PLATE 2.

Fig. 4. A "giant cell" from the ascending frontal convolution in man. Obtained from *recent brain* by means of the new freezing microtome. × 105 diameters.

Fig. 5. Section from the ascending frontal convolution in a case of senile atrophy. Obtained, by means of the new freezing microtome, from fresh brain. The proliferation of connective cells of the upper cortical layers is seen invading the vascular tracts and nervous elements. × 180 diameters.

Fig. 6. A cluster of amœboid connective cells from the third layer of the ascending frontal convolution, in a case of senile atrophy. Section obtained from *fresh brain* by pressure and teasing.

III. "On the Thickness of Soap Films." By A. W. REINOLD, M.A., Professor of Physics in the Royal Naval College, Greenwich, and A. W. RÜCKER, M.A., Professor of Physics in the Yorkshire College of Science, Leeds. Communicated by R. B. CLIFTON, F.R.S., Professor of Experimental Philosophy in the University of Oxford. Received June 13, 1877.

Attempts have from time to time been made by various physicists to obtain from the phenomena of capillarity, or from observations on liquid films, an indication of the magnitude of the radius of molecular attraction. The authors of this note have, with the same object in view, lately made a series of experiments to determine whether the law that the resistance offered to the electric current by a uniformly thick homogenous body varies inversely as the section is or is not apparently obeyed by liquid films, as any apparent departure from that law might be taken to indicate a want of homogeneity, or that the thickness of the film was comparable with the magnitude of the radius of molecular attraction.

Their investigations on this point are not as yet sufficiently advanced for publication; but in the course of their work they have made some observations on the forms of soap films, which they venture to lay before the Royal Society in a preliminary note.

A liquid film, inclined to the horizontal so as to become gradually thinner by the slow descent of the liquid, will, under favourable conditions, appear black, as in the central portion of Newton's rings. By optical methods it is only possible to obtain a superior limit to the thickness of the black portion of such a film; but there is no doubt that at the lower boundary of the black the thickness of the film increases with extraordinary rapidity.

As a general rule no trace of the blue of the first order can be perceived; but when the colour next below the black is the white of the first order, the line of separation between the two is very definite; and if the film be moved so as to change the angle of incidence of the light by

which it is viewed, the position of the boundary of the black remains fixed, though that of each of the other colours is altered with every motion. More frequently, however, the presence of the colours of the first order, and some or all of those of the second order, can only be detected by means of a microscope, and to the naked eye several tints appear to be wanting between the black and the colour which immediately succeeds it.

The constant recurrence of the phenomenon above mentioned, viz. the very rapid change in the thickness of the film in the immediate neighbourhood of the black, suggests an intimate connexion between the thickness of the latter and the molecular constitution of the liquid. Any investigation as to whether such a connexion exists must, it appears, be commenced by seeking the answers to the following questions:—

1. Do very rapid changes in thickness occur elsewhere in the films?
2. Is the black portion of the film uniform in thickness?
3. If so, is the thickness the same for all films formed with the same liquid?

The authors have, with a single liquid and with the form of apparatus employed, obtained results which, although not sufficiently numerous to enable them to give general answers to these inquiries, display, as they believe, a hitherto unsuspected constancy in the phenomena exhibited by liquid films thinning under the influence of gravity, and seem to merit further study.

The method employed was to measure simultaneously the electrical resistance of the films, and the breadths of the bands of colour they displayed.

The liquid used was M. Plateau's "liquide glycérique," made by dissolving 1 part by weight of oleate of soda in 40 parts of water, and adding 3 volumes of this liquid to 2·2 volumes of Price's glycerine. To improve the conductivity, 3 parts by weight of potassium nitrate were dissolved in every 100 parts of water along with the oleate of soda.

The films which were submitted to investigation were cylindrical in shape, and were formed between two platinum rings of the same diameter placed one vertically over the other. The mode of supporting these and making the electrical connexions was as follows:—A glass cylindrical vessel, about 16 centims. high and 9 centims. in diameter, was fitted with an ebonite cover divided into two unequal parts. Each of these would, if placed on the top of the vessel, remain *in situ*. A brass tube, which could be elevated or depressed, and was retained in its position by friction, passed through the larger portion of the cover, and was, when in position, in the centre of the glass vessel. A piece of india-rubber tubing provided with a pinchcock was attached to the upper end of this tube, and to the bottom of it was soldered a brass plate carrying the upper platinum cylinder. This latter was formed of stout platinum foil, the edges of which were welded together in order to avoid the introduction of any

foreign metal which might give rise to local galvanic action when the current passed through the film. At the bottom of the glass vessel was a small porcelain dish containing mercury, in which was placed a platinum crucible, the lower part of which was amalgamated to ensure good contact with the mercury. The diameter of the mouth of the crucible was as nearly as possible the same as that of the platinum cylinder above mentioned. To obviate the possibility of the film thinning by evaporation from its surface, a little of the liquid used was placed in the bottom of the glass vessel, and the platinum crucible was also filled with the liquid to within about 1 millim. of its upper edge. The cylindrical films were produced in the following manner. A plane film was formed on the platinum cylinder, which, when the cover was replaced on the vessel, was blown out into a bubble through the india-rubber tubing. This bubble, when large enough, adhered to the edge of the platinum crucible, and both the quantity of air within it and the position of the crucible were then so regulated as to make it as accurately cylindrical as possible. The edge of the platinum cylinder was levelled by altering the position of the whole apparatus until all points on its edge were, as was determined by the help of a cathetometer, in the same horizontal plane. With this arrangement it was easy to measure the resistance of the film. A binding-screw on the smaller portion of the ebonite cover was connected by a platinum wire with the mercury on which the platinum crucible rested, while another binding-screw on the brass tube formed the point of connexion with the upper electrode. The resistance was measured by means of a Wheatstone's Bridge (of the Post-Office pattern), and as it continually changed, and nearly always slowly but steadily increased, the known resistance was made up to a certain amount, and the moment when the unknown resistance reached that amount noted.

As the resistances to be measured were very large, a box of resistance-coils was introduced into one arm of the bridge containing ten resistance-coils of about 100,000 ohms each. The actual resistance available for purposes of measurement was thus rather greater than 1,000,000 ohms, and by the multiplying power of the instrument resistances up to 100 times this amount could be measured. The galvanometer used was a reflecting instrument of 5000 ohms resistance. In the arm of the balance which contained the film-resistance a commutator was placed, and every time the key was depressed the direction of the current was changed. By this means error due to polarization was reduced to a minimum. The battery consisted of three Grove's cells.

The electrical observations were made in a room adjoining that in which the film under experiment was placed, and at the same time a second observer measured with a cathetometer the breadths of the bands of colour exhibited by the film. This operation was repeated at least twice, and the time of each observation was noted, so that the rate of

descent of the line of separation between each pair of colours could be determined and its position calculated at the moment when any particular electrical observation was made.

In this calculation the velocity of the motion of each colour was supposed to be uniform between successive observations. This supposition was to a certain extent justified by the fact that when several measurements were taken, the calculated velocities for consecutive intervals of time differed but little for the thinner parts of the films. In the thicker parts a greater irregularity was observed, but the exact determination of the position of the boundary between two colours was also of less importance. In making these observations a screen was placed behind the film, and the telescope of the cathetometer was directed to a portion of the film illuminated by means of a mirror so placed that the direction of the incident light was perpendicular to the optic axis of the telescope. The film was thus viewed by light incident at 45°, and the refractive index of the liquid being known the thickness corresponding to any colour may be deduced by means of the table of Newton's rings given in Watts's 'Dictionary of Chemistry.'

The refractive index was found to be 1·395 for mean yellow rays. It was determined, not from the liquid used, which was made a week before the experiments began, but from another freshly made solution of precisely the same composition. This course was rendered necessary by the fact that such solutions rapidly become turbid, owing to the formation of a precipitate which cannot be sufficiently removed to enable the liquid to be used for such a purpose.

TABLE I.

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
	mm.	mm.	mm.									
Black / Yellow 2	286·90	286·25	·65	0·00	10·00	10·00	·065	286·25	3·45	401		
											2404	1·000
Orange 2	282·00	281·15	·85	0·75	11·00	10·25	·084	281·23	5·02	430		
											2096	1·005
Purple 3	280·35	279·75	·60	3·00	11·75	8·75	·069	279·87	1·36	525		
											1733	1·004
Green 3	279·00	278·50	·50	3·75	12·25	8·50	·060	278·63	1·24	630		
											1376	1·008
Red 3 / Green 4	277·30	276·75	·55	5·25	13·25	8·00	·069	276·98	1·65	825		
											1127	1·002
Green 4 / Red 4	276·35	275·90	·45	6·25	14·00	7·75	·058	276·13	0·85	950		
											987	1·003
Red 4 / Green 5	273·40	268·00	5·40	7·25	14·25	7·00	·771	271·27	4·86	1077		
Green 5									11·97	1150	870	1·000

Reading for the top of the liquid cylinder ...... 289·7
" " bottom " ...... 259·3

Table I. is an example of the way in which the results required were deduced from the experiments. Column I. gives the cathetometer readings in millims. for the positions of the colours whose names are placed opposite.

The orders of the colours are indicated by the numbers placed after them. The readings for a particular colour were obtained by placing the point of intersection of the cross wires of the telescope in the centre of the band of that colour. In some cases more definite readings could be obtained by taking the line of division between two colours, and such are indicated by writing the names of the two colours one above the other with a line between them. The last colour named is that of the portion of the film extending to the bottom from the point at which the last reading given was taken.

Column II. gives a similar set of readings taken a little later.

Column III. the differences between corresponding numbers in I. and II., or the distances through which the colours had moved.

Columns IV. and V., the number of minutes which elapsed between the first reading given in Column I. and the others given in Columns I. and II.

Column VI. gives the differences between corresponding numbers in V. and IV., or the number of minutes which elapsed between two observations of the same colour.

Column VII. gives the quotients of the numbers in III. by the corresponding numbers in VI., or the velocity of motion of each colour in millims. per minute.

Column VIII. gives the positions of all the colours ten minutes after the first observation (at which time an electrical observation was made), deduced from the previous Columns.

Column IX. gives the lengths of the sections of the cylinder comprised between the colours named. These are obtained by subtracting the top number in Column VIII. from the reading for the top of the cylinder, viz. 289·7, each of the other numbers from that above it, and by subtracting from the last number the reading for the bottom of the cylinder, or 259·3.

Column X. gives in millionths of a millim. the thicknesses for air corresponding to the various colours named.

These data were sufficient to enable the authors to represent graphically the shapes of the films, and they annex (pp. 340, 341) a number of curves drawn for this purpose, of which that numbered VII. is obtained from the set of experiments given in Table I.

Since the length of the liquid cylinder was 30·4 millims., it is necessary to represent the thickness on a much larger scale than that used for the length. No calculation is needed to allow for the refractive index or for the angle of incidence of the light, since, by introducing these corrections (which become necessary before the resistance of the film can be calcu-

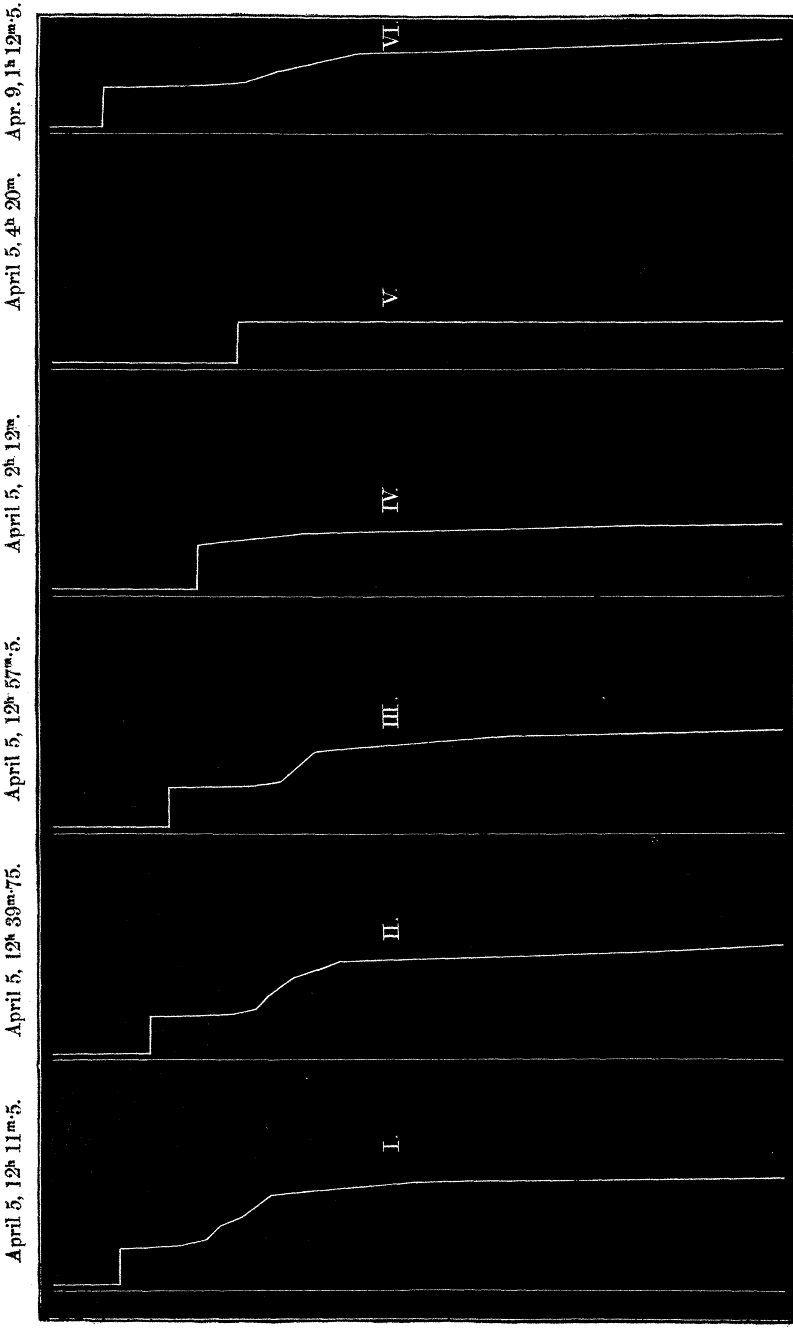
lated), the ratios of the numbers given in Column X. would not be altered; the only change would be in the scale on which the thickness is represented. The topmost and lowest points of the curves refer to the parts of the films in contact with the upper and lower cylinders, and the thicknesses at these and all intermediate points are represented by the horizontal distances between the points on the curves and the vertical lines drawn near them. For the reasons given above, lines representing the thicknesses of the cylinders are magnified 5000 times more than those representing their lengths.

Figs. I. to V. (p. 340) represent the successive forms assumed by a film at the hours named on them. They serve to illustrate the phenomena generally observed—namely, that after the formation of the black the colours of the portion of the film in contact with it change so as to indicate an increase in thickness, and are, as it were, absorbed by those immediately below them, although in no case does any portion of the film become thicker than any other part situated at a lower level than itself. At the same time, however, the lower part of the film continues to become thinner, so that at last the whole assumes one uniform tint, which changes but slowly, and sometimes in such a way as to show that the whole film is become thicker. This phenomenon is probably caused by absorption of moisture from the air, though the thickening of the upper part of the film above referred to may be due, at all events in part, to the fact that the formation of the black part of the film must necessitate the comparatively rapid removal of the superfluous liquid from that portion in immediate contact with it.

Fig. VI. represents a film in which the lower boundary of the black was, at the time of the observation, rising instead of descending.

The last four figures represent films which were not sufficiently thin to exhibit the black; the upper part of that shown in fig. X. was, at the time of the observation, becoming thicker.

An inspection of these figures is sufficient to prove that, as a general rule, the films did not increase uniformly in thickness from top to bottom, but that regions of comparatively rapid and slow increase of thickness alternated. The inclination of the outside layer of the film to the vertical seems often suddenly to become much greater at or about a thickness corresponding to 20 small divisions on the curve-paper or to the yellow of the second order, as seen through the telescope of the cathetometer, but none of the changes in thickness are so rapid as that which takes place at the lower edge of the black. One of the films observed (not one of those drawn) displayed a ring of black 7·82 millims. in breadth, while the rest of the film appeared to be a uniform green of the third order. If we suppose the upper part of the film to be as thick as is possible consistently with its appearing black when seen by light at normal incidence, the film must in this case have been increased to fourteen times


April 5, 12h 11m·5.
I.
April 5, 12h 39m·75.
II.
April 5, 12h 57m·5.
III.
April 5, 2h 12m.
IV.
April 5, 4h 20m.
V.
Apr. 9, 1h 12m·5.
VI.

April 12.

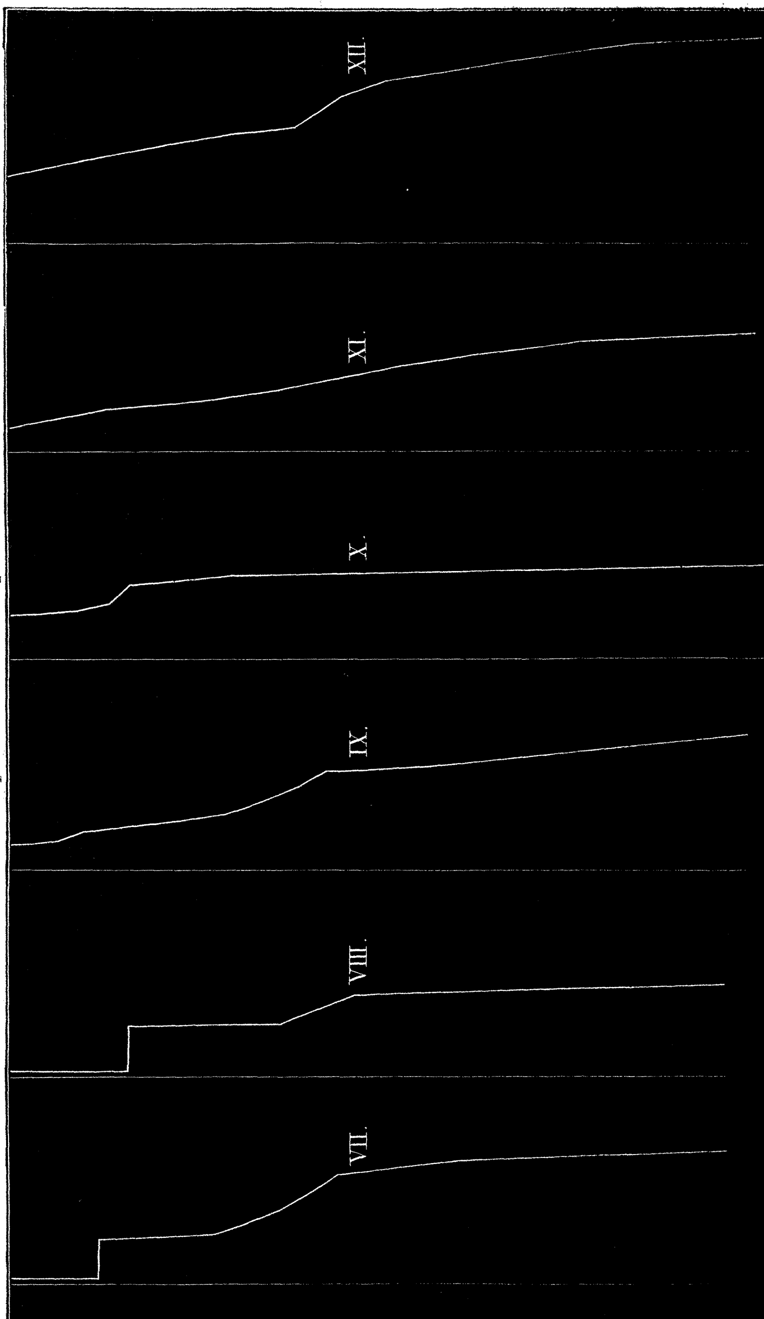
April 10.

April 9.

April 6.

April 10, 12h 54m.

April 10, 12h 10m.



that thickness so suddenly that the colours between the black and green of the third order were quite invisible.

The forms of the coloured portions of the films having been thus determined, the authors were now able to calculate their resistances. The resistance of a ring of the cylinder of length l, radius r, and of uniform thickness τ, might be taken to be $=\frac{\rho l}{2\pi r\tau}$, where ρ is the specific resistance of the liquid, which was determined to be 222 ohms at the temperature at which the experiments were conducted.

The radii, both of the platinum ring and crucible, were determined by means of the cathetometer. Both were found to be very nearly circular and their mean diameters were 33·26 millims. and 33·77 millims. respectively. The mean of these numbers, or 33·51 millims., was taken to be the value of $2r$. Since the thickness of each little ring was not uniform, the value taken for τ was the mean value of the thicknesses of its upper and lower edges. It can easily be shown that the resistances so calculated would be a little less than the true resistance, and a correction factor was introduced, the value of which is

$$\kappa=\frac{\tau_1+\tau_2}{2(\tau_1-\tau_2)}\log_e\frac{\tau_1}{\tau_2},$$

where τ_1 and τ_2 are the numbers above referred to as those the mean of which was taken to be the thickness of the film. The value of this factor, which depends only on the ratio of τ_1 to τ_2, was calculated for several values of that ratio, and the numbers required were obtained by interpolation. The values of $1\div\frac{\tau_1+\tau_2}{2}$ for the case considered are given in Column XI. of Table I., and Column XII. contains the values of the correction factor.

Hence, introducing the corrections for the oblique incidence of the light and for the refractive index, the resistance of the cylinder was given by the expression

$$222\times\frac{1\cdot395\times\cos i'}{2\pi r}\Sigma\frac{2l\kappa}{\tau_1+\tau_2}.$$

The resistance so calculated was subtracted from the total observed resistance of the film, and the number thus obtained was assumed to give the resistance of the black portion.

Table II. gives the results of the experiments.

Column I. gives the time at which the observations of the electrical resistance were made.

Column II. the breadth of the band of black.

Column III. the names and orders of the colours corresponding to the thinnest and thickest portions of the coloured parts of the cylinder when seen by light incident at 45°.

Column IV. gives the observed resistance of the cylindrical film expressed in megohms.

TABLE II.

Time of observation. I.	Breadth of ring of black in mm. II.	Colours of rest of cylinder. III.	Total resistance of cylinder in megohms. IV.	Resistance of coloured portions. V.	Resistance of black per mm. VI.
		Cylinder No. I., April 5, Temp. 13°·6.			
h. m.					
12 11·5	2·70	Blue 2 to red 4	5·92	·931	1·848
14	2·83		6·12	·929	1·834
17	2·99		6·42	·929	1·836
30	3·66		7·69	·925	1·848
36	4·04	Green 2 to red 4	8·23	·935	1·806
39·75	4·26		8·53	·936	1·783
45·75	4·55		8·83	·951	1·732
48	4·72	Green 2 to green 4	9·15	·970	1·733
51·5	4·90		9·45	·964	1·732
55	5·08		9·75	·968	1·729
57·5	5·20		10·00	·967	1·737
1 1	5·38		10·30	·964	1·735
5	5·59	Yellow 2 to green 4	10·70	·926	1·748
7·75	5·73		11·00	·923	1·759
2 12	6·30	Orange 2 to green 3	12·80	1·113	1·855
22·5	6·35	Orange 2 to purple 3	12·95	1·245	1·843
4 20	8·00	Yellow 2	15·30	1·434	1·733
Later ...	8·77	Orange 2	16·90	1·299	1·779
"	9·60	" "	18·30	1·176	1·784
		Cylinder No. II., April 6, Temp. 13°·9.			
	7·82	Green 3	14·50	·910	1·738
		Cylinder No. III., April 9.			
1 12·5	2·25	Yellow 2 to green 4	5·00	1·155	1·709
20	1·90		4·60	1·223	1·777
2 0	1·40	Orange 2 to green 3	3·77	1·325	1·746
3 30	2·50	Yellow 2	6·17	1·794	1·750
		Cylinder No. IV., April 10, Temp. 13°·3.			
12 1	2·86	Yellow 2 to green 5	5·76	·897	1·700
10	3·45		6·75	·901	1·695
17·25	3·60		6·86	·907	1·654
54	4·70	Orange 2 to red 3	8·87	1·064	1·661
59·5	4·72		9·17	1·074	1·715
1 3	4·80		9·27	1·054	1·712
42	5·55	Red 2 to purple 3	10·70	1·164	1·718
2 4·75	5·70	Red 2	11·18	1·408	1·714
2 38·5	6·15	Orange 2	11·97	1·454	1·710
4 9	7·25	Yellow 2	13·48	1·495	1·653
		Cylinder No. V., April 12.			
	8·90	Yellow 2	16·77	1·402	1·727
	11·87	Blue 2	22·24	1·350	1·760

Column V. the resistance of the coloured portion, calculated as above described, and also expressed in megohms.

Column VI. is obtained by subtracting the numbers in V. from the corresponding numbers in IV. and dividing by the corresponding numbers in II., and thus gives in megohms the resistance of a ring of the black portion of the film 1 millim. in breadth.

The numbers in Column II. are bracketed when they were obtained by means of the same two series of measurements with the cathetometer, and are thus dependent on the same observations. Numbers not bracketed were obtained by totally independent optical and electrical measurements.

It will be seen from this Table that five different films were studied upon five different days; and that, in all, 36 determinations of the resistance of the black portions of the films were made.

The highest and the lowest values obtained from individual experiments differ from one another by about 11 per cent. of the mean value; but the means of the results of each day's observations display a closer agreement. Thus the mean of the values obtained from

Cylinder I.	is	1·782,	deduced from	19	observations.
,, II.	,,	1·738,	,,	1	,,
,, III.	,,	1·745,	,,	4	,,
,, IV.	,,	1·693,	,,	10	,,
,, V.	,,	1·743,	,,	2	,,

The maximum discrepancy is about 5 per cent. of the mean between the greatest and least values.

The figures, however, give interesting results when grouped in different ways. Thus, taking the means of the values obtained when the lengths of the black part of the film lay between certain limits, it was found that when the black part was

> 0 and	< 2	millims.,	the mean	value was	1·761,
> 2 ,,	< 4	,,	,,	,,	1·764,
> 4 ,,	< 6	,,	,,	,,	1·734,
> 6 ,,	< 8	,,	,,	,,	1·760,
> 8 ,,	< 10	,,	,,	,,	1·756,
> 10 ,,	< 12	,,	,,	,,	1·760.

Again, grouping the results according to the thickness of the coloured portion of the film which appeared to be in immediate contact with the black, it was found that when the colour of that portion, as viewed through the telescope, was the

blue	of the	second order,	the mean	value was	1·826,
green	,,	,,	,,	,,	1·748,
yellow	,,	,,	,,	,,	1·719,
orange	,,	,,	,,	,,	1·756,
red	,,	,,	,,	,,	1·716,
green	of third	order,	,,	,,	1·738.

The first of these numbers is considerably larger than the others, as it is almost entirely deduced from the high values obtained during the early experiments on the first cylinder. These experiments are, however, deprived of the significance they might otherwise have seemed to possess by the fact that the only other observations taken with the blue of the second order in contact with the black gave for the resistance of the latter the normal value 1·760, while, on the other hand, high values were on one or two exceptional occasions obtained when the part of the film next the black was sufficiently thick to show the orange of the second order.

It is not easy to decide whether the different values obtained at various times correspond to real differences in the thicknesses of the black portions of the films or are due to errors of experiment.

The resistances measured were, as has been seen, very high; but experiments made for the purpose proved that the galvanometer was always sensitive to at least 1 per cent. of the total resistance measured. The most probable cause of error is the fact that the lower edge of the black does not always lie in a horizontal plane. Thus on one occasion one end of the boundary between the black and the coloured part of the film was observed to sink no less than 0·5 millim. in a few seconds. In one or two cases where this fact was noted, the number given for the length of the black is a mean of readings taken in different parts; but it was difficult to determine whether the edge furthest from the observer was or was not below that nearest to him. This source of error would, of course, be of greater importance as the breadth of the black portion of the film diminished; but the magnitude of these deviations from horizontality appeared to become greater as that breadth increased.

Without, however, making any allowances for these causes of error, the experiments certainly show, for the particular liquid and apparatus used,—

i. That the variations in thickness of the black portion of the films were but a small fraction of that thickness.

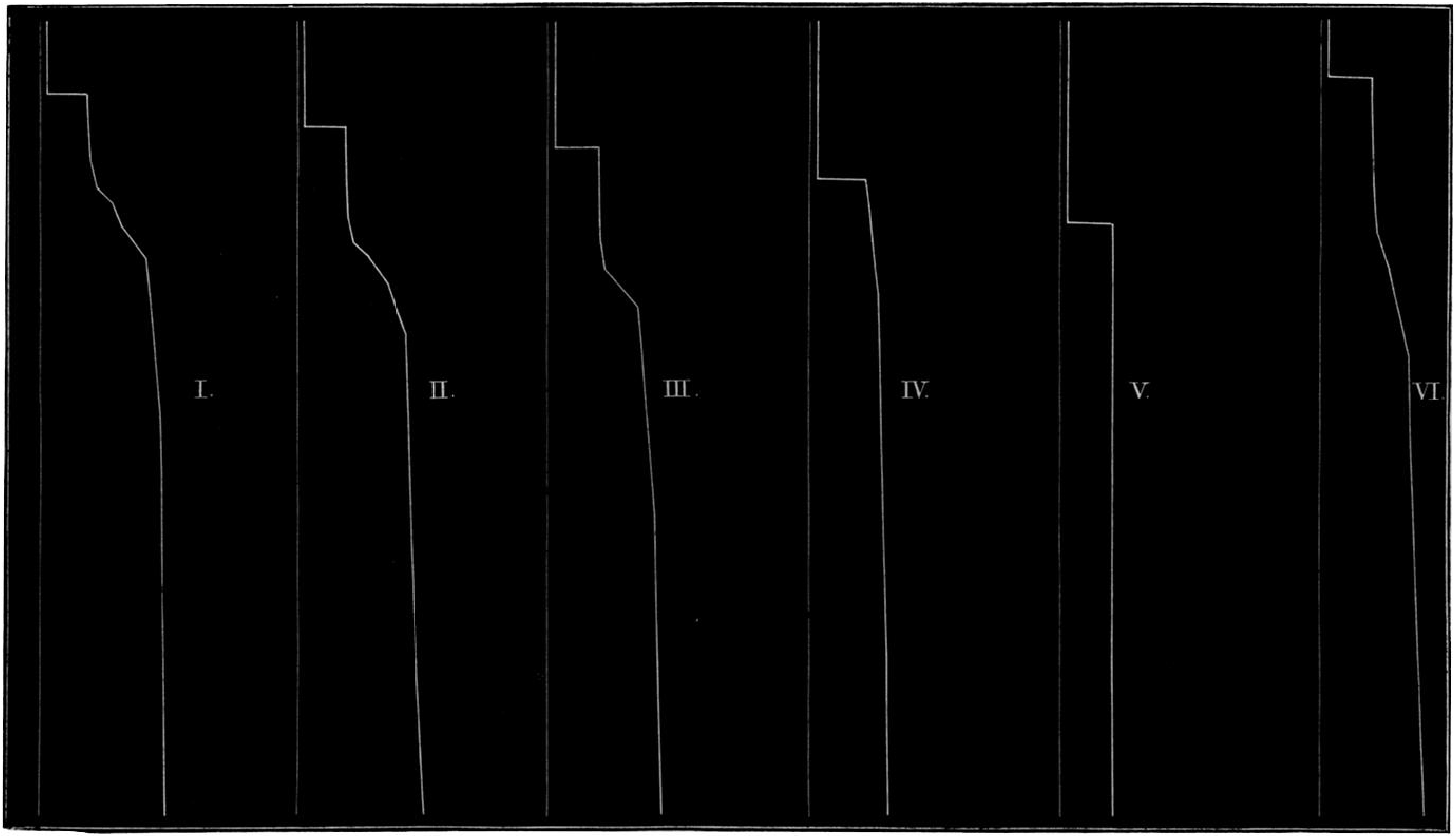
ii. That the thickness is independent of the breadth of the black ring.

iii. That it is also independent of the thickness of that portion of the film which appears to the naked eye to be in immediate contact with it.

The last question on which the authors propose to touch is that of the absolute thickness of the black portion of the films; and though their experiments only enable them to calculate that thickness on the assumption that Ohm's law holds good, the result may be interesting.

The mean of all their experiments gives for the resistance of a ring of the black film 1 millim. broad, 1,750,000 ohms; whence, since the diameter of the cylinder is 33·51 millims., it is easy to calculate that, if Ohm's law holds, the thickness of the film must be 12 millionths of a millimetre, or about one third of the thickness corresponding to the beginning of the black for the liquid submitted to experiment.

April 5, $12^{h}\ 11^{m}\cdot5$. April 5, $12^{h}\ 39^{m}\cdot75$. April 5, $12^{h}\ 57^{m}\cdot5$. April 5, $2^{h}\ 12^{m}$. April 5, $4^{h}\ 20^{m}$. Apr. 9, $1^{h}\ 12^{m}\cdot5$.



April 10, 12^h 10^m. April 10, 12^h 54^m. April 6. April 9. April 10. April 12.