Tide.</i>
$S = 23 \cdot 4$ inches.	$S = 7 \cdot 0$ inches.
$i_s = 5^h 12^m.$	$i_s =$
$M = 20 \cdot 9$ inches.	$M = 17 \cdot 0$ inches.
$i_m = 0^h 34^m.$	$i_m = -0^h 12^m.$